Localization of intense electromagnetic waves in plasmas

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We present theoretical and numerical studies of the interaction between relativistically intense laser light and a two-temperature plasma consisting of one relativistically hot and one cold component of electrons. Such plasmas are frequently encountered in intense laser–plasma experiments where collisionless heating via Raman instabilities leads to a high-energetic tail in the electron distribution function. The electromagnetic waves (EMWs) are governed by the Maxwell equations, and the plasma is governed by the relativistic Vlasov and hydrodynamic equations. Owing to the interaction between the laser light and the plasma, we can have trapping of electrons in the intense wakefield of the laser pulse and the formation of relativistic electron holes (REHs) in which laser light is trapped. Such electron holes are characterized by a non-Maxwellian distribution of electrons where we have trapped and free electron populations. We present a model for the interaction between laser light and REHs, and computer simulations that show the stability and dynamics of the coupled electron hole and EMW envelopes.

Keywords: electromagnetic waves; Maxwell’s equations; Vlasov equation

1. Introduction

The development of high-power lasers has made it possible to study the ultra-relativistic regime of laser–plasma interactions. This has a profound importance for laser-induced heating and inertial confinement fusion (Kodama et al. 2001, 2002), the production of GeV electrons and mono-energetic electron beams by wakefield acceleration (Bingham 2003; Faure et al. 2004; Geddes et al. 2004; Mangles et al. 2004; Tsung et al. 2004; Hogan et al. 2005), and gives the possibility to test fundamental physics where the intensity of the electromagnetic field approaches the Schwinger limit (Bulanov et al. 2003; Marklund & Shukla 2006). Extremely short and intense laser pulses are also studied experimentally in the framework of nonlinear optics (Brabec & Krausz 2000). The first theoretical studies of relativistic laser pulses in plasmas date back to the 1950s (Akhiezer & Polovin 1956), when it was recognized that the electrons may gain relativistic mass due to their quivering velocity in an ultra-intense electromagnetic wave (EMW). The instability of relativistically large amplitude EMWs has been studied for magnetized electron plasmas (Stenflo 1976, 1981), electron
beam plasma systems (Tsintsadze & Stenflo 1974), hot electron-ion plasmas (Tsintsadze et al. 1979) and magnetized hot plasmas (Tsintsadze et al. 1980; Shukla et al. 1986). The instability of relativistically strong laser light in an unmagnetized plasma has recently been revisited (McKinstrie & Bingham 1992; Sakharov & Kirsanov 1994; Guérin et al. 1995; Quesnel et al. 1997; Barr et al. 1999), where it was recognized that relativistic effects can decrease the growth rate of the instabilities (Decker et al. 1996). The saturation of the modulational instability is the formation of localized solitary waves in an overdense plasma. Analytic expressions for localized EMW envelopes were derived for linearly polarized EMWs in uniform and non-uniform plasmas (Kaw & Dawson 1970; Max & Perkins 1971), and a class of exact analytic expressions were derived for circularly polarized, standing solitary and periodic EMWs in a cold plasma (Marburger & Tooper 1975). For laser pulses propagating in underdense plasmas, there exist a class of solitary EMWs that are treated as a nonlinear eigenvalue problem that has a point spectrum (Kaw et al. 1992; Kuehl & Zhang 1993; Sudan et al. 1997; Saxena et al. 2006) for different excited eigenstates of localized EMW envelopes. In one dimension, explicit expressions for the wake created by a rectangular laser pulse have been derived (Berezhiani & Murusidze 1990). Theoretical and simulation studies have demonstrated the acceleration of electrons by the plasma wakefield behind short laser pulses (Tajima & Dawson 1979), where it was recognized that the lifetime of the laser pulse was limited by the Raman forward scattering instability, and the erosion and self-steepening of the front of ultra-relativistic laser pulses (Decker et al. 1996). Another acceleration mechanism of electrons is the reflection by the front of a laser pulse itself via the ponderomotive force (Liu & Tripathi 2005). Simulation studies of the Raman backward and forward scattering have shown efficient electron acceleration (Bertrand et al. 1995) as well as photon cascade and condensation (Mima et al. 2001) and the formation of large-amplitude electromagnetic spikes of backscattered light (Shvets et al. 1997) in underdense plasmas. The stimulated processes and self-modulation of short laser pulses in a three-dimensional axisymmetric geometry have been studied theoretically and numerically (Andreev et al. 1995). For long laser pulses, the dominating instability is the backward Raman scattering instability that may lead to the acceleration and heating of electrons in the plasma and to the reflection of laser light against electrostatic waves and kinetic structures involving electron phase space holes (Montgomery et al. 2002; Nikolic et al. 2002; Hur et al. 2005). Both in laboratory (Jones et al. 1975) and space plasmas (Ergun et al. 2001), one frequently encounters electron distributions that can be described as a superposition of populations with two distinct temperatures. The nonlinear interactions between intense EMWs and two-component electron plasmas have been shown to give rise to new classes of parametric instabilities (Shukla & Hellberg 2002) and coherent EMW structures (Shukla et al. 2002). A review of theoretical and simulation studies of localized EMWs is given by Farina & Bulanov (2005), and of various plasma-based particle acceleration mechanisms is given by Bingham et al. (2004).

