The interaction between inner and outer regions of turbulent wall-bounded flow

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The nature of the interaction between the inner and outer regions of turbulent wall-bounded flow is examined. Townsend’s theory of inactive motion is shown to be a first-order, linear approximation of the effect of the large eddies at the surface that acts as a quasi-inviscid, low-frequency modulation of the shear-stress-bearing motion. This is shown to be a ‘strong’ asymptotic condition that directly expresses the decoupling of the inner-scale active motion from the outer-scale inactive motion. It is further shown that such a decoupling of the inner and outer vorticity fields near the wall is inappropriate, even at high Reynolds numbers, and that a ‘weak’ asymptotic condition is required to represent the increasing effect of outer-scale influences as the Reynolds number increases. High Reynolds number data from a fully developed pipe flow and the atmospheric surface layer are used to show that the large-scale motion penetrates to the wall, the inner–outer interaction is not describable as a linear process and the interaction should more generally be accepted as an intrinsically nonlinear one.

Keywords: wall turbulence; self-similarity; inactive motion

1. Introduction

Owing to the need for accurate prediction of the drag in large transport vehicles and fluid pipelines, the Reynolds number scaling of turbulent wall flows is becoming ever more important. To perform this with accuracy, a thorough understanding of Reynolds similarity of wall turbulence is required, where the drag coefficient decreases indefinitely with increasing Reynolds number, because the small-scale motion near the wall is directly affected by the viscosity at any Reynolds number. Ideas concerning the concept of energy equilibrium (‘production = dissipation’) in turbulent wall layers (Townsend 1956) are at the very heart of our understanding and prediction of wall turbulence. Fundamental to the validity of the hypothesis of ‘local equilibrium’ is the assumption that the large eddies are weak, even when constrained by a solid surface, and that turbulence kinetic energy is dissipated close to where it is produced. However, it has also long been accepted that this is not entirely correct (Townsend 1961, 1976; Bradshaw 1972) and that the large eddies do, in fact, contribute a significant fraction of the Reynolds stresses, so that near a wall, the

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stresses do not scale simply, i.e. they do not scale with wall variables \((\nu/u_T, u_T)\), where \(u_T = \sqrt{\tau_w/\rho}\); \(\tau_w\) is the wall shear stress; and \(\nu\) is the kinematic viscosity. Moreover, the wall-parallel \((u, w)\) velocity components that are subject to the viscous constraint behave in a way that is entirely different from the behaviour of the wall-normal \((v)\) velocity component, which is subject to the impermeability constraint. This means that the \(v\)-component of eddy motion is ‘blocked’, approximately at wavenumbers that are inversely proportional to the eddy distance from the wall, and that the energy appears in the \(u\) and \(w\)-components. This failure of wall scaling (at least, in the case of the wall-parallel velocity components) persists at ‘high’ Reynolds numbers, when \(y^+ = yu_T/\nu \gg 1\), and the appropriate length- and velocity-scales are \(y\) and \(u_T\), respectively. In fact, there is now a substantial body of evidence indicating that the influence of the outer length-scale increases with Reynolds number, so that the local-equilibrium approximation, deduced by scaling the turbulence kinetic energy equation with \(y\) and \(u_T\) alone, is clearly a ‘no-interaction’ condition. The fundamental question is whether there are ranges in either real or wavenumber space, in which the turbulence exhibits universal features (e.g. a ‘constant-stress’ layer, \(-\bar{u}w \approx u_T^3\)), in the sense that they can be scaled by a single length-scale and a single velocity-scale.

These fundamental considerations led Townsend to propose the concept of ‘inactive motion’ (Townsend 1961; Bradshaw 1967a, b; Morrison et al. 1992, 2004), which, to a first-order approximation, contributes only to the wall-parallel components in the form of a low-frequency modulation. Inactive motion has therefore been used to explain the failure of the \(u\) and \(w\)-components to scale with wall variables. With the assumption of independence of widely spaced Fourier components and that the \(v\)-component is \(O(\nu y/w)\) (Bradshaw 1967b), it seems plausible to assume that, to a first-order approximation, the inactive component does not interact with the shear-stress \((\tau = -\rho \bar{u}w)\)-bearing (‘active’) motion, which constitutes equilibrium layers. Inactive motion can therefore be taken to be a low-frequency modulation linearly superposed on the active component. Bradshaw (1967b) also suggested that pressure fluctuations contribute to inactive motion, strictly, via the pressure–velocity transport term in the equation for the turbulence kinetic energy.

