A deterministic model for the sublayer streaks in turbulent boundary layers for application to flow control

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We present a relatively simple, deterministic, theoretical model for the sublayer streaks in a turbulent boundary layer based on an analogy with Klebanoff modes. Our approach is to generate the streamwise vortices found in the buffer layer by means of a vorticity source in the form of a fictitious body force. It is found that the strongest streaks correspond to a spanwise wavelength that lies within the range of the experimentally observed values for the statistical mean streak spacing. We also present results showing the effect of streamwise pressure gradient, Reynolds number and wall compliance on the sublayer streaks. The theoretical predictions for the effects of wall compliance on the streak characteristics agree well with experimental data. Our proposed theoretical model for the quasi-periodic bursting cycle is also described, which places the streak modelling in context. The proposed bursting process is as follows: (i) streamwise vortices generate sublayer streaks and other vortical elements generate propagating plane waves, (ii) when the streaks reach a sufficient amplitude, they interact nonlinearly with the plane waves to produce oblique waves that exhibit transient growth, and (iii) the oblique waves interact nonlinearly with the plane wave to generate streamwise vortices; these in turn generate the sublayer streaks and so the cycle is renewed.

Keywords: sublayer streaks; bursting cycle; turbulent boundary layers

1. Introduction

Now and over the next few decades, airframe manufacturers face one of their toughest challenges, namely the need for greatly reduced emissions, particularly of greenhouse gases, such as carbon dioxide. In the eyes of the general public, combating climate change is, of course, currently the prime motivation. It seems probable that these concerns will translate into ever-tougher political and regulatory action over the coming decades. Although improving the efficiency of the aeroengines can make a significant contribution, ultimately emissions can only be reduced by burning less fuel. This leads ineluctably to a requirement for

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large drag reductions of the order of 50%. In response to this challenge, the aircraft industry is beginning to think about how to achieve this formidably difficult goal.

The main sources of drag on an aircraft are due to skin friction, the formation of shock waves, the generation of lift and flow separation. These are usually termed skin-friction, wave, induced (or vortex) and form drag, respectively. For modern airliners, such as the Airbus A300-600, skin-friction drag accounts for approximately 50% of the total at cruise conditions. Most of the other drag are very roughly divided between induced and wave drag, with form drag making only a small contribution at cruise conditions. Not a great deal can be done about induced drag, as it is an essential by-product of generating lift for conventional aircraft (Kroo 2001). Small reductions can be made by fitting winglets to the wingtips, but these gains have already been largely made with existing aircraft. However, since induced drag is proportional to the square of the aircraft’s weight, the largest reductions are likely to come from reducing the total drag, thereby reducing the weight of fuel required. Wave drag is caused by the inevitable presence of regions of supersonic flow over the wing when flying at high subsonic speeds. Careful wing design can reduce the strength of the shock waves. However, although there is probable scope for further improvement, these gains have also already been largely made with the introduction of modern supercritical wings (Sobieczky & Seebass 1984). It seems, then, that a large drag reduction implies a large reduction in skin-friction drag.

Broadly, there are two main strategies for reducing skin-friction drag. Since laminar flow has much lower levels of skin friction than turbulent flow, one can seek to maintain laminar flow. Alternatively, one can seek to reduce the skin friction of the fully turbulent flow. It is conceivable that by careful design, natural laminar flow can be maintained over a substantial part of the wing. However, to go further than this using current proven technology would necessitate the use of laminar-flow control in the form of boundary-layer suction (Braslow 1999; Gad-el-Hak 2000). Compliant walls (Carpenter 1990; Gad-el-Hak 2000) offer a passive method for laminar-flow control but are not practical for use in aircraft (Carpenter et al. 2001). Conventional aircraft have fuselages for which little can be done to achieve laminar flow without resorting to boundary-layer suction. The alternative would be some sort of flying wing aircraft, but they also have serious drawbacks.

What, then, can be done to reduce skin friction in fully turbulent flows? One successful passive method that has already been used on commercial airliners is riblets. This technique was developed at NASA Langley in the USA (Walsh & Weinstein 1978) but it was also discovered independently in Germany through the study of the hydrodynamics of shark scales (Reif & Dinkelacker 1982; Bechert et al. 1985, 1997, 2000). Riblets take the form of minute streamwise ridges and valleys on the aircraft’s surface. A polymeric riblet film with triangular-shaped ridges and valleys is produced by the 3M company and used by Cathay Pacific on their Airbus A300-600 aircraft. To achieve a reduction in skin-friction drag, the spacing between adjacent ridges has to be in the range of 25–75 μm for flight conditions. Up to approximately 6% reduction in skin-friction drag can be achieved with the application of such riblets to the surface. Greater percentage reductions are possible with the more advanced riblet designs developed and investigated by Bechert et al. (1997). In addition, with other
surface roughness-type treatments, e.g. the randomized, chevron-shaped, ‘roughness’ elements of Sirovich & Karlsson (1997) (see also Carpenter 1997), approximately 10% drag reduction was achieved. Other examples are given by Jiménez (2004). Fransson et al. (2006) have even demonstrated that the right choice of roughness element and distribution can delay the onset of laminar–turbulent transition (see also Choi 2006). For all these various types of surface treatment, 10–12% is probably the ultimate reduction potentially achievable for skin-friction drag when applied to commercial airliners.

It seems, then, that active control of some kind will be required to achieve large reductions in skin-friction drag. For this, a good understanding of the flow physics of the near-wall region of the turbulent boundary layer is required. There are several types of flow structures in the near-wall region, which is usually defined as consisting of the viscous sublayer, the transitional layer and the inner part of the log-law region. The first type of flow structure identified historically was the hairpin vortices of Theodorsen (1952). Later, Kline et al. (1967) discovered the sublayer streaks. These are regions that are narrow in the spanwise and wall-normal directions but elongated in the streamwise direction, having streamwise velocities that alternate in the spanwise direction between being higher or lower than the mean velocity. Kline et al. (1967) postulated that they were created by a vertical ‘lift-up’ mechanism produced by the induced velocity due to the legs of the hairpin vortices. In fact, even a randomized field of wall-normal velocity perturbations can create streaks (Chernyshenko & Baig 2005b). However, in turbulent boundary layers, streamwise vortices in some form are usually considered to be responsible.

