A mixed contact model for an immersed collision between two solid surfaces

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Experimental evidence shows that the presence of an ambient liquid can greatly modify the collision process between two solid surfaces. Interactions between the solid surfaces and the surrounding liquid result in energy dissipation at the particle level, which leads to solid–liquid mixture rheology deviating from dry granular flow behaviour. The present work investigates how the surrounding liquid modifies the impact and rebound of solid spheres. Existing collision models use elastohydrodynamic lubrication (EHL) theory to address the surface deformation under the developing lubrication pressure, thereby coupling the motion of the liquid and solid. With EHL theory, idealized smooth particles are made to rebound from a lubrication film. Modified EHL models, however, allow particles to rebound from mutual contacts of surface asperities, assuming negligible liquid effects. In this work, a new contact mechanism, ‘mixed contact’, is formulated, which considers the interplay between the asperities and the interstitial liquid as part of a hybrid rebound scheme. A recovery factor is further proposed to characterize the additional energy loss due to asperity–liquid interactions. The resulting collision model is evaluated through comparisons with experimental data, exhibiting a better performance than the existing models. In addition to the three non-dimensional numbers that result from the EHL analysis—the wet coefficient of restitution, the particle Stokes number and the elasticity parameter—a fourth parameter is introduced to correlate particle impact momentum to the EHL deformation impulse. This generalized collision model covers a wide range of impact conditions and could be employed in numerical codes to simulate the bulk motion of solid particles with non-negligible liquid effects.

**Keywords:** mixed contact model; collision in liquids; elastohydrodynamic collision model

1. Introduction

In a flow of liquids plus solid particles, the momentum and energy transport depend on mechanisms not found in single-phase flows. Particle collisions, frictional contacts and viscous dissipation due to the relative motion between the solid and liquid phases dissipate the bulk kinetic energy of the flowing mixture. Direct particle collision, often taking place within a few milliseconds, is one

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One contribution of 11 to a Theme Issue ‘New perspectives on dispersed multiphase flows’.

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efficient mechanism to redistribute the bulk kinetic energy by reversing the respective particle motions. Such collision-induced momentum transfer is analogous to the idealized transport mechanism in classical gas dynamics that assumes nearly elastic point–force interactions between pair molecules. Consequently, a collision-rich granular flow with negligible interstitial liquid effects often exhibits a fluid-like macroscopic behaviour. However, while standard fluid mechanical theories have been successfully applied to describe the bulk behaviour of a granular flow, there always exist deviations between the motions of granular materials and a fluid. The impulsive nature of particle collisions can result in permanent deformation to and wave propagation within the solid particles, phenomena that are not present in gas kinetic theory. Furthermore, frictional surfaces and enduring contacts make a flowing granular material more dissipative and less dispersive than liquid or gas molecules. The surface friction thus serves as a second mechanism that transforms the bulk kinetic energy into heat, rather than collision-induced deformation energy. Recent increases in computational power have allowed the simulation of bulk granular flows employing only the dry coefficient of restitution, $e_{\text{dry}}$, and friction coefficient, $f$, but the lack of a feasible interaction model has prevented the successful modelling of more complex two-phase flows.

With the addition of a liquid phase, several researchers have reported that normal and tangential velocity components after a collision can change dramatically from those observed in a dry environment (Barnocky & Davis 1988; Joseph et al. 2001; Davis et al. 2002; Gondret et al. 2002; Joseph & Hunt 2004; Yang & Hunt 2006). The filling liquid modifies the dissipation mechanisms by lubricating and deforming the impact surfaces, as well as by exerting hydrodynamic forces on the moving particles. Thus, a model that accurately predicts the liquid-modified dissipation parameters would be of great use in simulating solid–liquid flows. In the literature, the wet coefficient of restitution, $e_{\text{wet}}$, is defined as the ratio of the relative velocity between two solid objects after they rebound to that before impact. This coefficient characterizes the energy loss during an immersed collision in both phases due to coupled particle–particle and particle–liquid interactions. Experimental investigations reveal a strong correlation between $e_{\text{wet}}$ and the particle Stokes number—a dimensionless number that indicates the ratio of the particle inertia force to the viscous force. A similar correlation is observed for the normal component of motion during a wet oblique collision. Moreover, experimental evidence suggests that the tangential motion can be well described as a ‘pseudo-dry’ contact, as long as a lubricated friction coefficient is used (Davis et al. 2002; Joseph & Hunt 2004; Yang & Hunt 2006). Yang & Hunt (2006) further conclude from their experiments that the tangential motion resulting from an immersed oblique collision can be approximated using a pseudo-dry collision model that depends only on $e_{\text{wet}}$. This is a useful result since the liquid-modified friction coefficient for an unsteady collision is usually difficult to measure and prone to errors.