However, recent laboratory data obtained at very high Reynolds numbers (Morrison et al. 2004; Zhao & Smits in press) show that the large eddies in the inner layer are not inactive, but rather they contribute to the energy production there, thus supporting the suggestion of Hunt & Morrison (2000) deduced from observations in the atmospheric surface layer that, at high Reynolds numbers, a ‘top-down’ influence is dynamically significant, in addition to a ‘bottom-up’ one, which is likely to be more prevalent at low Reynolds numbers. Morrison et al. (2004) suggest that the large-scale contribution in the inner layer increases both as the Reynolds number increases and as the distance from the wall decreases. While the Reynolds number, \(R^+\) (taking \(R\) as a general outer length-scale), is a measure of the influence of the large eddies relative to the direct effects of viscosity, it is important to distinguish between the two. Bradshaw & Huang (1995) have provided further evidence that, at low Reynolds numbers, the inner–outer interaction can lead to non-universality of even the shear stress. Simulation data show that, for example, although the limiting behaviour of the shear stress is expected to be \(-\bar{u}w^+/y^{+\beta}\), it is, in fact, quite markedly dependent on the...
Reynolds number. This is not unexpected, in the sense that Townsend’s hypothesis rather highlights the top-down effect of the large scales at high Reynolds numbers. More intriguingly, near-wall statistics at the same $y^+$ and $R^+$ also depend on the nature of the outer boundary conditions. Bradshaw & Huang (1995) have also modelled the effect of inactive motion as a Stokes layer at the wall, the thickness of which tends to zero as Reynolds number becomes very large. The difficulty with this picture is that, while it shows how the large-scale motion can drive a viscous layer near the wall, it omits any nonlinear interactions.

The foregoing model suggests that, even when $R^+ \to \infty$, a decoupling of the influence of the large scales from the direct effects of viscosity in the near-wall region is unlikely. Arguments concerning scale separation require care (for a fuller discussion, see McKeon & Morrison 2007); writing $R^+ = y^+/(y/R)$ indicates that scale separation may be achieved either by $y^+ \to \infty$ when $y \sim R$ or by $y/R \to 0$ when $y^+$ is finite. Therefore, while at face value it might appear that a very large Reynolds number offers a broad range of eddy sizes, one essentially has to determine whether the appropriate Reynolds number is $y^+$ (the active motion) or $R^+$ (the inactive motion). The extent to which active $y$-scale effects can be decoupled from inactive $R$-scale effects is considered further in §2.

In addition to increasing $R^+$, the inner–outer interaction can also be augmented by using roughness of height, $k$, so that the interaction may be gauged by $k/R$, assuming that, for simplicity, the wall layer is ‘fully rough’, and that $k^+ > 100$. The classical view of the way the roughness affects the structure of the boundary layer was also expounded by Townsend (1956, 1976), who argued that the major effect is simply to change the surface stress and that changes to the basic turbulence structure of the flow can be scaled by an increase in $u_\tau$ alone (e.g. Flack et al. 2005; Kunkel & Smits 2006). This classical view involves, essentially, an assumption that, once again, the inner and outer regions do not interact, in the sense that the direct effects of roughness do not appear outside the roughness sublayer, typically $y < 10k$. However, such a view is not universal (Krogstad & Antonia 1999; Bhaganagar et al. 2004). Jiménez (2004) suggests that Townsend’s hypothesis becomes invalid once $k/R > 2.5\%$. Intuitively, it would seem highly probable that Townsend’s similarity will break down for very large roughness elements.

In the case of roughness, scale separation may be examined by writing $R^+ = k^+/(k/R)$. For the case of large roughness, $k/R \to 1$, the bottom-up and top-down effects are directly coupled and, even when $k^+ \sim R^+ \to \infty$, scale separation of $k$ with respect to $R$ is not possible, so that Townsend’s hypothesis is unlikely to be valid. This therefore suggests that for inner–outer interactions augmented by $R^+ \to \infty$ on a smooth surface, the same situation will pertain over some range of $y$. In this case, the coupling is predominantly top-down. In both cases, simple scaling is likely to break down, but in the case of roughness, the extent of the layer thickness over which this is apparent is likely to be much greater.