There is an enormous literature on the various aspects of the near-wall region of turbulent wall layers. It is beyond the scope of this paper to review this literature in its entirety. Instead, we refer the reader to the recent excellent review of Panton (2001). Excellent, but more specialized, reviews are also contained in the introductions of Schoppa & Hussain (2002) and Chernyshenko & Baig (2005a). This provides the background for much of our work. However, we shall briefly describe the role that sublayer streaks are thought to play in the generation of surface shear stress. Initially, the low-speed sublayer streaks elongate and meander. Their downstream tips then rise above the wall. This process is usually termed ejection. The downstream tip of the streak then appears to develop fairly violent oscillations followed by what is often termed the ‘bursting’ process. The violent oscillations are usually attributed to Kelvin–Helmholtz-type instability resulting from the formation of local velocity profiles with points of inflexion. Immediately following bursting, there is a rush of high-speed fluid towards the wall. This is usually termed the ‘sweeping’ process. It is during the sweeps that the high levels of wall shear stress are generated. The whole process then begins again with streamwise vortex elements generating new sublayer streaks. Thus, there is a quasi-cyclic bursting process that is responsible for generating the high levels of wall shear stress. It is generally considered that a successful active control technique for reducing skin-friction drag would have to interfere with this quasi-cyclic bursting process and somehow disable it.

Much is known about the characteristics of the sublayer streaks. For example, the statistically mean spanwise spacing is usually quoted as $100 \nu / U_t$ (where $\nu$ is the kinematic viscosity of the fluid; $U_t = \sqrt{\tau_w / \rho}$ is the friction velocity; $\tau_w$ is the
mean wall shear stress; and $\rho$ is the fluid density). The streaks are approximately $10-20\, v/\bar{U}$ wide and extend up to $1000\, v/\bar{U}$ in the streamwise direction. There is less consensus on the values for the so-called bursting frequency and its inverse the bursting period. Rao et al. (1971) found experimentally that the bursting period scaled with outer variables. Later, Blackwelder & Haritonidis (1983) ascribed this result to the scale-dependent effect of the probes of Rao et al. (1971) on the measurements of bursting period. Most authors are inclined to believe that the period scales with inner variables, but it remains an open question. We shall follow Johansson et al. (1987) and assume the bursting frequency to be approximately $0.04\, U^2/\nu$.

The information on the characteristics of the sublayer streaks briefly reviewed above is relevant when considering their active control under flight conditions. For example, let us take the fuselage of an Airbus A340-300. A typical cruise speed is 880 km h$^{-1}$ or approximately 250 m s$^{-1}$ at a height of approximately 10 000 m. Assuming the International Standard Atmosphere, the kinematic viscosity and density of air at this height are $35.25 \times 10^{-6}$ m$^2$ s$^{-1}$ and 0.414 kg m$^{-3}$, respectively. The fuselage is approximately 64 m in length, giving an overall Reynolds number $R_e = U_\infty L/\nu = 4.5 \times 10^8$, where $L$ is the fuselage length and $U_\infty$ is the freestream speed, here assumed to be cruise speed. Using the Prandtl–Schlichting formula for estimating the coefficient of skin-friction drag, namely

$$C_{D_f} = \frac{1}{2} \rho U_\infty^2 A = \frac{0.455}{(\log_{10} R_e)^{2.58}},$$

where $A$ is the surface area of the fuselage, we estimate the friction velocity averaged over the fuselage surface to be $U_\infty^2 C_{D_f}/2 \approx 7.3$ m s$^{-1}$. Therefore, the near-wall length-scale $v/\bar{U} \approx 4.8$ μm, and a typical average spanwise streak spacing and streamwise length are 0.5 and 5 mm, respectively.

Temporal demands are also placed on the actuators and sensors by the bursting frequency, which indicates the order of frequency response required. Using the estimate due to Johansson et al. given above, we obtain a bursting frequency of 60 000 rad s$^{-1}$ (10 kHz) or a period of approximately 20 μs. If an actuator is to be successful in controlling single coherent structures, its duration should be a fraction of this period because the streaks convect quickly downstream and can drift in the spanwise direction. Johansson et al. (1987) calculated an average convection speed of $10\, \bar{U}$ or approximately 75 m s$^{-1}$ in the present example.

The streak dimensions estimated above give us an idea of the dimensions and spacing required for the actuators and sensors in a distributed control system. A provisional estimate would be that an array of actuators would be required, each of which is approximately 200 μm in diameter or less, with spanwise and streamwise spacings of approximately 500 μm and 5 mm, respectively. This implies an array of approximately $10^9$ actuators distributed over the surface of the fuselage.

The only feasible way such a large array of very small actuators and sensors could be manufactured is by using MEMS (Microfabricated ElectroMechanical Systems) technology. This term encompasses an area of technology and science that has emerged over the last two decades. It makes it possible to manufacture large numbers of very small sensors and actuators, with their necessary electronic
control systems on a common substrate using integrated circuit fabrication techniques. Such MEMS devices can be made very small (10–100 μm) and produced relatively cheaply in large numbers. They are currently used for such applications as sensors for automotive airbags, miniature sensors in medical applications and in ink-jet printer heads. However, a very substantial investment in research and development is needed before MEMS devices could be successfully developed for the reduction of skin-friction drag in flight conditions.

This brings us to one of the main motivations for the research described in this paper. Given the investment required, aircraft manufacturers will wish to have a firm indication that arrays of MEMS devices could potentially deliver the desired large drag reduction. The problem is that the great disparity in length- and time-scales between laboratory and flight conditions makes it difficult to obtain experimental verification. Equally, the use of direct numerical simulations is confined to much lower Reynolds number and will not reach flight-scale Reynolds numbers for the foreseeable future. Matters are not a great deal better with large eddy simulations, since in this case the upper bound to Reynolds number is determined by the scales of the near-wall flow structures. Most techniques for dealing with this limitation resort to either some sort of wall function or a hybrid method, such as detached eddy simulation (Travin et al. 1999; Spalart et al. 2003), which uses a RANS (Reynolds-averaged Navier–Stokes) method for the near-wall region (Piomelli & Balaras 2002). RANS methods can be used at flight-scale Reynolds numbers. However, because they depend so heavily on calibrated semi-empirical turbulence modelling, RANS methods are unlikely to be able to simulate the interaction between the near-wall flow structures and a control system. (See another paper by Gatski et al. (2007) in this special issue for a survey of the state of the art for turbulence modelling and RANS methods.) It seems that only some sort of low-order approximate modelling of the near-wall flow structures will be capable of simulating their response to MEMS-scale control in flight conditions. In this spirit, we present our deterministic model for the sublayer streaks in turbulent boundary layers.