In this paper, we consider the nonlinear interactions between intense EMWs and plasma containing two distinct groups of electrons: one low-energy component that is described by the relativistic electron hydrodynamic equations, and one high-energy hot electron component that is modelled by the relativistic

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Vlasov equation. The EMWs are coupled nonlinearly with the plasma, supported by the cold and hot electron components, via the relativistic ponderomotive force and the Poisson equation. Owing to a combination of relativistic mass increase and density depletion, relativistic electron holes (REHs) can also work as a resonance cavity that can trap EMWs (Shukla & Eliasson 2005; Eliasson & Shukla 2006). We study steady-state solutions of the governing nonlinear system of equations in order to understand the interplay between the cold and hot electron components on the properties of intense electromagnetic envelopes coupled with large-amplitude REHs. The stability of the coupled REH–EMW envelope solitons is investigated by means of direct simulations of the Vlasov–Maxwell system of equations.

2. The mathematical model

Let us present the relevant nonlinear equations describing the interaction between intense EMWs and a collisionless plasma consisting of hot and cold electron components. The EMW localization will be assumed to occur on a short time scale, in which the ions do not have time to respond to the electromagnetic and electrostatic fields. The equilibrium quasi-neutrality condition yields \( n_{i0} = n_{eh0} + n_{ec0} \equiv n_{e0} \), where \( n_{j0} \) is the unperturbed particle density of the species \( j \) ( \( j = i \) for the ions, \( e \) for the electrons, and \( eh \) and \( ec \) for the hot and cold electron components, respectively). The EMWs give rise to relativistic electron mass increase and produce electron density perturbations by the relativistic ponderomotive force

\[
F = -m_e c^2 \gamma \frac{\partial \phi}{\partial z},
\]

where \( m_e \) is the electron rest mass, \( c \) is the light speed and \( \gamma = \left( 1 + \frac{p_z^2}{m_e^2 c^2} + \frac{e^2 |A|^2}{m_e^2 c^4} \right)^{1/2} \) is the relativistic gamma factor. Here, \( p_z \) is the \( z \) component of the electron momentum, \( A \) is the perpendicular (to \( \hat{z} \), where \( \hat{z} \) is the unit-vector along the \( z \) direction in a Cartesian coordinate system) component of the vector potential of the circularly polarized EMWs and \( e \) is the magnitude of the electron charge. The dynamics is governed by the wave equation for the EMW, the relativistic Vlasov equation for the hot electrons, and the continuity and relativistic momentum equations for the cold electrons, which in normalized units take the form

\[
\frac{\partial^2 A}{\partial t^2} - \frac{\partial^2 A}{\partial z^2} + \Omega^2 A = 0, \tag{2.1}
\]

\[
\frac{\partial f_h}{\partial t} + \frac{p_z}{\gamma_h} \frac{\partial f_h}{\partial z} + \frac{\partial (\phi - \gamma_h)}{\partial z} \frac{\partial f_h}{\partial p_z} = 0, \tag{2.2}
\]