Compared with the extensive experimental work associated with determining $e_{\text{wet}}$, theoretical work on the liquid-modified collision process has been somewhat limited. The intent of this work is to address this deficiency and, as such, to develop a collision model for a general immersed collision beyond Stokes flow conditions. In developing this model, this paper examines the liquid effects on the contact mechanisms and identifies the parameters that characterize a fully immersed...
2. Elastohydrodynamic collision models

Davis et al. (1986) employed elastohydrodynamic lubrication (EHL) theory to describe the approach of two solid spheres in a Newtonian liquid. The lubrication force generated in the thin layer reduces the approach speed and deforms the solid surfaces. A dimensionless elasticity parameter, \( \varepsilon = 4E\mu U_{i0}a^{3/2}/\delta_0^{5/2} \), was used to quantify the tendency of elastic deformation under the influence of the lubrication force. In addition to the initial impact velocity and the initial film thickness, \( U_{i0} \) and \( \delta_0 \), respectively, this parameter also depends on the liquid viscosity, \( \mu \), the reduced sphere radius, \( a^* = (1/a_1 + 1/a_2)^{-1} \), and the reduced elastic modulus, \( E^* = \left( 1-v_i^2 \right)/\pi E_i + \left( 1-v_f^2 \right)/\pi E_f \), where \( a_i, v_i \) and \( E_i \) are the radius, Poisson’s ratio and Young’s modulus, respectively, of each of the two spheres (\( i = 1, 2 \)). The authors suggested \( \delta_0 = 0.01a \) as the distance at which the elastohydrodynamic effects become important for sphere motion at low gap Reynolds numbers, \( Re_\delta = 2\delta_0\rho_i U_{i0}/\mu \). The sphere motion terminates at a minimum approach distance \( \delta_t \) that can be estimated as \( \delta_{tID} \approx 1/3\delta_0^{2/5} \). The factor 1/3 is extrapolated by the current authors from the simulation results of the original paper; the capital \( D \) subscript refers to this specific estimation of the minimum approach distance. The solid surfaces were assumed by Davis et al. to be perfectly smooth. Rebound of the two spheres occurred through restoration of the elastic strain energy with no surface contact.

Subsequently, in investigating the approach of a solid sphere of density \( \rho_s \) and volume \( V \) towards a wall, Davis et al. (1986) assumed that the lubrication force was the dominant influence on the sphere motion. Using \( \delta(t) \) and \( U_i(t) \) to denote the gap width and the impact velocity for a sphere of mass \( m_s = \rho_s V \), these authors combined and integrated the two dynamic equations \( d\delta/dt = -U_i(t) \) and \( m_s dU_i/dt = -F_{hub} = -6\pi \mu a^2 U_i/\delta \) to obtain

\[
\frac{U_i(\delta)}{U_{i0}} = 1 - \ln \left( \frac{\delta_0}{\delta} \right)/St_0. \tag{2.1}
\]

The sphere velocity was found to decrease with gap width during the approach. The dimensionless number \( St_0 = m_s U_{i0}/6\pi \mu a^2 \) is the impact particle Stokes number characterizing the ratio of particle inertia to the liquid viscous force when a fully immersed sphere is in motion. In the solid–liquid flow literature, the particle Stokes number is conventionally presented in terms of the particle Reynolds number, \( Re_0 = 2a^* \rho_f U_{i0}/\mu \), and the solid-to-liquid density ratio, \( \gamma = \rho_s/\rho_f \), as \( St_0 = (\gamma/9)Re_0 \).

Since neither physical contact nor plastic deformation is assumed to occur during the collision, the sphere rebounds at the minimum approach distance, \( \delta_{tID} \), with velocity \( U_r = -U_i(\delta_{tID}) \).

Phil. Trans. R. Soc. A (2008)
In further work, Barnocky & Davis (1988) assumed a physical contact at a rebound distance that is commensurate with the mean height of surface asperities on the sphere, $\delta_c$. The instantaneous rebound velocity was proposed as $U_r = -e_{dry} U_i(\delta_c)$, using the dry coefficient of restitution, $e_{dry}$, to account for the energy loss due to the asperity contact. In a later work by Joseph et al. (2001), Barnocky’s analysis was extended for the post-collision sphere motion, during which time a dominant lubrication force was also assumed. When the sphere had rebounded back to $\delta_0$, the rebound velocity, $U_{r0}$, was found to be

$$\frac{U_{r0}}{U_{i0}} = \frac{U_r}{U_i} - \ln \left( \frac{\delta_0}{\delta} \right) / St_0. \quad (2.2)$$