The discussion above makes it clear that few unresolved questions in fluid mechanics and aerodynamics are more important than identifying and modelling the mechanism whereby the near-wall structures of turbulent wall layers generate high levels of shear stress at the wall. It holds the key to developing successful control strategies for the reduction of skin-friction drag. It is equally important for the opposite situation, namely controlling the flow in order to raise the levels of turbulence for postponing flow separation or enhancing exchange processes. There are also many unresolved subsidiary questions, for example: what gives rise to the quasi-periodic bursting cycle; what determines the spanwise spacing and strength of the sublayer streaks; how are the streamwise vortical components responsible for generating these streaks created; and what, if any, is the role of the outer part of the turbulent boundary layer in the generation of wall shear stress?

For some years, research teams at the University of Warwick, Cardiff University and the Indian Institute of Technology, Delhi have been working on a combined theoretical, computational and experimental, collaborative research programme aimed at answering these questions. Our programme is ongoing and we have by no means found satisfactory answers to all the unresolved questions. Certainly, our deterministic model will need further development before it is capable of fully simulating all the desired flow physics. Nevertheless, our research
has reached the stage where useful theoretical models have been developed and many of the questions have received at least partial answers. For example, our theoretical model for the near-wall structures has recently been used to undertake a study of the use of massless-jet microactuators to control the sublayer streaks and thereby the levels of wall shear stress in a turbulent boundary layer (Lockerby et al. 2005). We have also used it recently to investigate the reduction of turbulent skin friction due to the use of compliant walls (Ali & Carpenter 2005). In this paper, we shall also briefly describe our theoretical model for the quasi-periodic bursting process, but shall mainly focus on modelling the generation of the sublayer streaks.

As remarked above, we shall not attempt to review in its entirety the enormous literature on the near-wall structures of turbulent wall layers. However, our approach to the modelling of the sublayer streaks and the bursting process did not come out of the blue. Accordingly, we shall review the previous papers on allied approaches. Our model for the quasi-periodic bursting cycle involves near-wall structures in the form of propagating waves, as well as the usual streamwise vortices and streaks. These wave-like flow structures have been somewhat overlooked in the past, so we shall also review the relevant literature on waves in turbulent wall layers and modelling the bursting cycle.

The rest of the paper is set as follows. The numerical methods are described in §2. We then review the relevant literature on modelling the sublayer streaks, describe our model for the sublayer streaks and present and discuss some results in §3. The modelling of the sublayer streaks is then put in context in §4, with a brief description of our overall model for the quasi-periodic bursting process preceded by a brief review of the relevant literature on modelling the processes in the bursting cycle. Finally, some brief conclusions follow in §5.

2. Numerical methods

The governing equations and numerical methods are almost identical to those described by Lockerby et al. (2005), in which our model for the sublayer streaks was first introduced. We use the velocity–vorticity method of Davies & Carpenter (2001). The coordinate system (x, y, z) corresponds respectively to the streamwise, spanwise and wall-normal directions. The domain is semi-infinite, such that z=0 defines the lower boundary representing the solid surface that, since we use a compliant wall as one of our illustrative examples, could in general move. We assume that there is a specified, undisturbed flow, represented by the dimensionless mean velocity, \( \bar{U} = (U, 0, 0) \), and vorticity, \( \bar{\Omega} = (0, 0, \Omega_z) \), where \( U \) and \( \Omega_z \) correspond to the mean turbulent profiles. The freestream velocity, \( U_\infty \), and boundary-layer displacement thickness are used as the reference quantities for non-dimensionalization. The total dimensionless velocity and vorticity fields are then decomposed into \( U = \bar{U} + u \) and \( \Omega = \bar{\Omega} + \omega \), where \( u = (u, v, w) \) and \( \omega = (\omega_x, \omega_y, \omega_z) \) represent perturbations from the prescribed mean flow.

We shall consider the governing equations for the perturbation flow variables only. These are divided into two sets, namely the primary variables \( \{\omega_x, \omega_y, w\} \) and the secondary variables \( \{\omega_z, u, v\} \). The secondary variables may be defined explicitly in terms of the former, and thus, in principle, be eliminated from...
consideration. It can be shown (Davies & Carpenter 2001) that the evolution of the three primary variables is governed by the following three equations only:

\[
\frac{\partial \omega_x}{\partial t} + \frac{\partial N_z}{\partial y} - \frac{\partial N_y}{\partial z} = \frac{1}{R} \nabla^2 \omega_x - \frac{\partial F_y}{\partial z},
\]

(2.1)

\[
\frac{\partial \omega_y}{\partial t} + \frac{\partial N_x}{\partial z} - \frac{\partial N_z}{\partial x} = \frac{1}{R} \nabla^2 \omega_y,
\]

(2.2)

\[
\nabla^2 w = \frac{\partial \omega_x}{\partial y} - \frac{\partial \omega_y}{\partial x},
\]

(2.3)

where \( \mathbf{N} = \vec{\Omega} \times \mathbf{u} + \omega \times \vec{U} + \mathbf{N}' \), \( \mathbf{N}' = \omega \times \mathbf{u} \) and \( R = U_\infty \delta^*/\nu \). The last term on the right-hand side of equation (2.1) represents the gradient of a fictitious body force; it acts as a vorticity source and will be explained in more detail below.

Because we are interested only in the fundamental coherent structures of the turbulent boundary layer (the streaks), we have adopted an undisturbed base flow corresponding to a time-averaged turbulent profile, with the aim of investigating linear perturbations from this. Thus, for the simulations presented in this paper, the governing equations are linearized by setting \( N_0 = 0 \). Strictly, then, our present approach is only valid, provided the perturbations are of small amplitude. There is, however, considerable evidence to suggest that the development of the streaks is governed by a linear mechanism (Landahl 1980, 1990). Moreover, their control can be modelled by a linear process (Kim & Lim 2000), so this linear approach is justified, even though developed turbulence is characteristically nonlinear.