Integration of the local-equilibrium condition and the assumption of a constant-stress region yield the log law. If the inner–outer interaction is significant such that the local-equilibrium approximation were invalidated, an implication might be that the log law is also invalid. Simple arguments (see

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Bradshaw 1967b or, for a recent account, Davidson 2004) show that for an root-mean-square inactive fluctuation of 20% of the local mean velocity, the increase in mean velocity due to the inactive contribution is only about 2%. Moreover, the region immediately adjacent to the wall to which the outer contribution is confined inevitably becomes thinner, and therefore below the lower limit for which the log law is likely to be valid, $600 < y^+ < 0.12R^+$ (McKeon et al. 2004; Morrison et al. 2004).

Wei et al. (2004) have recently used the momentum equation to define four regions in wall flows, each one characterized by the balance of two dominant terms (see also Sreenivasan & Sahay 1997). The interaction between the inner and outer regions is likely to be most significant in the region in which $\partial\tau/\partial y \approx 0$; physically, one might expect that, here, both the momentum flux due to low-momentum eruptions from the sublayer (‘ejections’, $-\overline{uv}_2$) and from high-momentum inrushes (‘sweeps’, $-\overline{uv}_4$) reach a maximum, and that therefore this region should be the focus of some attention. Note also that this aspect of the interaction involves nonlinear momentum-flux terms. Adrian and co-workers (Adrian et al. 2000; Christensen & Adrian 2001) have documented in detail the growth of packets of hairpin vortices, the most energetic modes of which carry roughly half the energy and shear stress near the surface and have large wall-normal extent. Packets have also been observed at the very high Reynolds numbers of the atmospheric surface layer (Hommema & Adrian 2003). Very-large-scale motions (VLSMs) have also been proposed as an agglomeration or coherent alignment of packets (Kim & Adrian 1999; Guala et al. 2006). However, much depends on the meandering of packets in and out of a streamwise vertical plane and the extent of the meandering is likely to be somewhat greater in a boundary layer (‘super-structures’; Hutchins & Marusic in press) than in an internal flow. Clearly, packets and superstructures/VLSMs are likely to dominate inner–outer interactions, and they appear to persist at very high Reynolds numbers. Yet, direct associations between structures and imposed scales require care. Bradshaw (1967b) noted that, even though a large structure (of scale $R$) may contribute significantly to the horizontal velocity components near the wall, its motion near the eddy ‘centre’ (and of scale $y$) can be considered as active. Superstructures/VLSMs are likely to conform to this two-scale description. Careful choice of correlation length-scales offers a more rational approach for extracting length-scales from the shear-stress-bearing motion. For example, Morrison et al. (1992) used the conditional correlation between the wall shear stress and the $\overline{uv}_2$ and $\overline{uv}_4$ products.

As we have seen, inactive motion is a first-order approximation of the true processes involved in the interaction between the inner and outer regions—it is essentially a large-scale (and therefore quasi-inviscid) linear modulation of the near-wall turbulence. While earlier work suggested that such a model becomes more accurate as the Reynolds number increases, it would now appear that the opposite is, in fact, the case. Moreover, the interaction is inherently nonlinear, so an understanding of the limitations of the theory of inactive motion is needed. The nature of the inner–outer interaction is important, because it explains many of the perceived differences between, for example, internal and external flows, and this is the purpose of this paper.

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2. Nature of the interaction

Initial attempts at quantifying the effect of the outer-layer motion on near-wall turbulence come from Townsend’s seminal work concerning the self-similar structure of ‘attached wall eddies’ and the way in which the larger ones generated the so-called inactive motion. It is clear that the nature of the interaction depends significantly on the form of the outer-region large eddies; in particular, Townsend (1961) supposed that the shear-stress-bearing eddies scale with $u_t$ and could therefore ‘in a sense, (be) attached to’ the wall. This assumption permeates much of the later work, but it became clear that a model based on the attached wall eddy alone (see Perry et al. 1986) could not accurately reproduce the Reynolds stresses without the inclusion of other types of eddy motion (Perry & Marusic 1995) that reflect an additional influence of the large scales near the wall. Recent developments in the attached wall-eddy model are given by Kunkel & Marusic (2006). Zagarola & Smits (1998) proposed that a better outer velocity-scale is $U_{c1} = \frac{K}{\gamma^2}$, suggesting that outer-region eddies appear in the form of detached freely moving structures (see also Morrison et al. 1992).

Clearly, attached wall eddies and detached freely moving ones would make very different contributions to an inner–outer interaction. These ideas are applicable to both fully developed internal flows and external boundary layers.

