Numerical modelling of MHD waves in the solar chromosphere

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Acoustic waves are generated by the convective motions in the solar convection zone. When propagating upwards into the chromosphere they reach the height where the sound speed equals the Alfvén speed and they undergo mode conversion, refraction and reflection. We use numerical simulations to study these processes in realistic configurations where the wavelength of the waves is similar to the length scales of the magnetic field. Even though this regime is outside the validity of previous analytic studies or studies using ray-tracing theory, we show that some of their basic results remain valid: the critical quantity for mode conversion is the angle between the magnetic field and the k-vector: the attack angle. At angles smaller than 30° much of the acoustic, fast mode from the photosphere is transmitted as an acoustic, slow mode propagating along the field lines. At larger angles, most of the energy is refracted/reflected and returns as a fast mode creating an interference pattern between the upward and downward propagating waves. In three-dimensions, this interference between waves at small angles creates patterns with large horizontal phase speeds, especially close to magnetic field concentrations. When damping from shock dissipation and radiation is taken into account, the waves in the low–mid chromosphere have mostly the character of upward propagating acoustic waves and it is only close to the reflecting layer we get similar amplitudes for the upward propagating and refracted/reflected waves. The oscillatory power is suppressed in magnetic field concentrations and enhanced in ring-formed patterns around them. The complex interference patterns caused by mode-conversion, refraction and reflection, even with simple incident waves and in simple magnetic field geometries, make direct inversion of observables exceedingly difficult. In a dynamic chromosphere it is doubtful if the determination of mean quantities is even meaningful.

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1. Introduction

The existence of a wide variety of wave-like phenomena is evident from solar observations. Where the sound speed and the Alfvén speed are disparate, the fast...
and slow magneto-acoustic-gravity (MAG) waves are decoupled, with one having a dominant magnetic character and the other being mostly acoustic in nature. When the phase speeds are comparable, the waves become alike and this is the cause of the mixing. The mixing region is thus where the sound speed is equal to the Alfvén speed. This is also close to where the plasma-\(\beta\), defined as the ratio of the thermal pressure to the magnetic pressure (\(\beta = 8\pi p/B^2\)), is close to unity since the ratio of the speeds is \(c^2/a^2 = \gamma 4\pi p/B^2 = (\gamma/2)\beta\) and thus \(\beta = 1.2\) where \(c=a\) for \(\gamma = 5/3\). We will refer to the region where the sound speed is equal to the Alfvén speed as the magnetic canopy, but we will also refer to this mixing layer as \(\beta \approx 1\). The fast mode in the high-\(\beta\) plasma of the photosphere is mainly acoustic in nature but when it propagates up into the magnetic canopy it may undergo mode conversion and emerge as a slow mode wave. These processes have been extensively studied analytically in the limit of small wavelengths compared with length scales in the medium. Analytical solutions were given for MAG waves in an isothermal atmosphere in an oblique magnetic field by Zhugzhda & Dzhalilov (1984a,b) with the solutions given in a simpler form by Cally (2001). Studies have also been made using ray-tracing theory in two-dimensions giving transmission and mode conversion coefficients as functions of the wave frequency and attack angle (Cally 2005). However, in the real solar atmosphere the dominant waves have wavelengths comparable with the length scales of the magnetic field and it is unclear how much of the analytical results are directly applicable to the solar case. Also, realistic magnetic field configurations are not amenable to analytic solutions. For these reasons it is important that analytic work is supplemented with numerical work. There is also a large literature on numerical studies of MHD waves under solar conditions, see Bogdan et al. (2003) for references. Numerical simulations are not without their particular problems: it is very difficult to construct boundary conditions that do not influence the interior in unphysical ways. To study the conversion between modes one must include the upper atmosphere of the sun where the Alfvén speed increases rapidly, forcing very small timesteps in the simulation. Other computational challenges include the problem of including realistic physics with non-local, nonlinear radiation and the fact that waves in the chromosphere grow to large amplitude such that linearized equations cannot be used and shock formation has to be treated.

The outline of this paper is as follows: in §2 we set the scene by giving some examples of what observations tell us, in §3 we discuss how waves from the photosphere will interact in the ‘magnetic canopy’ in the form of mode conversion, reflection and refraction and in §4 we give our conclusions.