The base flow we have adopted is based on the law-of-the-wall velocity profile given in inverse analytical form by Spalding (1961) combined with the law of the wake due to Coles (1956). Since there is no corresponding analytical form for the mean wall-normal velocity and no straightforward way of obtaining one, the use of Spalding’s approximate analytical velocity profile obliges us to assume a boundary layer of constant thickness. This, perhaps, has some advantage, since it has the merits of simplicity and separating out the other effects from those of boundary-layer growth. In fact, for typical simulations, the change in boundary-layer displacement thickness is very small. For example, if \( R = 5000 \) and the streamwise extent of interest for studying the development of streaks is less than 1000 wall units, using standard semi-empirical methods, one can estimate that the displacement thickness would increase by only approximately 1.25% over this distance.

Let us consider the case where the wall can move in the wall-normal direction but not in the streamwise and spanwise directions. Then, the boundary condition at the wall for small wall displacement, \( \eta \), on the wall-normal perturbation velocity is given by

\[
w(x, y, 0, t) = \frac{\partial \eta}{\partial t}(x, y, t).
\]

(2.4)

The other two velocity components obey the no-slip condition, which for small values of \( \eta \) (linearized boundary conditions) take the form (Carpenter & Garrad 1985)

\[
u(x, y, 0, t) = 0.
\]

(2.5b)
In Davies & Carpenter (2001), the definition of vorticity with use of conditions (2.5a) and (2.5b) leads rigorously to the following integral conditions:

\[
\int_0^\infty \omega_y dz = \eta \frac{dU}{dz}(0) - \int_0^\infty \frac{\partial w}{\partial x} dz,
\]

\[
\int_0^\infty \omega_x dz = \int_0^\infty \frac{\partial w}{\partial y} dz
\]

that are fully equivalent to the no-slip boundary conditions (2.5a) and (2.5b). Therefore, they can be viewed as constraints on the evolution of the primary variables \( \omega_y \) and \( \omega_x \), respectively. For the usual case of a rigid wall, \( \eta = 0 \). The only reason for introducing the possibility of non-zero \( \eta \) is so that we can consider the case of the compliant wall as one of the illustrative examples.

The inflow boundary conditions take the form of undisturbed flow, so that \( \omega_x = \omega_y = w = 0 \) there. These are also the boundary conditions at the upper boundary, \( z \to \infty \). The computational domains used in our simulations extended sufficiently far in the streamwise direction, such that the flow perturbations were negligible at the downstream outflow boundary.

A source term is added to the right-hand side of equation (2.1) for the streamwise vorticity perturbation, \( \omega_x \). This source term takes the form of \( \nabla \times \boldsymbol{F} \), where \( \boldsymbol{F} = (0, F_y, 0) \) is a fictitious body force; the source term can be written as

\[- \frac{\partial F_y}{\partial z} = -G \mathcal{F}(x - x_f, z - z_f) \cos \beta y, \]

where

\[ \mathcal{F}(x - x_f, z - z_f) \equiv \frac{\sqrt{ab}}{\pi} \exp \{-a(x - x_f)^2 - b(z - z_f)^2\}, \]

and \( G \) is the source strength; \((x_f, z_f)\) is the location of the source; \( a \) and \( b \) are the parameters specifying the width of the Gaussian function; and \( \beta \) is the dimensionless spanwise wavenumber of the force. The function \( \mathcal{F} \) in equation (2.8) could be regarded as a numerical approximation to the double delta function, \( \delta(x - x_f) \delta(z - z_f) \). The form (2.8) used for the vorticity source will be further discussed in §3.

The computational domain is semi-infinite, stretching from a uniform wall at \( z = 0 \) to infinity in a direction normal to it, and periodic in the spanwise direction. The governing equations are discretized using a centred finite-difference scheme in the streamwise direction \( (x) \) and a series of Chebyshev polynomials in the direction normal to the wall \( (z) \). The discretization in the spanwise direction \( (y) \) is in the form of a Fourier-series expansion. The wall boundary conditions are implemented in exactly the same way as those in Lockerby et al. (2005).

For the implementation of temporal discretization, the convective terms \( \mathbf{N} \) in equations (2.1) and (2.2) are calculated explicitly using the predictor–corrector scheme. The Adams–Bashforth scheme was used for the predictor stage, except for the viscous term involving wall-normal derivatives for which the Crank–Nicolson scheme was used. The Crank–Nicolson was used for all the terms in the corrector stage. In order to calculate the convective terms, the secondary variables must be evaluated, requiring some wall-normal integration of the
primary variables. This integration of the Chebyshev polynomials leads to a pentadiagonal set of equations that is solved using the Thomas algorithm. To calculate the product terms in \( N \), a fast Fourier transform is used to convert the variables to the spatial domain where the products can be evaluated efficiently. Then, the FFT is reapplied to transform the product back to the spectral domain. The wall-normal viscous components on the right-hand sides of equations (2.1) and (2.2) are treated implicitly, whereas the other viscous terms are treated explicitly. De-aliasing was not found to be necessary for the current linear simulations, but may well be required when nonlinear simulations are undertaken.

3. Sublayer streak modelling

We shall now discuss our approach to modelling the sublayer streaks. To do this, we draw an analogy between the sublayer streaks in the turbulent wall flow and the streak-like flow structures driven by freestream turbulence in laminar boundary layers. This approach draws on the concepts of hydrodynamic stability theory, receptivity and algebraic growth. The existence of sublayer streaks in turbulent wall layers was originally discovered by Kline et al. (1967), so it actually predates the discovery by Klebanoff (1971) of streak-like flow structures in boundary layers undergoing laminar–turbulent transition. Many authors since then have studied them.