Here, we assess the nature of the interaction by using the simplified equations of motion. With the neglect of the advection terms, Townsend (1961) shows that the equation for the turbulence kinetic energy, $\frac{1}{2}q^2$, may be written as

$$\frac{\partial U}{\partial y} = \frac{\tau^{1/2}}{\kappa y} \left( 1 - B \frac{y}{\tau} \frac{\partial \tau}{\partial y} \right),$$

(2.1)

where $B = 3(3/2)\kappa a_2 a_1^{-(3/2)}$ and $a_1 = -\overline{uv}/q^2$. The transport term is modelled by the diffusion relation

$$\frac{1}{2} q^2 v + \frac{p'v}{\rho} = -a_2 (q^2)^{(3/2)} \text{sgn} \left( \frac{\partial q^2}{\partial y} \right),$$

(2.2)

where both $a_1$ and $a_2$ are constants. Equation (2.1) is then a statement of approximate local equilibrium, in the sense that some turbulent transport is effected by the large eddies and represented by an additional term in which $B$ is $O(0.1)$ at $y/R \approx 0.1$, but is $O(1)$ at $y/R \approx 0.4$. Writing the momentum equation for pipe/channel flow as

$$\left( 1 - \frac{y}{R} \right) u_r^2 = -\overline{uv} + v \frac{\partial U}{\partial y},$$

(2.3)

and neglecting the viscous term, the transport term in equation (2.1) becomes simply $B y/(R - y)$. It increases monotonically with $y$ from the outer edge of the viscous sublayer until near the edge of the layer, becoming significant ($O(0.1)$) at the outer edge of the inner layer. However, the interaction is rather more complicated than this simple analysis would suggest, i.e. inactive motion decreases $a_1$ near the surface (Bradshaw 1967a) so that $B$ increases there. Moreover, the model assumes that the active and inactive components do not interact, and that the large scales make no contribution to production.

At high Reynolds numbers, the influence of viscosity well outside the sublayer has already been recognized, as the suggestion of a ‘mesolayer’ by Long & Chen (1981) and Wosnik et al. (2000) makes clear. Both equations (2.1) and (2.3)
ignore the direct effects of viscosity. Morrison et al. (2004) distinguish the receding influence of viscosity from the increasing influence of the large scales as $R^+$ increases. While many workers recognize $y^+ \approx 100$ as an outer limit to viscosity affecting the motion directly, McKeon et al. (2004) show that, in pipe flow, the mean velocity follows a power law out to $y^+ \approx 600$. In this context, it is interesting to note that, for internal flows, the locus of the peak in shear stress is given approximately by $y_p = 1.8(R^+)^{0.52}$, which can be interpreted as the locus of the mesolayer and hence effectively determines a lower limit to the extent of the log law. Wei et al. (2004) have termed this the ‘meso viscous/advection balance layer’.

In the boundary layers, Morrison et al. (1992) noted that, at a momentum-thickness Reynolds number of $1.5 \times 10^4$, ejection and sweep lengths scale on the Kolmogorov length-scale out to $y^+ \approx 1000$. The effect of the large-scale motion near the wall may be assessed more quantitatively by writing (Tennekes & Lumley 1972)

$$\frac{\partial}{\partial y}(-\overline{uv}) = \overline{v\partial u_x} - \overline{u\partial v_y} + \frac{1}{2} \frac{\partial}{\partial x} \left[ \overline{u^2} - \overline{v^2} - \overline{w^2} \right].$$

Following Davidson (2004), we decompose the velocity field so that $u = u_i + u_o$, where the subscripts ‘$i$’ and ‘$o$’ refer, respectively, to the inner and outer contributions. With a similar decomposition for the vorticity field and noting that $u_0 = 0$, equation (2.4) becomes

$$\frac{\partial}{\partial y}(-\overline{uv}) = [u_i \times \omega_i]_x + [u_o \times \omega_i]_x,$$

where, appropriate for a fully developed internal flow, the last term in equation (2.4) has been neglected. With the assumption of a sufficient separation of scales, the last term in equation (2.5) may also be neglected. Note that the removal of the cross product for the outer velocity effectively linearizes its influence, and it is for this reason that Townsend (1961) referred to inactive motion as a ‘meandering or swirling made up from attached eddies of large size...’. Therefore, $\overline{u^2} = \overline{u_i^2} + \overline{u_o^2}$, where $\overline{u_i^2} = F_i(y^+)$ and the outer influence appears as a linear superposition, $\overline{u_o^2} = F_o(R^+)$. Noting that $v_o \sim u_o y/x$, a consistent level approximation indicates that the outer contribution to $\overline{v^2}$ and $\overline{w^2}$ is negligible and that they are $F_i(y^+)$ only.