2. Observations

The granular flow in the solar convection zone sweeps the magnetic field into intergranular lanes increasing the field-strength to a few kilogauss. These regions of increased magnetic field are in rough pressure balance with the weak field surroundings and thus of lower density. The low density makes the magnetic structures look bright because the light comes from deeper, hotter regions than in the regions outside the magnetic structure. These high-intensity tracers of the magnetic field can be observed with especially high contrast and high spatial
resolution in the G-band spectral feature (due to CH molecular absorption) close to 430 nm.

With the commissioning of the Swedish 1-m solar telescope (Scharmer et al. 2003a) solar observations at a resolution of 0.1 arc s (70 km on the solar surface) have become available (see e.g. Scharmer et al. 2002, 2003b; Berger et al. 2004). Observations at high spatial resolution of the dynamic behaviour of the magnetic elements (Rouppe van der Voort et al. 2005) show that the granular flow often concentrates the flux into sheets that are visible as thin bright features in the filtergrams. Weak upflows are found in the flux sheets and strong downflows in the immediate surroundings. The flux sheets often become unstable to a fluting instability and their edges buckle. The sheets tend to break up into strings of bright points. These smaller flux concentrations have sizes down to the resolving power and they are continuously buffeted by the granules.

It is well known that the convection gives rise to stochastic generation of acoustic waves. In view of the presence of strong sheer at magnetic flux sheets, constant buffeting of small flux concentrations and instabilities present in the photospheric magnetic fluid we also expect a multitude of magnetic modes to be excited by the solar convection.

What happens to these waves as they propagate from the photosphere into the chromosphere? Observational clues have been obtained by McIntosh and co-workers.

McIntosh et al. (2001) and McIntosh & Judge (2001) find a clear correlation between observations of wavepower in SOHO/SUMER observations and the magnetic field topology as extrapolated from SOHO/MDI observations. These results were extended to the finding of a direct correlation between reduced oscillatory power in the two-dimensional TRACE UV continuum observations and the height of the magnetic canopy (McIntosh et al. 2003) and the authors suggest using TRACE time-series data as a diagnostic of the plasma topography and conditions in the mid-chromosphere through the signatures of the wavemodes present. Such helioseismic mapping of the magnetic canopy in the solar chromosphere was performed by Finsterle et al. (2004) and in a coronal hole by McIntosh et al. (2004).

The chromosphere is very inhomogeneous and dynamic. There is no simple way of inverting the above observations to a consistent picture of the chromospheric conditions. One will have to rely on comparisons with forward modelling.

3. Propagation and mode coupling

We have seen that we expect a multitude of waves and wave modes to be excited in the solar convection zone. In most of the convection zone, the excited waves will be predominantly acoustic in nature. When acoustic waves reach the height where the Alfvén speed is comparable to the sound speed they undergo mode conversion, refraction and reflection. In an inhomogeneous, dynamic chromosphere this region of mode conversion will be very irregular and change in time. We thus expect complex patterns of wave interactions that are highly variable in time and space. Simplified treatments imposing symmetries may lead to erroneous conclusions. However, this complex nature of the interactions also
necessitates the study of simplified geometries to examine the validity of analytic results and to build up the interpretation framework for more realistic magnetic topologies. We thus start by examining some simplified cases before we attempt to disentangle what is happening in a realistic three-dimensional magnetic field configuration.

Rosenthal et al. (2002) made two-dimensional simulations of the propagation of waves through a number of simple field geometries in order to obtain a better insight into the effect of differing field structures on the wave speeds, amplitudes, polarizations, direction of propagation, etc. In particular, they study oscillations in the chromospheric network and internetwork. They find that acoustic, fast mode waves in the photosphere become mostly transverse, magnetic fast mode waves when crossing a magnetic canopy where the field is significantly inclined to the vertical. Refraction by the rapidly increasing phase speed of the fast modes results in total internal reflection of the waves.

Bogdan et al. (2003) reported on similar simulations in a field geometry similar to a sunspot but scaled down by a factor of 10 for reasons of computational cost and exigency. Four cases are studied; excitation by either a radial or a transverse sinusoidal perturbation and two magnetic field strengths—either an ‘umbra’ at the bottom boundary with a field strength of 2750 Gauss or a weak-field case with a field strength four times smaller. In the strong field case the plasma $\beta$ is below unity at the location of the piston and the upward propagating waves do not cross a magnetic canopy. Because the field is not exactly vertical at the location of the piston, both longitudinal and transverse waves are excited. The longitudinal waves propagate as slow mode, predominantly acoustic, waves along the magnetic field. The transverse waves propagate as fast mode, predominantly magnetic, waves. These waves are not confined by the magnetic field and they are refracted towards regions of lower Alfvén speed. They are thus turned around and they impinge on the magnetic canopy in the ‘penumbral’ region. Where the wave vector forms a small angle to the field-lines, the waves convert to slow waves in the lower region; where the attack angle is large there is no mode conversion and the waves continue across the canopy as fast waves. The simulations show that wave mixing and interference are important aspects of oscillatory phenomena also in sunspots.