The ideas behind our model of the sublayer streaks are not, of course, entirely new. It has been known for some time that three-dimensional disturbances in a shear flow can grow algebraically, termed as such to distinguish it from exponential growth. This was demonstrated by Ellingson & Palm (1975) and Landahl (1980) for three-dimensional perturbations in an inviscid shear layer. The inviscid algebraic growth, together with a viscous-induced damping, constitutes what is sometimes called transient growth. This phenomenon has been demonstrated in viscous shear flows by Hultgren & Gustavsson (1981), Butler & Farrell (1992) and Henningson et al. (1993). Transient or algebraic growth is believed to be the cause of bypass transition in boundary layers. It is thought that small perturbations might grow sufficiently during the transient growth phase for nonlinear effects, such as secondary instability, to induce transition, hence bypassing traditional mechanisms based on exponential growth of Tollmien–Schlichting waves. The Klebanoff mode (Klebanoff 1971; Kendall 1985; Westin et al. 1994; Andersson et al. 1999; Lieb et al. 1999; Fasel 2002), generated by freestream turbulence in laminar boundary layers, is generally thought to be a manifestation of algebraic or transient growth (Herbert & Lin 1993; Bertolotti 1997; Goldstein & Wundrow 1998). This hypothesis is consistent with the findings of Landahl (1980, 1990), Breuer & Haritonidis (1990) and Henningson et al. (1993), who have described how the lift-up effect (a phrase introduced by Kline et al. (1967) and subsequently used by Landahl to describe the mechanism of algebraic growth) leads to longitudinal streaky structures similar to those observed in the Klebanoff mode. More recently, the focus of the model of Schoppa & Hussain (2002) is on transient growth of the streaky background flow.

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Landahl (1980, 1990) and Butler & Farrell (1992) made a connection between the longitudinal streaks in a fully turbulent boundary layer and algebraic growth. Therefore, there is an implied equivalence between the Klebanoff mode in the transitional boundary layer and the streamwise sublayer streaks in a fully turbulent boundary layer. The algebraic-growth mechanism induces streaks in both cases; it is forced by freestream turbulence in the transitional case and for a turbulent wall layer by the streamwise vortical structures in the buffer and log-law regions. This is the essence of our streak modelling (figure 1), which will be described in more detail below. It also bears some similarity to the optimal perturbation approach of Chernyshenko & Baig (2005a, b). Butler & Farrell (1992) and other investigators based their approach on the concept of an optimal disturbance. But this approach is best exemplified in the very recent papers by Chernyshenko & Baig (2005a, b). Their initial perturbations consist of streamwise vortices of fixed kinetic energy and zero streamwise velocity. The vortices decay

Figure 1. The form of the sublayer streaks in a turbulent boundary layer in zero streamwise pressure gradient at $t^+ = 39$. (a) side view ($y^+ = 0$); (b) plan view ($z^+ = 12.0$); (c) front view ($x^+ = 1225.0$). Full and dashed lines indicate respectively locally positive and negative non-dimensional streamwise velocity perturbation. Parameters: $R = 5161; \, \delta^* U_\infty/\nu = 200.1; \, U_\infty/U_r = 25.8; \, \beta = 16.3; \, x_f = 5.0; \, y_f = 0.06; \, a = 22.0; \, b = 2923; \, G = 0.124 \times 10^{-3}; \, t_f^+ = 15$. Computational parameters: streamwise step-length, $\Delta x = 0.1\delta^*$; streamwise length of domain is $20.0\delta^*$; time-step, $\Delta t = 0.05\delta^*/U_\infty$; total time of computation is $20\delta^*/U_\infty$.

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with time while the streamwise velocity develops in the form of alternating high- and low-speed streaks, grows to a large amplitude and then decays. The optimum perturbation is the one for which the ratio of the kinetic energy to its initial value grows to the largest possible value before decaying. Chernyshenko & Baig found a reasonably good agreement between the streak spacing corresponding to their optimal perturbation and data from direct numerical simulations and experiments.

We have mentioned that our sublayer streak model can be regarded as an analogy with the Klebanoff modes generated in laminar boundary layers by freestream turbulence. Accordingly, initially by way of verification, we ran our boundary-layer code with the Blasius laminar velocity profile as the undisturbed base flow and located the vorticity source equation (2.8) just above the boundary-layer edge. In a uniform flow, it can be shown (Kudar 2004) that the type of vorticity source given by equation (2.8) corresponds to a narrow sheet of streamwise vorticity with sinusoidally varying strength in the spanwise direction. Thus, when placed above the boundary-layer edge, it is a crude representation of the zero-frequency component of the streamwise vorticity in the freestream. It generates elongated streaky structures (Klebanoff modes) within the laminar boundary layer. In Lockerby et al. (2005), it was shown that for a fixed value of \( \gamma \), there was an optimal value of spanwise wavenumber \( \beta \) in equation (2.8) that generated the strongest streaks. (Our measure of streak strength here and subsequently is the maximum streamwise velocity perturbation within the

\[ \gamma^+ \]

\[ \lambda^+ \]

Figure 2. The height of the vorticity source above the wall plotted against the corresponding optimal non-dimensional spanwise wavelength (streak spacing) for three different streamwise pressure gradients in terms of the wake strength parameter, \( \Pi \), of Coles (1956). Open squares, \( \Pi = 0.895, \delta U_r/\nu = 191.3, U_\infty/U_r = 27.0 \) (adverse pressure gradient); open circles, \( \Pi = 0.550, \delta U_r/\nu = 200.1, U_\infty/U_r = 25.8 \) (zero pressure gradient); open diamonds, \( \Pi = 0.205, \delta U_r/\nu = 208.8, U_\infty/U_r = 24.7 \) (favourable pressure gradient). Remaining parameters are the same as those given in figure legend 1.
boundary layer at any given time.) This most receptive wavenumber was found to be very close to the dominant spanwise mode observed in the experimental study of Klebanoff (1971) (as reported by Herbert & Lin 1993; Bertolotti 1997).

