Experimental evidence of phase coherence of magnetohydrodynamic turbulence in the solar wind: GEOTAIL satellite data

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Magnetohydrodynamic (MHD) turbulence is commonly observed in the solar wind. Nonlinear interactions among MHD waves are likely to produce finite correlation of the wave phases. For discussions of various transport processes of energetic particles, it is fundamentally important to determine whether the wave phases are randomly distributed (as assumed in the quasi-linear theory) or have a finite coherence. Using a method based on the surrogate data technique, we analysed the GEOTAIL magnetic field data to evaluate the phase coherence in MHD turbulence in the Earth’s foreshock region. The results demonstrate the existence of finite phase correlation, indicating that nonlinear wave–wave interactions are in progress.

Keywords: magnetohydrodynamic turbulence; nonlinear interactions; phase coherence; solar wind

1. Introduction

The solar wind, a hot supersonic and super-Alfvénic plasma flow streaming from the Sun, provides an ideal environment for the study of nonlinear plasma phenomena. A wealth of nonlinear phenomena can be found in the solar wind, especially in the vicinities of the planetary bow shock and interplanetary shocks wherein the magnetic, electric and velocity fields show turbulent fluctuations. In particular, the low-frequency magnetohydrodynamic (MHD) or Alfvénic turbulence (Chian et al. 1998, 2006, 2007; Goldstein & Roberts 1999; Rempel & Chian 2005) is commonly found in the solar wind. Such MHD turbulence is considered to play crucial roles in heating of plasma and acceleration of energetic particles.

Transport of energetic particles by an MHD turbulent field has been analysed mainly in terms of the quasi-linear theory (Sagdeev & Galeev 1969). Two major
assumptions are integral to the theory. The first is that the amplitudes of the magnetic field perturbations are sufficiently small, so that truncation of terms at the second power of the wave amplitude is possible. The second assumption is the so-called random phase approximation, which excludes any effect of wave coherence due to random phase mixing. However, the MHD turbulence in space does not necessarily satisfy these assumptions. Such MHD turbulence often has amplitude comparable to or larger than the ambient magnetic field, and its waveforms are not likely to be stochastic. For instance, the so-called shocklets, commonly found in the upstream region of the Earth’s bow shock (Hoppe et al. 1981) and the bow shocks of other planets (Fairfield & Behannon 1976; Hoppe & Russell 1981) and near comets (Tsurutani 1991), have a magnetic field amplitude comparable to or even a few times larger than the average local magnetic field and nonlinear waveform. They are frequently accompanied by monochromatic whistler wave trains, which presumably are reminiscent of soliton trains. The shocklets have been shown to be the consequence of nonlinear evolution of obliquely propagating nearly monochromatic waves by numerical simulation studies (Hada et al. 1987; Omidi & Winske 1990).

Another typical example suggesting that the two central assumptions made in the quasi-linear theories may be violated is the short large-amplitude magnetic structures, detected upstream of quasi-parallel shock waves (Schwartz & Burgess 1991; Schwartz et al. 1992). They are short-duration, ultra-low-frequency waves (approx. 10 s), characterized by a well-defined single magnetic structure. Similar structures are reproduced in numerical simulations (Akimoto et al. 1991; Scholer 1993). The possibility of non-classical diffusion of charged particles by coherent large-amplitude MHD waves in terms of numerical simulations has been suggested (Kirk et al. 1996; Kuramitsu & Hada 2000). From the viewpoint of the aforementioned observations and numerical simulations, we expect that in MHD turbulence there exists a state in which the quasi-linear theory is no longer valid.

The purpose of this paper is to discuss the experimental evidence of nonlinear wave–wave interactions in MHD turbulence detected in the solar wind by the GEOTAIL satellite shown in figure 1, with emphasis on the calculation of the phase coherence in the magnetic field data measured by the magnetometers. The GEOTAIL mission is a collaborative project undertaken by the Institute of Space and Astronautical Science (ISAS) in Japan and the National Aeronautics and Space Administration (NASA) in the USA. Its primary objective is to study the dynamics of the Earth’s magnetotail over a wide range of distance, extending from the near-Earth region to the distant tail. We focus on the GEOTAIL data collected when the satellite travelled from the upstream region of the Earth’s bow shock towards the magnetopause from 18.00 UT 8 October to 06.00 UT 9 October 1995, as shown in figure 2.

We also discuss a method to evaluate the phase coherence in MHD turbulence quantitatively in §2. By applying this method, we evaluate the phase coherence in MHD turbulence using the magnetic field time-series observed by the GEOTAIL satellite in §3. Finally, we summarize the results in §4.