To further investigate the complex interactions in a realistic solar magnetic field we now turn to full three-dimensional simulations. The initial atmosphere is still isothermal with a pressure scale height of 158 km but at the bottom of the computational domain we impose a magnetic field taken from MDI observations. The computational domain is $18 \times 18$ Mm in horizontal extent and from 0.2 Mm above optical depth unity at 500 nm to 2.6 Mm height discretized onto a $120 \times 120 \times 80$ mesh. The maximum magnetic field strength is 205 Gauss and $\beta > 1$ everywhere at the bottom boundary. The observed magnetic field at the bottom boundary is extended by a potential field extrapolation. At the lower boundary we specify a 10 mHz vertical, planar wave with $2.5 \text{ m s}^{-1}$ amplitude. The amplitude is taken much smaller than observations indicate in order to first study the linear behaviour.

Figure 1 shows the magnetic field at the bottom of the computational box and the acoustic energy flux as function of height and time at three locations. The locations have been chosen to have small, intermediate and large angles between the magnetic field and the vertical direction. Since the waves start out as planar

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Phil. Trans. R. Soc. A (2006)
acoustic waves this angle is the attack angle of the waves and this is the crucial parameter for what happens to the waves. At an attack angle of 5° (lower left panel) the waves go through as acoustic waves changing over from being fast mode waves in the high-$\beta$, lower part, to being slow mode waves in the upper part. There is some downward propagating waves visible as an interference pattern in the later part. At an attack angle of 35° (lower right panel) we have both transmission and reflection. There is also some influence from refracted

Figure 1. (a) Magnetic field in a 120×120 grid-point (18×18 Mm) field of view. Solid contours give the locations where the angle between the magnetic field and the vertical at the $\beta=1$ height is 10 and 30°. Dashed contours give the locations where this angle is 80° (=almost horizontal). The other three panels show the velocity times the square root of the product of the density and the soundspeed (square root of the acoustic energy flux with the sign of the velocity) as function of height above the bottom of the computational domain (situated 0.2 Mm above optical depth unity at 500 nm) and time at the three locations shown in (a): $x=20$, $y=20$ (c), $x=30$, $y=60$ (b) and $x=30$, $y=20$ (d). The photospheric vertical magnetic field in Gauss and the angle with the vertical at $\beta=1$ are given above the panels. The vertical lines show the locations of $\beta=[100,10,1,0.1]$. From Carlsson & Stein (2002).

Numerical modelling of MHD waves

Phil. Trans. R. Soc. A (2006)
waves from regions of higher field strength. At an attack angle of 52° (upper right panel) there is hardly any transmitted slow mode wave. The three-dimensional configuration should also allow Alfvén waves to exist. An attempt has been made to identify three separate wave modes in the simulations. At each point in the computational domain we decompose the velocity field into three orthogonal directions; along the magnetic field, along the radius of curvature of the field (principal normal) and in the third direction orthogonal to the first two (binormal). Figure 2 shows such a decomposition at a vertical slice through the strongest magnetic field component, in the $x$–$z$ plane at $y = 40$. Below the $\beta = 1$ surface such a decomposition is not meaningful and we should there concentrate on the plots in the left panels. The vertical velocity shows the planar wave propagating upwards. Where the field is nearly vertical in the magnetic canopy (meaning small attack angle) in the left part of the slice, the mode conversion is nearly complete and the waves go through as longitudinal slow mode acoustic waves propagating along the field lines. Above $\beta = 1$ these waves are clearly seen in the component along the field line (top right panel). Where the attack angle is large (middle and right part of the slice), the waves stay as fast mode waves having a polarization transverse to the field and being refracted towards right. This is visible in the transverse decompositions (middle and lower right panel). Below the $\beta = 1$ surface these refracted waves interfere with the upward propagating waves creating a standing

![Figure 2](http://rsta.royalsocietypublishing.org/)

Figure 2. $x$–$z$ slice through the three-dimensional simulation at $y = 40$ and $t = 280$ s. Vertical velocity ($a$), density perturbation ($c$), temperature perturbation ($e$), velocity along the magnetic field ($b$), principal normal velocity (along the radius of curvature of the field: $d$), binormal velocity (perpendicular to the field and the radius of curvature of the field: $f$). Black lines show projection of the magnetic field lines, white lines show lines of constant plasma $\beta$. All perturbations have been scaled with the square-root of the density.