Here, we refer to this hypothesis as a ‘strong’ or ‘quasi-inviscid’ asymptotic condition, namely one in which the separation of scales $y$ and $R$ at $R^+ = \infty$ is made explicit, and no interaction is possible. Writing $R^+ = y^+ / (y/R)$, it is clear that, for the inactive motion, $R^+ = \infty$ at $y^+ = 0$, i.e. ‘the large eddies are weak’, in the sense that they bear none of the shear stress (Townsend 1956) and hence initiate no interaction with the near-wall, active turbulence. Alternatively, for the active motion, $R^+ = \infty$ at $y^+ = \infty$ and there is no outer influence. Such a result also implies that, at the same $R^+$, the internal and external flows would exhibit the same Reynolds stresses when normalized by $u_2^2$. Such a strong condition is obviously not physical; in fact, the difference between the impermeability and no-slip constraints leads to large anisotropy and inter-component transfer, which clearly indicate that the linear superposition of separate scale effects is unphysical.

However, application of the strong condition is illuminating. Townsend further suggested that ‘neglecting this possibility of outside influence’, $u$-component

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velocity spectra in the inner region will demonstrate complete (or self-) similarity if they collapse using inner scales only, i.e.
\[ \phi_{11}(k_1, y, u_r) = u_r^2 y \psi_{11}(k_1 y), \] (2.6)
where \( \psi \) is a universal function. Independence of the scaled spectra from \( y \) implies that \( \phi_{11} \propto u_r^2 k_1^{-1} \), \( k_1^{-1} \) being the only available length-scale. Morrison et al. (2004) refer to this condition as complete similarity. A \( k_1^{-1} \)-range offers further simplifications; it implies a hierarchy of non-interacting self-similar (statistical) eddies of infinite extent in both \( x \) and \( z \) directions (since the deduction of self-similarity in \( \phi_{33} \) is directly analogous to that in \( \phi_{11} \)).

Since \( [\mathbf{u} \times \omega]_x \) includes the wall-normal velocity component, it is relevant to examine the effect of blocking on the \( v \)-component, assuming that the large-scale influence may be linearly superposed on the near-wall motion and that the large- and small-scale components do not interact. With these assumptions, it can be shown (Hunt et al. 1989; Hunt & Carlotti 2001) that the two-point correlation of the \( v \)-component velocity becomes
\[ R_{22}(y) = \frac{v(y)v(y_1)}{v^2(y_1)} = \frac{y}{y_1}, \] (2.7)
where \( y_1 \) is a fixed location in the outer layer. Noting that \( k_1 \phi_{11} \propto u_r^2 \), similar reasoning suggests that \( k_1 \phi_{22} \propto u_r^2 k_1 y \) (Hunt & Morrison 2000), where blocking of the wall-normal component occurs at \( k_1 y \sim 1 \). For simplicity, the differences between the streamwise and wall-normal wavenumber directions (other than the effects of blocking), or any effects of aliasing, are ignored.

Integration of the self-similar spectral form for \( \phi_{11} \) in the range \( 1/y < k_1 < 1/R \) for the \( u \)-component gives
\[ \overline{u^2}^+ = B_1 - A_1 \ln \left[ \frac{y}{R} \right]. \] (2.8)
Integration of \( \phi_{22} \) in the range \( 1/y < k_1 < 1/A \), where \( A \approx 10 R \), a measure of the largest scales (Kim & Adrian 1999), gives
\[ \overline{v^2}^+ = B_2 \left[ 1 - \frac{y}{A} \right]. \] (2.9)
Equation (2.8) is that provided by Townsend (1976) and, in terms of the attached eddy model, has been the subject of much development (e.g. Kunkel & Marusic 2006). Equation (2.9) is similar to that provided by Townsend, but includes a weak \( y/A \) dependence. The effect of the blocking is embodied in the constant term, i.e. the blocking model stipulates that, at each \( y \), the dominant contribution to \( \overline{v^2}^+ \) comes from eddies of wavenumber \( k_1 < y^{-1} \). The experimental evidence for the veracity of these ideas is examined in §3.