\[
\frac{\partial N_c}{\partial t} + \frac{\partial}{\partial z} (N_c V_{cz}) = 0, \quad V_{cz} = \frac{P_{cz}}{\gamma_c}, \tag{2.3}
\]

\[
\frac{\partial P_{cz}}{\partial t} + \frac{\partial \gamma_c}{\partial z} \frac{\partial P_{cz}}{\partial z} = \frac{\partial \phi}{\partial z}, \tag{2.4}
\]

together with the Poisson equation for the electrostatic potential

\[
\frac{\partial^2 \phi}{\partial z^2} = \mu \int_{-\infty}^{\infty} f_h \, dp_z + (1 - \mu) N_c - 1. \tag{2.5}
\]
Here $\Omega^2 = \mu \int_{-\infty}^{\infty} \left( \frac{f_h}{\gamma_h} \right) dp_z + (1 - \mu) N_c / \gamma_c$ represents the squared local electron plasma frequency that accounts for the electron density variation and the relativistic electron mass increase, $\mu = n_{\text{ch0}} / n_{\text{c0}}$, and $\alpha = V_{\text{Teh}} / c$ where $V_{\text{Teh}} = (T_{\text{eh}} / m_e)^{1/2}$ is the electron thermal speed, and $T_{\text{eh}}$ is the temperature of the relativistically hot electrons. The vector potential $\mathbf{A}$ is normalized by $m_e c^2 / e$, the scalar potential $\phi$ by $m_e c^2 / e$, the momenta $p_z$ and $P_{cz}$ by $m_e c$, the relativistic electron distribution function $f_h$ by $n_{\text{ch0}} / c$, the cold electron density $N_c$ by $n_{\text{c0}}$, the space $z$ by $c/\omega_{\text{pe}}$, and the time $t$ by $\omega_{\text{pe}}^{-1}$, where $\omega_{\text{pe}} = (4\pi n_{\text{c0}} e^2 / m_e)^{1/2}$ is the electron plasma frequency. The gamma factors for the hot and cold electrons are $\gamma_h = (1 + p_z^2 + |\mathbf{A}|^2)^{1/2}$ and $\gamma_c = (1 + P_{cz}^2 + |\mathbf{A}|^2)^{1/2}$, respectively. In equation (2.1), we have used the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$, and have excluded the longitudinal ($z$) component $\partial^2 \phi / \partial t \partial z = j_z$, where $j_z$ is the parallel electron current density, since it is equivalent to the Poisson equation (2.5) (Shukla & Eliasson 2005).

In order to investigate the properties of steady state, driven REHs that move with a constant speed $v_0$ relative to the observer (bulk plasma) frame, we assume that $N_c$, $P_{cz}$ and $\phi$ depend on $\xi = z - v_0 t$ only, where $v_0$ is the constant

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**Figure 1.** The profiles of the scalar potential, the EMW envelope, and the cold and hot electron number densities (i–iv) associated with a (a) cold plasma EMW soliton and (b) REH–EMW soliton with large-amplitude trapped EMWs and (c) REH–EMW soliton with small-amplitude trapped EMWs, propagating with the speed $v_0 = 0.4$. The total electron density is $N_e = \mu N_{\text{ch}} + (1 - \mu) N_{\text{c}}$. For the EMW soliton (a), we used $\mu = 0$ and $\lambda = 0.4592$, for the REH–EMW soliton (b), we used $\mu = 0.9$, $\alpha = 0.4$, $W_{\text{max}} = 1.0$ and $\lambda = 0.224$ and for the bare REH (c), we used $\mu = 0.9$, $\alpha = 0.4$, $W_{\text{max}} = 0.01$ and $\lambda = 0.0114$. Adapted with permission from Shukla & Eliasson (2006).