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wave pattern. Because of the angle of the refracted waves, this standing wave pattern shows a large horizontal phase speed. The two transverse components show different patterns above $\beta = 1$ which is interpreted as signs of the existence of two transverse wave modes. In particular there exist waves propagating along the field lines with the Alfvén speed most clearly visible in the third component (bottom right panel) where the field is strongest. However, the two wave modes with transverse polarization are not easily separable.

The simulations described so far indicate that we should expect standing wave patterns in the chromosphere as soon as the attack angle is larger than $30^\circ$. As can be seen from the contours in figure 1 this is most of the area. However, it is important to note that these simulations do not contain any radiation and the wave amplitude is small so there is no dissipation in shocks. The amplitude of the downward propagating wave is, therefore, comparable in size to the upward

Figure 3. Same as figure 1 at $x=20$, $y=20$ (a) and $x=30$, $y=60$ (b). (c, d) The results of a simulation with a more realistic photospheric velocity amplitude (20 times larger than in a, b) leading to shock formation. This simulation also includes radiative damping. From Carlsson & Stein (2002).

Phil. Trans. R. Soc. A (2006)
propagating wave. In the real sun we have rather strong radiative damping in the photosphere and chromosphere and waves will steepen and form shocks, often before reaching the $\beta = 1$ layer. The downward propagating wave will, therefore, have much smaller amplitude than the upward propagating wave and the wave field will predominantly have the character of upward propagating acoustic waves in the lower chromosphere. The amplitudes will be more equal close to the $\beta = 1$ layer and we will there see more the character of standing waves. This is shown in figure 3. The simulation shown there has the same initial atmosphere as in the previous simulations but the amplitude of the piston is 20 times larger and radiation is included in the form of a cooling term proportional to the difference between the instantaneous temperature and the initial temperature multiplied by an exponential optical depth factor.

The shock formation is clearly seen in the lower panels as a sharp boundary between upward and downward velocity. The shock dissipation and the radiative cooling are active in the same region of the atmosphere and in sum lead to a decrease in the acoustic energy flux. The reflected/refracted wave thus has less energy flux than the upward propagating wave and the standing wave pattern we found in the no-dissipation case is no longer pronounced.

These three-dimensional simulations start to contain enough physics that a comparison with observations is meaningful. Do we see evidence for the suppressed oscillatory power at locations of low lying magnetic canopy as reported by McIntosh et al. (2003)? Figure 4 shows the oscillatory power in the simulation at $z = 0.4$ Mm, a height similar to the formation height of the TRACE UV intensity. We see that the power is indeed lower where the magnetic canopy height is below 1.75 Mm: the lowest power is outside the magnetic field.
concentrations while inside the region of small attack angle (see above) the power is higher. In the region between canopy heights 1.75 and 2 Mm we have a ring of increased power. This region of increased power moves outwards with increasing height but not as fast as the spreading of the magnetic field. These patterns result from the interference between upward propagating waves and refracted waves. These simulations were made with a monochromatic piston—in a more realistic situation with a multitude of waves propagating upwards we would expect the regions of constructive interference to be more spread out in the volume. One word of caution is also in order here: the computational box has a totally reflective lower boundary and periodic boundaries horizontally. These boundaries affect the result in a box that is only 18×18 Mm in size; the concentration of power just right of the middle of the box seems to be the result of the interference of waves travelling outward from the magnetic field concentration to the right and the waves leaving the box towards left and entering again from the right boundary. A detailed comparison with observations will have to be made with a larger box simulation.

4. Conclusions

The solar chromosphere is harbouring a multitude of wave modes. This is also the region where the phase speeds of different modes become comparable and the modes are not distinct any more. We have shown that even simple geometries and incident waves give rise to complex interference patterns that are not simple to interpret. More realistic three-dimensional magnetic field geometries and highly varying thermodynamic properties (in space and in time) will make inversions of observational data exceedingly difficult. It is even very questionable whether the deduction of mean quantities is meaningful in the highly dynamic chromosphere. However, without a better understanding of this magnetic transition region it will be difficult to bridge the gap between the lower atmosphere, where we may get detailed observational information like velocities and full vector magnetic fields, and the upper atmosphere, where the magnetic field is close to potential. The further exploration of the properties of the solar chromosphere will have to rely on the detailed comparisons between observations and realistic forward modelling.

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