In order to examine the consequences of the strong asymptotic condition, we also propose a ‘weak’ asymptotic condition, one that reflects more accurately the fact that the inner–outer interaction is not negligible, specifically that the outer influence is not inactive, but interacts with the inner component and contributes to the production of turbulence kinetic energy. Moreover, we note that the influence of the large scales (i) increases with \( R^+ \) and (ii) increases with decreasing distance from the wall (Morrison et al. 2004). Otherwise, no assumptions concerning the weakness of the outer-layer motion are made, so that equation (2.5) becomes
\[ \frac{\partial}{\partial y}(u_i \omega_i)_x = [u_i \times \omega_i]_x + [u_o \times \omega_i]_x + [u_o \times \omega_i]_x + [u_o \times \omega_i]_x. \] (2.10)
Noting that both $u_i$ and $u_o$ scale with $u_r$, $\omega_i$ with $u_r/y$ and $\omega_o$ with $u_r/R$, it is now apparent that the third and fourth terms on the right-hand side of equation (2.10) are $y/R$ times smaller than the others. However, it is probable that there will be a significant interaction between the low-wavenumber end of the vorticity field and the high-wavenumber end of the velocity field, suggesting that $[u_o \times \omega_i]_r$ is unlikely to be negligible and that arguments concerning separation of scales should be used with care. This indicates that the above linear decomposition (equation (2.5)) is not possible and that

$$
\overline{u^2}^{+} = F(y^{+}; y/R) = G(y^{+}; R^{+}),
$$

where, in the case of the wall-normal component, the outer influence is understood to be smaller than in the case of the wall-parallel components. Equation (2.11) therefore illustrates that, in order for the $\phi_{11}$ spectrum to exhibit self-similarity, simultaneous collapse with inner and outer variables is required, as suggested by Morrison et al. (2004), a situation first realized by Townsend (1976).

The logic of equation (2.11) is far-reaching. While both the strong as well as the weak asymptotic conditions lead to complete similarity, it is not clear whether such a condition has yet been observed, even in the atmospheric surface layer. For an opposing view, see Nickels et al. (2005) who suggest the appearance of self-similarity in $\phi_{11}(k_1)$ for about one-third of a decade in the region $100 < y^{+} < 200$. Clearly, an assessment of the importance of the inner–outer interaction may be made by a close examination of the available data.

3. An appraisal of some data

(a) Wall-pressure fluctuations

That part of inactive motion which derives from the pressure field has so far been ignored. Figure 1 shows two wall-pressure spectra: one taken beneath a smooth-wall boundary layer and the other beneath a boundary layer, following a rough-to-smooth change in surface condition, both in zero pressure gradient. Table 1 shows the relevant scaling parameters. Location ‘A’ is immediately before the end of the rough wall and location ‘B’ on the smooth wall, 1.5 m downstream of ‘A’.

The change in surface condition of the rough-to-smooth layer provokes a very strong interaction; while there is a log law that conforms to a smooth surface condition, such that the inner layer is in a state of ‘moving equilibrium’ (Townsend 1966), there is a significant transport of turbulence kinetic energy towards the surface. In the wall-pressure spectrum, this appears as a large increase in spectral density at frequencies below those of the straight-line slope of $\omega^{-1}$. Clearly, in the rough-to-smooth layer, there is a very strong interaction between the turbulence generated over the rough wall and that of the new internal layer developing on the smooth wall (for further details, see Morrison et al. 1992).

Bradshaw (1967b) showed that, by assuming that the surface pressure depends only on $\rho$ and $u_r$, dimensional arguments alone require the spectrum to be of the form

$$
\phi_{pp}(k_1) = A \frac{r^2_w}{k_1},
$$

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for the range of wavenumbers between approximately the reciprocal of the sublayer thickness and the reciprocal of the log-layer thickness. Owing to the wavenumber dependence of the pressure–field convection velocity, the $u_{K1}^{-1}$-dependence is approximate. Bradshaw (1967b) suggests that the frequency spectrum varies as $u_{K0}^{-0.7}$. Integration of equation (3.1) in this range gives

$$
\frac{\overline{p^2}}{\tau_w^2} = B \ln[R^+] + C,
$$

where $B = A \log e$ and $C$ is a constant of integration. Estimates of $B$ vary slightly: Farabee & Casarella (1991) obtained $B = 1.86$; Spalart (1988) suggests ‘about 2’; and the present spectra give $B = 1.61$. This indicates that, even when

**Figure 1.** The wall-pressure spectra.

**Table 1.** Principal parameters of datasets.

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<th>‘A’</th>
<th>‘B’</th>
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considering the surface pressure due to the active motion alone, wall-pressure fluctuations increase with $R^+$ and that, therefore, the large scales penetrate to the wall.