We were struck by the apparent similarity between the Klebanoff modes and the sublayer streaks in a turbulent boundary layer. Accordingly, in an attempt to generate sublayer streaks, we located the vorticity source within the boundary layer. The sheet of streamwise vorticity it produces can now be regarded as a crude representation of the vortex elements found in the buffer layer. We now vary both the height, $z_f$, of the source above the wall and its spanwise wavenumber, $b$, to find the optimal combination that generates the strongest streaks for fixed amplitude $G$. The form of the streaks corresponding to the optimum combination is shown in figure 1 at a particular stage in their development. Figure 2 shows how the maximum streak perturbation velocity varies with $z_f$ (the location of the vorticity source in wall units, i.e. $z_f = z_f / \delta^* U_f / \nu$, where the friction velocity, $U_f \equiv \sqrt{\tau_w / \rho}$, where $\tau_w$ is the shear stress at the wall and $\rho$ and $\nu$ are respectively the density and kinematic viscosity of the fluid).

For the laminar boundary layer, the vorticity source remains fixed for all time, $t > 0$. It is found in this case that the maximum perturbation velocity associated with a high-speed streak rises until it reaches a steady-state value. We then seek the value of $b$ that maximizes this steady-state value. For the turbulent boundary layer, the forcing term is constrained to act only for a discrete non-dimensional time, $t_f^+$ (where dimensional time is made non-dimensional using a reference time of $\nu / U_f^2$), rather than continuously. Figure 3 shows how the maximum streak perturbation velocity varies for three different values of $t_f^+$. We have followed the example of Gad-el-Hak & Hussain (1986) in the experimental study of artificially generated
streaks and have chosen to use $t^f = 15$. In their experimental study, they generated streaks by means of streamwise-oriented suction strips with spanwise separation of 100 wall units. A suction time of $15v/U^2$ gave bursting characteristics that were the most similar to real turbulent flows. In our case, this choice for $t^f$ led to an optimal streak spacing reasonably close to the experimentally observed values. We shall see in §4 that there is possibly a less arbitrary justification for the choice.

Figure 4 shows how the strength of the streaks varies with $\lambda^+ = 2\pi/\beta^+ (\beta^+ \equiv \beta v/(U_\tau \delta^+))$ for the boundary layer developing under zero pressure gradient, as well as for those in favourable and adverse pressure gradients. It can be seen that the optimum value is approximately 78, which is within the range of mean streak spacing seen in experiments (see the review of previous experimental studies given by Zacksenhouse et al. 2001). The precise optimum streak spacing found in our simulations depends also on the values of the source parameters $a$ and $b$. Precisely, what values should be chosen is unclear. Presumably, the value of $b$ is linked to the dimensions of the typical streamwise vortices found in the buffer layer. (The wall-normal thickness of the vortex sheet is proportional to $1/\sqrt{b}$, and $1/\sqrt{a}$ is a measure of the streamwise spread of the vorticity source.) Surprisingly, we found that the results were not greatly sensitive to the precise value of $b$, but were sensitive to the value of $a$. There does not appear to be any clear guidance on physical grounds for the choice of value for $a$. When $a$ is sufficiently large, the optimal streak spacing does not change with further rise in the value of $a$. However, very large values of $a$ lead to numerical problems. The value of $a$ we used here was sufficiently large such that the optimal streak spacing had reached its asymptotic constant value, but small enough to avoid numerical problems. The fact that we use different values of $a$ and $b$ from those used by Lockerby et al. (2005; their values are given in the legend of figure 7) is the reason why our results and theirs differ.

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In the real turbulent boundary layer, the streamwise vortical structures that generate the sublayer streaks are in fact travelling downstream. We therefore carried out some simulations with a moving vorticity source. Typical results are shown in figure 5, which plots maximum perturbation velocity versus the ratio of the source convection speed \( u_f \) to the local mean velocity, \( U_f \) at \( z_f \). It can be seen that the optimum convection speed equals the local mean velocity. At \( z_f^+ = 18 \), \( U = 12 U_\tau \), which is in reasonably close agreement with the experimental observations of Johansson et al. (1991), corresponding to \( 13 U_\tau \), and with the value of \( 10 U_\tau \) found in the direct numerical simulations of Kim & Spalart (1987). Elsewhere in the paper, the results are based on a stationary vorticity source.

The mean velocity profile of the turbulent boundary layer varies with Reynolds number. As a consequence, assuming a fixed value of \( \tau_f^+ = 15 \), the optimum streak spacing, \( \lambda_{\text{opt}}^+ \), and the maximum velocity perturbation associated with the streaks vary with Reynolds number as shown in figure 6. The evidence on the variation of average streak spacing with Reynolds number is mixed, with some experimental studies suggesting that it is invariant with Reynolds number and others showing a slight rise with Reynolds number. For example, Klewicki et al. (1995) obtained an average spacing of \( 93\nu/ U_\tau \) at \( R_\theta = 1.5 \times 10^6 \) (where \( R_\theta \) is the Reynolds number based on boundary-layer momentum thickness), which is within the range of experimental values at very much lower Reynolds numbers. The range in values of \( \lambda_{\text{opt}}^+ \) found in figure 6 lies within the range of experimental values for average streak spacing reported by Zacksenhouse et al. (2001). The variation of optimal streak strength with Reynolds number is also plotted in

**Figure 5.** The variation of maximum non-dimensional streak strength for a moving vorticity source with the non-dimensional source convection speed. Parameters: \( z_f^+ = 0.09 \); \( \beta = 12.6 \), all other parameters are the same as given in figure legend 1.
Figure 6. Non-dimensional optimum spanwise wavelength (streak spacing) and non-dimensional maximum streak strength as a function of Reynolds number. Open diamonds, \( \lambda_{opt}^+ \); open circles, \( u_{\text{max}}^+/U_r \). Other parameters are as the same as given in figure legend 1.

In both cases, it should be remembered that the theoretical predictions are based on a fixed value of \( t_f^+ \). There is experimental evidence that the bursting period (in wall units) rises with Reynolds number (Rao et al. 1971).

Results for an adverse and a favourable streamwise pressure gradient are also plotted in Figure 4. It can be seen that the optimum streak spacing varies little with pressure gradient, although the streak strength is considerably less in the favourable pressure gradient than in the adverse one. This behaviour is consistent with the limited experimental evidence. For instance, Kline et al. (1967) stated that streaks were more likely to burst in adverse pressure gradients and more quiescent in favourable ones. More quantitatively, Fernholz & Warnack (1998) found that the velocity and shear stress fluctuations in a favourable pressure gradient were 10% below those for zero pressure gradient.