Substituting equation (2.1) and $U_r = -e_{dry} U_i(\delta_c)$ into equation (2.2), a wet coefficient of restitution was estimated as

$$e_c = \frac{U_{r0}}{U_{i0}} = e_{dry} + \frac{1}{St_0} e_{dry} \ln \left( \frac{\delta_c}{\delta_0} \right). \quad (2.3)$$

The subscript ‘c’ in equation (2.3) indicates an asperity contact at $\delta_c$—such a collision process will be referred to hereinafter as the EHL asperity collision model. A second EHL film collision model is also defined in this work, in which the sphere rebounds from $\delta_f$ with no surface contact. By replacing $\delta_c$ with $\delta_f$ and $e_{dry}$ with 1.0 in equation (2.3), the resulting wet coefficient of restitution is found to be

$$e_f = 1 + \frac{2}{St_0} \ln \left( \frac{\delta_f}{\delta_0} \right), \quad (2.4)$$

with the subscript ‘f’ denoting a film contact. These two EHL collision models allow the calculation of a wet coefficient of restitution as a function of $St_0$, $e_{dry}$ and a length-scale ratio, $\delta_c/\delta_0$ or $\delta_f/\delta_0$, which depends on the surface properties. In figure 1, predictions from equations (2.3) and (2.4) are compared with experimental data to illustrate the dependence of $e_{wet}$ on the impact Stokes number. Two sets of experimental results are plotted: the sphere-on-wall collision data by Joseph et al. (2001) and the binary collision data between spheres of equal size by Yang & Hunt (2006). A constant $\delta_c/\delta_0 = 10^{-3}$ is adopted based on a measured surface roughness, $\delta_c = 0.1 \mu m$, and $\delta_0 \approx 0.01a \ (a = 2-5 \ mm)$. Following the on-wall dry collision measurements by Joseph et al. (2001), a value of $e_{dry} = 0.97$ is used. For the calculation of $e_f$, a range of minimum film thicknesses, $\delta_f/\delta_0 = 1.5 \times 10^{-4}$ to $4 \times 10^{-3}$, is used, according to $U_{i0} = 20-500 \ mm \ s^{-1}$ and $\mu = 1-40 \ cP$ measured in the experiments.

As may be observed, as $St_0$ increases from a critical value of approximately 10–20, both EHL collision models are able to capture the rapid rise of $e_{wet}$. However, there are notable discrepancies between the measured and predicted $e_{wet}-St_0$ correlation, especially in the range of $40 < St_0 < 400$. For $15 < St_0 < 30$, most of the experimental data fall between the two $e_f$ curves but start to scatter from the model predictions as $St_0$ further decreases. Both models predict a zero $e_{wet}$ below a certain value of $St_0$ corresponding to collisions with no rebound, a phenomenon that is also observed in the experiments. In such circumstances, all the particle kinetic energy is dissipated by solid contact, surface deformation and
viscous dissipation. These mechanisms combine to result in a critical impact condition, usually characterized by a critical Stokes number, $St_c$, in the multiphase flow literature. The disagreement between the predicted and measured $St_c$ is probably due to dissipation mechanisms not considered in the two EHL collision models, such as interactions between the asperities and the interstitial liquid. Thus, Joseph et al. (2001) suggested that using $e_{\text{dry}}$ alone may not fully represent the energy loss during an asperity contact in wet collisions. For impact at higher $St_0$, from 30 to 800, the corresponding $Re_0$ lies between 60 and 650, with $\gamma = 1.4$ to 7. At these Reynolds numbers, lying outside the Stokes flow regime, new flow phenomena may develop to affect the collision process, such as a change in $\delta_0$ or the formation and shedding of vortex rings. Moreover, since a collision process is intrinsically unsteady, hydrodynamic forces such as the added mass force and the Basset history force may also become important. Addressing some of these neglected liquid effects would thus allow the development of a more general collision model. In the following, the liquid effects on the contact scheme are investigated to develop a subsequent hybrid EHL collision model.