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For the EMW soliton (a), we used $\mu = 0$ and $\lambda = 0.4592$, for the REH–EMW soliton (b), we used $\mu = 0.9$, $\alpha = 0.4$, $W_{\text{max}} = 1.0$ and $\lambda = 0.224$ and for the bare REH (c), we used $\mu = 0.9$, $\alpha = 0.4$, $W_{\text{max}} = 0.01$ and $\lambda = 0.0114$. Adapted with permission from Shukla & Eliasson (2006).
propagation speed, and that \( f_h \) depends on \((\xi, p_z)\) so that \( \Omega \) is a function of \( \xi \) only. We insert the ansatz \( \mathbf{A} = (1/2) W(\xi)(\hat{x} + i\hat{y})\exp[-i(\omega_0 + \theta)t + i(k_0 + K)z] + \) complex conjugate into equation (2.1). By choosing \( K = \omega_0(v_0 - v_5) + v_0\theta \), all coefficients in the resulting equation become real-valued, and we can assume that

\[
\begin{align*}
\text{Figure 2. (a) The separatrices between trapped, reflected and free particles for a cold plasma EMW soliton are indicated with dashed lines, while the solid line shows the value of the functions } p_{\pm}(\phi, W). \text{ We used } v_0 = 0.4, \mu = 0 \text{ and } \lambda = 0.4592. \text{ The dotted line shows the cold fluid momentum } P_{cz}(W, \phi). \text{ (b) The separatrices between trapped and free electrons for a REH-EMW soliton, given by the function } p_z(\phi, W), \text{ are indicated with solid lines. We used the parameters } v_0 = 0.4, \mu = 0.9, \alpha = 0.4 \text{ and } \beta = -0.5. \text{ The EMW amplitude was } W_{\text{max}} = 1.0, \text{ giving } \lambda = 0.224. \text{ The dotted line shows the cold fluid momentum } P_{cz}(W, \phi). \text{ Adapted with permission from Eliasson & Shukla (2006).}
\end{align*}
\]
Figure 3. Phase space plots of the electron distribution function (a) and the modulus of the electromagnetic field (b) at times (i) $t=0$, (ii) $t=50$, (iii) $t=125$ and (iv) $t=162$. Parameters used in the initial conditions (a(i),b(i)) are $\alpha=0.4$, $\beta=-0.5$ and $\mu=1$. The left (smaller) REH has a speed of $v_0=0.28$ and the right (larger) hole $v_0=-0.12$. Adapted with permission from Shukla & Eliasson (2005).

$W$ is a real-valued function. Equation (2.1) then takes the form

$$\frac{d^2W}{d\xi^2} - \lambda W - \gamma_0^2(\Omega^2 - \Omega_p^2) W = 0,$$

(2.6)

where $\omega_0 = (\Omega_p^2 + k_0^2)^{1/2}$ is the EMW frequency, $\Omega_p^2 = \mu \int_{-\infty}^{\infty} \tilde{f}_0(p_z)/\sqrt{1 + p_z^2} dp_z + 1 - \mu = \mu K_0(\alpha^2)/K_1(\alpha^2) + 1 - \mu$ is the normalized (by $\omega_{pe}$) relativistic plasma frequency at equilibrium, $K_j$ is a modified Bessel function of second kind and $v_g = \partial \omega_0/\partial k_0 = k_0/\omega_0$ is the group speed. Here $\lambda = -\theta^2 - 2\omega_0 \theta + \gamma_0^2 \omega_0^2 (v_g^2 - \Omega_p^2)$ represents a nonlinear eigenvalue of equation (2.6), where we have denoted $\gamma_0 = 1/\sqrt{1-v_g^2}$. We restrict ourselves to the case $v_g = v_0$ so that $k_0 = \gamma_0 v_0 \Omega_p$, $\omega_0 = \gamma_0 \Omega_p$ and we have $K = v_0 \theta$ and the nonlinear frequency shift $\theta = -\omega_0 + (\omega_0^2 - \lambda)^{1/2}$. The gamma factors are now expressed as $\gamma_0 = (1 + p_z^2 + W^2)^{1/2}$ and $\gamma_c = (1 + P_{cz}^2 + W^2)^{1/2}$. The continuity and momentum equations (2.3) and (2.4) can be integrated once to yield $-v_0 N_c + N_c V_{cz} = -v_0$ and $-v_0 P_{cz} + \gamma_c - 1 - \phi = 0$, respectively, where we have used the boundary conditions $N_c = 1$, $P_{cz} = 0$, $|A| = 0$ and $\phi = 0$ at $|\xi| = \infty$. The cold particle number density is thus $N_c = v_0/(v_0 - V_{cz})$, where the fluid speed is $V_{cz} = P_{cz}/(1 + P_{cz}^2 + W^2)^{1/2}$, and the momentum is $P_{cz} = \gamma_0^2 v_0 (1 + \phi) - \sigma \gamma_0^2 (1 + \phi)^2 - 1 - W^2)^{1/2}$, where