We turn now to the turbulent boundary layer over a compliant wall. This is a good illustration of the predictive capability of our model of the sublayer streaks. We use the plate–spring theoretical model of a compliant wall introduced by Carpenter & Garrad (1985). In this case, the compliant wall is represented by a thin stiff plate (characterized by a flexural rigidity, \( B \), density, \( \rho_m \), and plate thickness, \( b \)) supported on a continuous spring foundation (characterized by a spring stiffness, \( K \)). The equation of motion for this type of wall is linked to equations (2.1)–(2.3) by requiring continuity of velocity at the wall through the conditions (2.4), (2.6) and (2.7). Continuity of tractive force is also required at the wall. In the present case, this is equivalent to requiring continuity of pressure because with the plate–spring model, the wall is only free to move in the wall-normal direction.

Lee et al. (1993) have carried out an experimental study comparing the characteristics of the streaks in turbulent boundary layers over rigid and compliant walls. Figure 7 compares the variation according to our theoretical

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model of optimal streak spacing as a function of Reynolds number with the experimental statistically averaged values measured by Lee et al. (1993). Also plotted are the theoretical results corresponding to the Kramer-type compliant wall studied by Carpenter & Garrad (1985); such a wall has a good laminar-flow capability. It can be seen that reasonably good agreement is found between the theoretical model and the experimental data of Lee et al. (1993). This is possibly partly due to chance because the compliant wall in the experiments consisted of a single homogeneous viscoelastic layer and the plate–spring parameters chosen to model this compliant wall were based on the guidance provided by Duncan (1986). Nevertheless, it could at best be only an approximate model. The values of the source parameters $a$ and $b$ selected were the same as those of Lockerby et al. (2005) and different from those used earlier in this paper. This is the reason for the differences between figure 7 and figures 4 and 5. At the very least, figure 7 shows that our theoretical model is capable of predicting the correct trends for the effects of wall compliance. It also predicts that the streak strength greatly decreases as the wall compliance rises. This is also in line with the experimental observations of Lee et al. (1993). Further details can be found in Ali & Carpenter (2005).

4. Modelling the quasi-periodic bursting process

Several previous authors (Hamilton et al. 1995; Jiménez & Pinelli 1999; Schoppa & Hussain 2002; Kim 2005) have proposed theoretical models for the self-sustaining regenerative process for the bursting cycle. Typically, three stages are envisaged:
(i) streamwise vortices that generate, (ii) streamwise streaks that then break down via some sort of instability or transient growth mechanism, and (iii) generate streamwise-dependent three-dimensional disturbances that in turn generate the streamwise vortices. Our approach to understanding and modelling the near-wall dynamics of turbulent wall layers is also firmly grounded in hydrodynamic stability theory and associated concepts on receptivity and transient growth. Our deterministic model of the bursting cycle is based on the following deterministic elements of the near-wall region:

(i) plane propagating Tollmien–Schlichting-like waves,
(ii) alternating high- and low-speed sublayer streaks,
(iii) low-frequency, predominantly streamwise vortex elements,
(iv) oblique propagating TS-like waves, and
(v) other higher-frequency vortical elements.

In our model, the streamwise vortices (iii) generate the streaks (ii). At the same time, other vortical elements (v) generate the plane waves (i). The plane waves interact nonlinearly with the streaks to generate oblique waves (iv). This results in algebraic growth of the waves (Sen et al. 2006). There is probably also algebraic growth of the streaks due to this interaction, but we have not yet analysed this. The oblique and plane waves will also interact nonlinearly to generate streamwise vortices. We thus have the basis for the regeneration of the streamwise vortices and the plane waves.

It used to be conventional wisdom that turbulent boundary layers do not support propagating waves. This view stemmed from the theoretical and experimental study of Hussain & Reynolds (1972) and Reynolds & Hussain (1972). However, around the same time as this study, Morrison et al. (1971) found experimental evidence of the existence of wave-like structures in the sublayer region of fully developed pipe flow. Much later, Sirovich et al. (1990, 1991) and Zhou & Sirovich (1992) carried out analyses of direct numerical simulations based on proper orthogonal decomposition, showing that a significant component of the near-wall region in turbulent plane channel flow consists of plane and oblique propagating waves. Moreover, the phase speeds $c_p$ and $c_o$ of the plane and oblique waves, respectively, are related by

$$c_o = c_p \cos \theta,$$

where $\theta$ is the angle of propagation of the oblique waves. This indicates that the conditions exist for strong interaction between the plane and oblique waves. Further analysis shows that such interactions are associated with the bursting events and the most significant modes propagate at 65° to the streamwise direction. One feature of their results that puzzled Sirovich et al. was that the critical layer based on the linear instability of the turbulent mean velocity profile should be located at a height of $56\nu/U_\tau$ above the wall and well removed from the buffer layer. This apparent anomaly has been explained by the theoretical and experimental research of the IIT Delhi team (Sen & Veeravalli 1998, 2000a,b; Sen et al. 2006). Briefly, they recognized that the near-wall turbulence environment is highly anisotropic. Accordingly, they used an anisotropic eddy-viscosity model in their stability analysis of the mean turbulent profile. This crucial departure from the analysis of Reynolds & Hussain (1972) had profound implications.

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Unlike Reynolds & Hussain, Sen & Veeravalli found that the turbulent boundary layer could support growing propagating waves. Moreover, the critical layer was found to be located at a height of approximately $18 \nu / U_t$, nearly consistent with the POD analysis of Sirovich et al. More tellingly, Sen & Veeravalli obtained a good agreement between their theory and experiments on driven waves in turbulent channel flow. Among other things, a close agreement was found between the theoretical and experimental eigenfunctions (disturbance velocity profiles). The experimental study also confirmed the existence of growing plane waves.