### 2. Phase coherence index

When we attempt to obtain phase information from data, the Fourier transform has traditionally been the starting point for this purpose. This transformation of
A time series $x(t)$ is defined as

$$\hat{X}(\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t} \, dt,$$  \hspace{1cm} (2.1)\]

where $\omega$ indicates the angular frequency, which provides the information of amplitude $|\hat{X}(\omega)|$ and phase distribution $\phi(\omega) = \tan^{-1}(\text{Im}(\hat{X}(\omega))/\text{Re}(\hat{X}(\omega)))$. An example is given in figure 3. In space plasma physics research, the amplitude (power spectrum) has been discussed in the literature over many years, for example the classifications of geomagnetic pulsations (Saito 1969) and the power-law type spectrum of magnetic field turbulence in the solar wind (Goldstein & Roberts 1999) and in geomagnetic activities (Tsurutani et al. 1990). The phase distribution $\phi(\omega)$, on the other hand, has not received much attention in space plasma applications. A possible reason may be that the phase distribution in Fourier space appears to be almost completely random, as seen in figure 3c.

Furthermore, the distribution of the wave phase depends on the choice of the coordinate origin, which is arbitrary. We show two solitary waveforms in figure 4a, b and their phases in Fourier space in figure 4c, d. These waves are exactly the same except that they are shifted differently in the horizontal direction. The distribution of phases for figure 4a is coherent at low frequency. On the other hand, when the shift
Figure 2. The orbit of the GEOTAIL satellite from 18.00 UT 8 October to 06.00 UT 9 October in 1995 (thick solid line). The thin solid curves correspond to the nominal location of the bow shock and the magnetopause, respectively (Fairfield 1971).

Figure 3. Fourier transform of a time-series: (a) the time-series (magnetic field), (b) the power spectrum and (c) the phase distribution.
is 2404 sampling periods, which is an arbitrary number, then the phase distribution appears to be almost completely random, as shown in figure 4b (note the periodic boundary conditions). This may be the reason why the phase information has been overlooked in the past. In order to avoid the influence of the choice of the coordinate origin, we need to depart from the estimation in the Fourier space. To this end, let us turn our attention to the waveform in real space.

Hada et al. (2003) and Koga & Hada (2003) introduced a method to evaluate the degree of phase coherence among the Fourier modes quantitatively. Here, we explain the method in detail. Suppose we have a sequence of data, \( x(t) \), for instance a measurement of magnetic field by spacecraft in the solar wind. From the original data (ORG), we can make two surrogate data (see figure 5). Firstly, we decompose the original data into the power spectrum and the phases using the Fourier transform. We then randomly shuffle the phases, but keep the power spectrum unchanged, and, from these two pieces of information in the Fourier space, we perform the inverse Fourier transform to create the phase-randomized surrogate (PRS). Likewise, we can make the phase-correlated surrogate (PCS), in which the phases are all made equal. The three data, ORG, PRS and PCS, share exactly the same power spectrum, while their phase distributions are all different. We note that the calculation of PRS is performed using the average over 100 realizations of the phase shuffling in order to reduce the uncertainty.

The distribution of phases of the ORG data looks almost as random as that of the PRS in figure 5, due to the arbitrary choice of the coordinate origin. However, we can characterize the difference in the phase distribution by the difference in the waveform in real space, instead of the Fourier space. Let us define the path length as,

\[
L(\tau) = \sum_t |x(t + \tau) - x(t)|,
\]

\[2.2\]
where $\tau$ is a measure characterizing the coarse-graining of the data. When the phases are correlated, the path length of the data tends to be shorter than the case when the phases are random. Using the path length, we can evaluate the degree of phase coherence as the difference of geometrical characteristics of each set of data, without being influenced by the coordinate origin. Figure 6 shows the path length of each set of data (ORG, PRS and PCS). In general, the path length of PRS is longer than that of PCS, and they quantify the extreme values of the path length. To evaluate the degree of phase coherence, we therefore define the phase coherence index,

$$C_{\phi}(\tau) = \frac{L_{\text{PRS}}(\tau) - L_{\text{ORG}}(\tau)}{L_{\text{PRS}}(\tau) - L_{\text{PCS}}(\tau)}. \quad (2.3)$$

If the original data have random phase, $C_{\phi}$ should be approximately 0, whereas $C_{\phi}=1$ if the phases are completely correlated. The phase coherence index $C_{\phi}$ is shown in figure 6 (right axis).

### 3. Applications

In this section, we evaluate the phase coherence in MHD turbulence using the GEOTAIL 16 Hz magnetic energy data observed from 18.00 UT 8 October to 06.00 UT 9 October 1995. During this period, the GEOTAIL was approaching the Earth’s bow shock from far upstream, passing through the shock and entering into the magnetosphere. We separated the entire period into 37 datasets, each with 16,384 samples and about 17 min duration. First of all, we examined the evolution...
of $C_f$ for the whole sequence of the GEOTAIL data where $C_f$ is evaluated only for $\tau=1$ s in order to compare the results for different datasets. The results are shown in figure 7; the blank in the figure indicates a data gap. As the GEOTAIL approaches the Earth’s bow shock, the magnetic energy (turbulence level) gradually increases, becomes large inside the magnetosheath, and decreases again as the GEOTAIL enters into the magnetosphere. The evolution of $C_f$ approximately follows the evolution of the magnetic field turbulence level. This is a natural consequence of the phase coherence generated by nonlinear wave–wave interactions among finite-amplitude MHD waves of different scales.