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\( \sigma = 1 \ (\sigma = -1) \) for \( v_0 > 0 \ (v_0 < 0) \). Equation (2.2) takes the form
\[
\left( -v_0 + \frac{p_z}{\gamma_h} \right) \frac{\partial f_h}{\partial \xi} + \frac{d(\phi - \gamma_h)}{d\xi} \frac{\partial f_h}{\partial p_z} = 0, \tag{2.7}
\]
and the Poisson equation (2.5) takes the form
\[
\frac{d^2 \phi}{d\xi^2} = \mu \int_{-\infty}^{\infty} f_h \, dp_z + (1 - \mu) N_c - 1. \tag{2.8}
\]
Using the technique outlined by Shukla & Eliasson (2005), an exact solution of equation (2.7) can be constructed of the form
\[
f_h = \left\{
\begin{array}{ll}
  a_0 \exp\left[-\alpha^{-2}\left(\sqrt{1 + \tilde{p}_+^2(\mathcal{E}_h)} - 1\right)\right], & p > p_+,
  \\
  a_0 \exp\left[-\alpha^{-2}(\gamma_0 - 1 + \beta \mathcal{E}_h)\right], & p_- \leq p \leq p_+,
  \\
  a_0 \exp\left[-\alpha^{-2}\left(\sqrt{1 + \tilde{p}_-^2(\mathcal{E}_h)} - 1\right)\right], & p < p_-,
\end{array}
\right.
\tag{2.9}
\]
where \( \mathcal{E}_h = -v_0 p_z + \gamma_h - 1/\gamma_0 - \phi \) is the conserved energy of the electron trajectories of equation (2.7). The separatrices in momentum space between the free and trapped electrons are given by \( p_\pm = \gamma_0 v_0 (\gamma_0 \phi + 1) \pm \gamma_0 [(\gamma_0 \phi + 1)^2 - 1 - W_2]^1/2 \) and are obtained by setting \( \mathcal{E}_h = 0 \) and solving for \( p \). The distribution function for the trapped electrons (in the interval \( p_- < p < p_+ \)) is determined by the trapping parameter \( \beta \), which gives an excavated distribution.
function for negative values. We note that when \( \phi = W = 0 \) far away from the REH, then the integration limit \( p_+ = p_- \), and the integrated number density of the trapped electrons vanishes. The functions \( \tilde{p}_\pm(E_h) = \gamma_0 v_0(\gamma_0 E_h + 1) \pm \gamma_0[(\gamma_0 E_h + 1)^2 - 1]^{1/2} \) are chosen so that they take the value \( p \) where \( \phi = W = 0 \), and thus the electron distribution function (2.9) equals the Jüttner–Synge distribution function (de Groot 1980) \( \tilde{f}_0(p_z) = a_0 \exp[-(\sqrt{1 + p_z^2} - 1)/\alpha^2] \), far away from the REH, where the normalization constant is \( a_0 = \left\{ \int_{-\infty}^{\infty} \exp[-(\sqrt{1 + p_z^2} - 1)/\alpha^2] dp_z \right\}^{-1} = \exp(-\alpha^{-2})/2K_1(\alpha^{-2}) \). Integrating the distribution function for the trapped and free particles (2.9) over the momentum space, we obtain the total electron number density as a function of \( \phi \) and \( W \), which are calculated self-consistently by means of the Poisson and Schrödinger equations, respectively.