(b) Linear superposition

Figure 2 shows measurements of the $R_{22}$ correlation (2.7) made in the atmospheric surface layer at the 300 m mast at the Boulder Atmospheric Observatory. These data are reported by Hunt et al. (1989). While there are always uncertainties associated with measurements in the surface layer, it is clear to this author that many of the data do not exhibit the direct proportionality between $R_{22}$ and $y$; rather, measurements of $R_{22}$ are less than estimates from the model that assumes linear superposition.

Figure 3 shows $\overline{v'^2}$ from high Reynolds number pipe flow (Zhao & Smits in press). Simple blocking arguments suggest that $\overline{v'^2}$ is approximately constant, but that, at sufficiently large $y$, it becomes unimportant and $\overline{v'^2}$ decreases. For $y/R > 0.1$, the same data scaled on outer variables ($u_τ, R$) collapse onto a single line, exhibiting similarity, in conformity with Townsend’s theory. The remarkable feature of these data is the increase in $\overline{v'^2}$ for $y^+ < 200$ and this is not explained by the linear model. There is also an implication that, as in the case of $\overline{u'^2}$, there is a near-wall peak, presumably in the viscous sublayer where the direct effects of viscosity will reduce $\overline{v'^2}$. Intriguingly, while the near-wall peak in $\overline{u'^2}$ may be explained by the maximum in production of turbulence kinetic energy near $y^+ ≈ 15$, no such explanation is possible in the case of $\overline{v'^2}$, for which the corresponding production term is zero. Usually, energy in the $v$-component may be explained by the effects of pressure–strain correlations and transport towards the wall. This appears to be the case here also; it should be noted that these are
nonlinear effects. These data show a strong inner–outer interaction; these suggest that it occurs only at the two lowest Reynolds numbers. However, resolution will affect the extent to which this region can be resolved.

(c) Spectral self-similarity

Figure 4 shows streamwise-velocity spectra calculated using the data of Högström & Bergström (1996) taken on the ‘Laban’s Mills’ site. The mean wind velocity is typically 7–10 m s\(^{-1}\) and data were recorded at three heights simultaneously, 1.6, 3.1 and 6.3 m for 8 h. Particular attention is paid to the wavenumber resolution at low wavenumbers; further details are given in McKeon & Morrison (2004). Noting that simultaneous collapse of spectra using both inner and outer variables along a horizontal straight line in figure 4 is required for a demonstration of self- (complete) similarity, it appears evident that there is no such demonstration by these data. The boundary-layer thickness, \(\delta\), is estimated to be 500 m, that is, \(\delta^+ \approx 10^7\). This easily meets the minimum requirement of \(\delta^+ > 52\ 500\) suggested by Nickels et al. (2005), necessary for a \(k_1^{-1}\)-range to be observable. Moreover, the direct effects of viscosity at the measurement positions are negligible. However, the interpretation of atmospheric data can often be challenging. In the present case, while the surface layer is neutrally stable, the site is at a latitude of 57° N, so that the effects of Coriolis forces may not be insignificant.

Figure 5 shows premultiplied \(\phi_{22}\) spectra from high Reynolds number pipe flow (Zhao & Smits in press) for \(y/R = 0.051\) and 0.096, that is, sufficiently close to the wall (\(y/R < 0.1\)) for the effects of blocking to be apparent, yet sufficiently remote from the wall (\(500 < y^+ < 2000\)) for the nonlinear effects apparent in figure 3 to be absent. The spectra show approximate collapse with inner scales except at the highest wavenumbers. Simple blocking arguments suggest \((k_1y\phi_{22}(k_1y))/u_r^2 \propto k_1y\) in the region of \(k_1y \sim 1\). The same spectra plotted on linear–linear axes show that this is the case approximately for \(k_1y < 0.5\) only and figure 5 (in which both
area and ordinate are directly proportional to energy) shows that, in terms of energy, the effects of linear blocking are relatively modest. The most interesting aspects of the wall-normal component are those at $y^+ \approx 200$, which, based on the evidence of figure 3, show pronounced effects of the inner–outer interaction in the near-wall region where the direct effects of viscosity are also prevalent.