The profile of $C_f$ as a function of $\tau$ for the time indicated by the arrow in figure 7 in the upstream region of the Earth’s bow shock is shown in figure 8. The value of $C_f$ increases as $\tau$ decreases from $\tau \sim 10$ to 1 s. The range where the $C_f$ increases corresponds, approximately, to a frequency range of approximately $0.1\Omega_i - \Omega_i$, where $\Omega_i$ denotes the local ion-cyclotron frequency evaluated in the upstream region (approx. 1 Hz). Figure 9 shows the local intermittency measure $I(a, \tau)$ for the same data which Farge (1992) introduced in order to extract non-stationary or intermittent events using a continuous wavelet transform $W(a, \tau)$,

$$W(a, \tau) = \frac{1}{\sqrt{a}} \int f(t) \psi^* \left( \frac{t-\tau}{a} \right) dt, \quad \text{and} \quad (3.1)$$

$$I(a, \tau) = \frac{|W(a, \tau)|^2}{\langle |W(a, \tau)|^2 \rangle_\tau}, \quad (3.2)$$

where the asterisk indicates the complex conjugate and $a$ and $\tau$ show the wavelet scale and time, respectively. The brackets denote the ensemble average of $W(a, \tau)$ for a scale $a$. Figure 9 shows that there exist several localized intermittent energy concentrations within the characteristic band from

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approximately 1 to 10 s. Indeed, if one excludes these intermittent energy concentrations from the data, then the $C_f$ index decreases considerably. Thus, these results prove that the finite phase coherence is the origin of the intermittent energy bursts (He & Chian 2003; Rempel & Chian 2007).

Figure 10 shows a scenario for the nonlinear evolution of MHD turbulence in the upstream region of the Earth’s bow shock. In the region, ion beams originating from the bow shock are frequently observed (Sonnerup 1969;
Once ion beams are injected into the foreshock region, quasi-monochromatic low-frequency MHD waves (in this study approx. 0.1 $U_i$) are excited due to ion-beam instabilities (Fairfield 1969; Gary et al. 1984; Narita et al. 2003). Although excited waves propagate against the solar wind flow, they are convected back to the shock because their phase speed is much less than the solar wind speed. During the course, they can evolve nonlinearly. Indeed, such nonlinear evolution of MHD waves has been reported in several observations (Hoppe et al. 1981; Schwartz et al. 1992) and numerical simulations (Hada et al. 1987; Omidi & Winske 1990; Akimoto et al. 1991; Scholer 1993). Therefore, these results indicate that such nonlinear evolution of MHD turbulence and its high $C_f$ region are related to each other. The decrease of the $C_f$ index below 1 s (approx. $\Omega_i$) in figure 8 implies that nonlinear interactions among MHD waves become weak due to energy dissipation processes such as Landau damping.

4. Summary

In this study, the phase correlation in MHD turbulence observed by the GEOTAIL satellite near the Earth’s bow shock was evaluated. We found that there exists finite phase coherence ($C_\phi > 0$) in MHD turbulence in the upstream region of the Earth’s bow shock. The value of $C_\phi$ at $\tau = 1$ s is approximately 0.5, but sometimes it can be as large as approximately 1.0 near the shock. Furthermore, we found that the phase coherence is mainly generated in the characteristic frequency band $0.1 \Omega_i \leq \omega \leq \Omega_i$ in the upstream region.

We would like to point out that the assumptions used traditionally in the quasi-linear theory, i.e. small-amplitude and random phase approximation, are not valid for the MHD turbulence in the Earth’s foreshock region. Since...
nonlinear interactions among MHD waves lead to coherent and intermittent fields therein, we expect that the non-classical particle transport transcending the quasi-linear theory should be considered, as reported by Kirk et al. (1996) and Kuramitsu & Hada (2000).

The analytical signal concept (Rosenblum et al. 1996) provides a powerful tool to analyse nonlinear signals. The instantaneous amplitude and phase have a clear physical meaning only if a signal is narrowband, whereas turbulence such as the GEOTAIL data analysed in our paper is broadband consisting of a wide range of scales. In order to investigate nonlinear wave–wave interactions, we need to decompose the broadband turbulent signal. From this point of view, the phase information obtained from the Fourier transform is still convenient for our analysis because it permits the wave number/frequency decomposition, albeit we have to implicitly assume weak nonlinearity. We plan to apply a combination of wavelet transform and Hilbert transform (Le Van Quyen et al. 2001) or the Hilbert–Huang transform (Huang et al. 1998) in our future analysis of turbulence data.

This research is supported by the Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), Brazil (04/15942-2) and CNPq. The GEOTAIL magnetic field (MGF) data were provided by Prof. S. Kokubun and Prof. T. Nagai. The authors are grateful to ISAS/JAXA in Japan for their permission to use a figure of the GEOTAIL.

References


Phil. Trans. R. Soc. A (2008)


Phil. Trans. R. Soc. A (2008)