3. Numerical results

In figure 1, we have displayed the numerical solution of the coupled Schrödinger and Poisson system of equations (2.6) and (2.8). It shows the profiles of the potential \( \phi \), the EMW envelope \( W \) and the hot and cold electron number densities, both for the case with only cold electrons and for the case with 90% hot electrons and 10% cold electrons (\( \mu = 0.9 \)). Both EMW solitons in a cold plasma (Kaw et al. 1992) and trapped EMW envelopes in REHs (Shukla & Eliasson 2005) can have different levels of excited states where the ground state is associated with a symmetric, bell-shaped envelope, the first excited state is associated with an antisymmetric envelope with one maximum and one minimum, etc. We here consider an antisymmetric EMW envelope with one maximum and one minimum, and will discuss the similarities and differences between the cold plasma soliton and the REH–EMW soliton. Both the cold plasma soliton and REH–EMW soliton are associated with a large-amplitude positive potential, which can trap the electrons. In the case of only cold electrons, all electrons are free and we do not have any trapped electrons, but when a hot electron population is included, some of the thermal electrons will be trapped in the positive potential hump. The question remains whether these two types of solitary waves are fundamentally different or whether they are the same kind of localized EMWs, but in different parameter regimes. One difference between the two cases is that the electromagnetic envelope soliton involving only cold electrons has a point spectrum (Saxena et al. 2006), i.e. for a given speed of the EMW soliton there is one eigenvalue \( \lambda \) and one amplitude/shape of the EMW envelope (see figure 1a(i)–(iv)), while for the REH–EMW soliton there is a continuous range of values on \( \lambda \) depending on the amplitude of the EMW envelope (see figure 1b(i)–(iv),c(i)–(iv)). Another difference is that the electron trajectories have a different topology for the two cases, which is illustrated by following the trajectory for a test electron in phase space. The relativistic equation of motion for the test electron has the conserved energy integral \( -v_0 p_z + \gamma - 1 - \phi = \mathcal{E} \), which for \( \mathcal{E} = 0 \) gives the electron momentum that equals the cold fluid momentum, \( p_z = P_{ez} \), and for \( \mathcal{E} = 1/\gamma_0 - 1 \) gives the separatrix \( p = p_\pm \) between trapped and free electrons in the REH solution discussed above. In figure 2a, we have assigned different values of \( \mathcal{E} \) for an EMW soliton in a cold plasma. The dotted curve illustrates a test electron with the same energy (\( \mathcal{E} = 0 \)) as for the cold fluid.
particles, yielding $p_z = P_{cz}$. The solid line represents the separatrix between the trapped and free electrons in a REH for which $\mathcal{E} = 1/\gamma_0 - 1 \approx -0.0835$. Finally, the dashed lines are for $\mathcal{E} \approx -0.0455$, which is the largest energy that trapped or reflected electrons can have; electrons with larger energies are free (untrapped and unreflected), and electrons with lower energies are associated with either trapped electrons inside the solitary pulse or reflected on either side of the electromagnetic pulse. In figure 2b, we have used the fields associated with an REH–EMW soliton and calculated the particle trajectories for a test electron. Here, the solid curve, for $\mathcal{E} = 1/\gamma_0 - 1 \approx -0.0835$, describes the separatrix between trapped and free electrons in the REH–EMW soliton. We see that the width of the trapped region vanishes for large values of $|\xi|$, and for the REH–EMW soliton, we do not have any reflected electrons. Thus, the localized structures are fundamentally different for the cold plasma soliton and the REH–EMW soliton cases. The cold plasma electromagnetic soliton will reflect thermal electrons that are travelling with a speed close to the soliton speed, and the soliton will lose energy to those electrons. In a sense, the cold plasma EMW soliton has a long-range effect in the plasma via the reflected regions indicated in figure 2a. As a contrast, the REH–EMW soliton has no reflected electrons, but only a population of trapped electrons that are absent far away from the soliton. This soliton is thus more ‘localized’, and turns out to be more stable than the cold plasma EMW soliton. We next investigate how the inclusion of a cold plasma population influences the dynamics of the REH–EMW soliton, by solving the coupled system of equations (2.6) and (2.8) for different values of $\mu = n_{e0}/n_{ed}$. Below a critical value of $\mu \approx 0.8$, we could not find numerical solutions for localized REH–EMW solitons of the type shown in figure 2b. The stability of electromagnetic solitons of different types in a cold plasma has been examined by Saxena et al. (2006) via fluid simulations.