4. Discussion

The data presented here all show the effects of a nonlinear inner–outer interaction, which, in the transport equations, appears through the nonlinear transport and pressure–strain terms. Such an interaction can be made very

\[ Re = 4.8 \times 10^5 \]

- $y^+ = 500; \ y/R = 0.051$
- $y^+ = 949; \ y/R = 0.096$

\[ Re = 1.0 \times 10^6 \]

- $y^+ = 1079; \ y/R = 0.051$
- $y^+ = 2046; \ y/R = 0.096$

Figure 5. The wall-normal velocity spectra, $(k_1y\phi_2(k_1y))/u_r^2$ versus $k_1y$ (from Zhao & Smits in press).
significant by using, for example, surface roughness or free-stream turbulence. The data of figure 1 and parameters in table 1 might appear to be the results of a somewhat artificial experiment, in the sense that, while of practical significance, the experiment has been designed essentially to provoke a large interaction for which the theory will fail. Nevertheless, such an experiment involving a rough-to-smooth change in surface roughness produces values of $\sqrt{p^2/\tau_w}$ that would only appear at much higher Reynolds numbers in an unperturbed flow.

Examination of simpler flows, and, in particular, their comparison, reveals rather more subtle effects. Both Bradshaw & Huang (1995) and Nieuwstadt & Bradshaw (1997) have suggested that there are real differences between the internal and external wall layers at the same $R^+$, and even between the channel and pipe flows. Statistically significant differences appear to become more perceptible in the higher moments. Nieuwstadt & Bradshaw (1997) suggest that these differences become greater than the probable experimental error for third-order moments and above. Much of their comparison involves the use of DNS data, from which one can deduce that the differences arise owing to the differences in boundary conditions, specifically those at the edge of the boundary layer, or the channel or pipe centreline. However, simulations inevitably restrict comparisons to low Reynolds numbers, but the indications of this paper are that measurable differences would also appear in the second-order moments if the Reynolds number were high enough. The measurement of such small differences in the higher moments will undoubtedly require fully resolved measurements of high accuracy. Some progress has recently been made (see Kunkel et al. 2006). Otherwise, further experiments and simulations in which the interaction may be stimulated without the need for very high Reynolds numbers would avoid many of the difficulties that such experiments at high Reynolds numbers entail.

By extension, there are other comparisons that can be illuminating, but which depend on a finer degree of comparison. In practice, there is an unlimited number of ways of generating, for example, a boundary layer of given $R^+$ in zero pressure gradient on a smooth surface, which may be summarized as being either at low velocity, long fetch, ($x^+$), or at high velocity, short fetch. Excluding extraneous influences, dimensional reasoning suggests that any differences between these two boundary layers would have to arise through the influence of other length or velocity scales associated with initial conditions. Often experimenters choose to avoid these by the use of a trip, the residual effects of which are expected to be negligible at a sufficiently high Reynolds number (Erm & Joubert 1991). But what is a ‘correctly stimulated’ boundary layer?

Much recent progress has been made in the mapping of multiple solutions to the Navier–Stokes equations for pipe flow (Hof et al. 2004), the number of which increases with Reynolds number. These solutions are unstable and the observations of Hof et al. (2004) suggest that the turbulent state is organized around a few dominant travelling waves. While direct evidence that unstable travelling waves are relevant to turbulent flows is still lacking, they are persistent and, intriguingly, have features (streamwise vortices and streaks) that are fundamental to wall turbulence at all Reynolds numbers. In terms of dimensional analysis, the question is therefore at what non-dimensional streamwise distance, $x^+$, does any functional dependence disappear?
The nature of the interaction between the inner and outer regions of turbulent wall motion has been assessed; the local-equilibrium approximation is identified as a no-interaction condition and inactive motion, used by many workers to explain the failure of wall scaling to collapse the Reynolds stresses, while admitting the influence of the large scales near the wall, is equivalent to a weak interaction in which the large scales appear as a linear superposition on the active, shear-stress-bearing motion at the wall. The implication is that, at the same Reynolds number, internal flows will exhibit the same statistics as boundary layers. There is evidence that this is not the case, even at high Reynolds numbers. It seems probable therefore that the true nature of the inner–outer interaction is inherently nonlinear, which may be used to explain perceived differences between internal and external flows. Other questions, such as the influence of initial conditions, remain open. Recent developments in high-quality accurate instrumentation will enable detailed comparisons.

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References


The turbulent wall-bounded flow


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