Some preliminary calculations of the nonlinear interaction between the plane TS-like wave and the streaks were presented by Sen et al. (2006) based on stability theory. They wrote the flow perturbation, $\phi = [u, v, w, p]$ in the form

$$\phi = \phi_s + \phi_p + \phi_o,$$

(4.2)

where

$$\phi_s = [\hat{u}_s(z) \cos \beta y, \hat{v}_s(z) \sin \beta y, \hat{w}_s(z) \cos \beta y, \hat{p}_s(z) \cos \beta y],$$

(4.3)

corresponds to the sublayer streaks;

$$\phi_p = [\hat{u}_p(z), \hat{v}_p(z), \hat{w}_p(z), \hat{p}_p(z)]e^{i(\alpha_p x - \omega t)},$$

(4.4)

corresponds to the plane waves; and

$$\phi_o = [\hat{u}_o(z) \cos \beta y, \hat{v}_o(z) \sin \beta y, \hat{w}_o(z) \cos \beta y, \hat{p}_o(z) \cos \beta y]e^{i(\alpha_o x - \omega t)},$$

(4.5)

corresponds to the oblique waves generated by the nonlinear interaction between $\phi_s$ and $\phi_p$. In equations (4.4) and (4.5), $\alpha_p$ and $\alpha_o$ are the non-dimensional streamwise wavenumbers for the plane and oblique waves, respectively, and $\omega$ is their common frequency. It can be shown (Sen et al. 2006) that the governing equation for $\hat{w}_o$ takes the form

$$L(\alpha_o) \hat{w}_o = R_f e^{i(\alpha_p - \alpha_o) x},$$

(4.6)

where $L$ is an extended Orr–Sommerfeld-type operator. It can also be shown that for weak streaks, the interaction results only in a correction to the eigenfunction $\alpha_o$ of the form

$$\Delta \alpha_o \propto e^{i(\alpha_p - \alpha_o) x}.$$

(4.7)

The difference in the real parts of the wavenumbers contributes to a change in phase, while the difference in the imaginary parts leads to alternating amplification and attenuation depending on sign. This corresponds to the intermittent bursting observed by Fasel (2002) in his numerical simulations of the interaction between plane Tollmien–Schlichting waves and Klebanoff modes in a laminar boundary layer. For stronger streaks, the behaviour is qualitatively different because explosive algebraic growth occurs.

Preliminary numerical calculations in Sen et al. (2006) demonstrate that both scenarios are possible and suggest that algebraic growth should start to dominate when the streak perturbation velocity exceeds a threshold value that is less than $0.1 U_\infty$. In both cases, at $R=5000$, there is a low-frequency modulation having a period of approximately $36 \nu / U_t^2$. According to our theoretical model,
this corresponds to the period of the bursting cycle. It is very close to the value of 36.6 measured by Klewicki et al. (1995) for a boundary layer at very large Reynolds number and of the same order of magnitude as the value of 80.5 found by Rao et al. (1971) and the value of 72 measured in the original facility of Kline et al. (1967); as quoted by Rao et al. (1971). It is also of the same order of magnitude as the value of 80.5 found by Rao et al. (1971) and the value of 72 measured in the original facility of Kline et al. (1967); as quoted by Rao et al. (1971). It is also of the same order of magnitude as the value of 60 obtained by Kim & Spalart (1987) in their direct numerical simulations. Probably, coincidentally, the maximum in figure 3 is also close to 36 when our value of $t_{f}^{+} = 15$ is used. Furthermore, in order for the relation (4.1) to be satisfied to give a strong interaction between the oblique and plane waves, our preliminary analysis shows that it is necessary for the spanwise wavenumber, $\beta$, to correspond to a wavelength of approximately $100\nu/U_{\tau}$. This implies that the interaction will generate streamwise vortices with a characteristic spanwise spacing of the order of 100 wall units. Thus, we have here another mechanism for selecting the streak spacing observed in experiments. It is also noteworthy that the angle of propagation, given by $\theta = \arctan(\alpha_{o}/\beta)$, for the oblique waves in our theoretical model is approximately $60^\circ$, close to the angle observed by Sirovich et al. (1990, 1991). Our model also provides an explanation for their observation of an apparent anomaly regarding the location of the critical layer; it is shown in Sen et al. (2006) that both the theoretical and experimental location for the critical layer is close to $z^{+} = 18$, which is reasonably in accordance with the observations of Sirovich et al. (1990, 1991).

To sum up, our theoretical model provides a rational basis for both the selection of bursting period (and hence the choice of $t_{f}^{+}$) and the spanwise spacing of the streaks. However, a much more detailed study is required before this can be fully confirmed.

5. Conclusions

We present first a relatively simple, deterministic, theoretical model for the sublayer streaks in a turbulent boundary layer based on an analogy with Klebanoff modes. Our approach is to generate the streamwise vortices found in the buffer layer by means of a vorticity source in the form of a fictitious body force. Provided the source is active only for a fixed time of $t_{f}^{+} = 15$, we show that the strongest streaks correspond to a spanwise wavelength of approximately $78\nu/U_{\tau}$ that lies within the range of the experimentally observed values for the statistical mean streak spacing. We also present results for sublayer streaks in adverse and favourable pressure gradients. In accordance with experimental observation, we find that the streamwise pressure gradient has little effect on the streak spacing, but the streaks are stronger in adverse pressure gradients than in favourable ones. We modify the model so that the vorticity source is moving. In agreement with empirical evidence, we find that the strongest streaks correspond to a source moving at a speed equal to the local mean velocity. We also investigated the effect of Reynolds number on the streaks and found a weak variation with the non-dimensional streak spacing increasing and the associated non-dimensional perturbation velocity decreasing with a rise in Reynolds number. Finally, we showed that our theoretical predictions for the effects of wall compliance on the streak characteristics agreed well with experimental data.

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We also described our proposed theoretical model for the quasi-periodic bursting cycle that places the streak modelling in context. We envisage this cycle as being initiated by streamwise vortices and other vortical elements that generate respectively the sublayer streaks and plane Tollmien–Schlichting-like waves propagating in the near-wall region. When the sublayer streaks grow to a sufficient magnitude, they interact nonlinearly with the plane waves to form oblique waves. Depending on the strength of the interaction, either intermittent growth is generated or strong transient growth results from the interaction. In both cases, the interaction is characterized by a low-frequency modulation, with a period similar in magnitude to the experimentally observed bursting period. Thus, either scenario could be a model for the bursting process, but we prefer the strong interaction as the candidate. The bursting cycle is closed by the nonlinear interaction between the oblique and plane waves to generate the streamwise vortices. These in turn generate the sublayer streaks anew.

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