In order to study the dynamics of interacting solitary structures composed of localized REHs loaded with trapped EMWs, we have numerically solved the time-dependent, relativistic Vlasov equation (2.1) together with the wave equation (2.2) and the Poisson equation (2.3) by methods developed by Shukla & Eliasson (2005) and Eliasson & Shukla (2006), and have displayed the results in figures 3–5. The initial conditions are obtained from the solutions of the quasi-stationary equations described above, where the left REH initially has the speed $v_0 = 0.28$ and is loaded with EMWs with $W_{\text{max}} = 1.5$, while the right REH has the speed $v_0 = -0.12$ and is loaded with EMWs with $W_{\text{max}} = 2.5$. We have used $k_0 = v_0 = 0$ in the initial condition for $A$ and in the solution of the EMW equation (cf. Eliasson & Shukla 2006). In figure 3, we display the phase space distribution of the electrons and the electromagnetic field amplitude at different times. In the simulation, the REHs loaded with trapped EMWs collide, merge and finally split into two REHs, while there remain two strongly peaked EMW envelopes at $z \approx 10$ and 30 after the splitting of the REH. A population of electrons are accelerated to large energies, seen at $z=40$ in figure 3a(iv). In figure 4, we show the time development of the EMW amplitudes, REH potential, the squared local plasma frequency and the electron number density. We see the time development of the fields under the collision and splitting of the REHs and the creation of the two localized EMW envelopes at $z \approx 30$; clearly visible in a(i)–(iv) at $t>150$. The two localized EMW envelopes are created by the EMW energy that the collapsing REHs have deposited into the plasma. The interacting REHs also excite large-amplitude electrostatic Langmuir waves, seen as oscillations in the
REH potential for \( t > 100 \). In order to investigate the robustness of a REH–EMW soliton with both hot and cold electrons, we have numerically solved the time-dependent Vlasov–Maxwell–Poisson system of equations (2.1), (2.2) and (2.5) for a REH–EMW soliton that travels from a region containing a small fraction of cold electrons, \( m_e = 9 \), into a region having only cold electrons, \( m_e = 1 \). The cold electrons have \( \alpha = 0.1 \) and the hot electrons have \( \alpha = 0.4 \). The numerical result is displayed in figure 5, where we see that the REH is quite robust against streams of cold electrons and wave turbulence, and so is the EMW envelope. However, small-amplitude EMWs are leaving the REH in its trailing end at the latest time when the REH has entered the cold plasma region. In simulations with only hot electrons, the REH–EMW soliton is very robust and does not show any signature

Figure 5. The profiles of the electromagnetic field amplitude \( |A| \) (a) and phase space plots of the electron distribution function \( f \) (b) at times (i) \( t = 0 \), (ii) \( t = 50 \), (iii) \( t = 100 \) and (iv) \( t = 150 \). In the initial condition, we used \( v_0 = 0.4, \mu = 0.9, \beta = -0.5 \) and \( W_{\text{max}} = 1.0 \). The plasma near the EMW soliton has \( \alpha = 0.4 \) (cf. figure 2), while the plasma at \( |z| > 20 \) is colder with \( \alpha = 0.1 \). Adapted with permission from Eliasson & Shukla (2006).
of instability. This robustness is in line with experimental observations of non-REHs (Saeki et al. 1979; Bertsche et al. 2003), where the electron holes were very long-lived despite the three-dimensional geometry of the experiments.

4. Conclusions

In conclusion, we have presented theoretical and numerical studies of the localization of EMWs in a plasma consisting of a hot and cold electron population. This kind of plasma is common in laser-produced plasmas, where collisionless heating by the Raman scattering instabilities leads to a high-energy population of electrons and non-Maxwellian electron distributions. Our computer simulations of the Maxwell–Vlasov–Poisson system reveal the stability of localized electromagnetic pulses accompanied by localized electrostatic potentials associated with a charge separation that is created by the relativistic ponderomotive force of the localized light, as well as by the self-trapped electron population in the positive potential.

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