3. Hybrid EHL collision model

(a) Mixed contact scheme

In previous EHL collision models, the particle rebounds at one of the two different length scales: the actual asperity height, $\delta_c$, or the minimum approach distance, $\delta_f$. Since $\delta_f$ is closely related to the impact conditions, there exist
situations in which the two length scales are of commensurate sizes. Under such circumstances, both contact mechanisms may become equally important in dissipating particle kinetic energy, resulting in a new contact mechanism—a mixed contact. As depicted in figure 2, a small amount of liquid may be trapped between surface asperities in the contact area upon impact. Such a surface feature will be termed a liquid well. The trapped liquid is squeezed through the final stage of a compression process until the sphere rebounds. By considering the trapped liquid as a local lubrication film of thickness $\delta_c$, the hindering force that develops may be approximated as $f = 6\pi \mu a^2 U_{i0}/\delta_c$. The number of such dissipating liquid wells, $N$, will depend on the size of the contact area and the asperity number density of the deforming surface. To estimate the overall resistant force, the linearity of creeping flow is applied to superpose the liquid effects in each of the wells, resulting in $F = Nf$. These side-by-side liquid wells can be interpreted as $N$ parallel nonlinear springs distributed across the contact area, generating a total restoration force $F = Nf$ to impede the surface approach. Over a forcing duration, $\Delta t \sim \delta_c/U_{i0}$, the impacting sphere will lose momentum due to the viscous impulse $T = F \Delta t = N \sigma a^2$. Following Yang & Hunt’s (2006) interpretation of the particle Stokes number as a measure of available particle momentum to overcome the viscous impulse, a new dimensionless number, $\sigma = m_p U_{i0}/N \delta_c \mu a^2$, is defined in a similar manner to estimate the persistence of particle motion against the viscous impulse $T$. The inverse of $\sigma$ may thus be used to characterize the particle momentum change due to $T$ as $m_p \Delta U_{i0} \sim 1/\sigma = N/St_0$. Furthermore, by correlating the particle kinetic energy loss to its momentum change, a recovery factor, $\eta$, can be defined as

$$\eta = \frac{m_p \Delta U_c - m_p \Delta U_{i0}}{m_p \Delta U_c},$$

(3.1)

**Figure 2.** Illustration of the collision-induced liquid motion in the inter-asperity liquid wells on the contact area.
characterizing the energy loss in the liquid wells during a mixed contact. In equation (3.1), the actual momentum change is scaled by a critical value, \( m_p \Delta U_c \sim 1/\sigma_c = N/St_c \), corresponding to a terminated sphere motion. In other words, the parameter \( \sigma_c \) characterizes complete energy dissipation by the surface liquid wells and its correlation to the critical particle Stokes number, \( St_c \), is assumed to follow that of \( \sigma \) to \( St_0 \). The recovery factor can be manipulated into

\[
\eta = 1 - \frac{St_c}{St_0}.
\]

(3.2)

The value of critical Stokes number \( St_c \) under various impact conditions has been investigated by many researchers. There is no strong evidence to support a universal \( St_c \), but a range of \( 5 < St_c < 20 \) is widely accepted. Davis et al. (2002) have proposed several models expressing \( St_c \) as a function of \( e_{\text{dry}} \), \( \delta_0 \), and \( \delta_t \) or \( \delta_c \); however, the uncertainty in \( \delta_0 \) often results in a prediction deviating from the actual measurement. To negate this uncertainty when evaluating \( \eta \), an experimentally determined \( St_c \) under the corresponding impact conditions is employed here. According to equation (3.2), if two solid surfaces collide at a value of \( St_0 \) that greatly exceeds \( St_c \), the recovery factor will approach unity, indicating negligible liquid-well influences. However, at low \( St_0 \), the particle inertia is small compared with the viscous impulse, resulting in significant energy loss and a correspondingly low \( \eta \). A zero \( \eta \) in a mixed contact represents total energy dissipation by the squeezed inter-asperity liquid motion. To develop a rebound scheme with a mixed contact mechanism, for the case in which \( \delta_t \approx \delta_c \), the rebound velocity is proposed to be

\[
U_r = -\eta e_{\text{dry}} U_i(\delta_t).
\]

(3.3)

In addition to the recovery factor, this rebound velocity also depends on \( e_{\text{dry}} \) that measures the kinetic energy loss due to the solid asperity contact. To incorporate collisions for \( St_0 < St_c \), the recovery factor is set to zero, corresponding to the expected situation of a fully stopped sphere in front of the target surface. With the mixed contact mechanism, a new model for the wet coefficient of restitution can be obtained by replacing the \( e_{\text{dry}} \) in equation (2.3) with \( \eta e_{\text{dry}} \), resulting in

\[
e_m = \eta e_{\text{dry}} + \frac{1 + \eta e_{\text{dry}}}{St_0} \ln \left( \frac{\delta_t}{\delta_0} \right).
\]

(3.4)

The subscript ‘m’ refers to the implementation of the mixed contact mechanism in the resulting EHL mixed collision model. Equation (3.4) predicts \( e_m \) as a function of four collision parameters: \( e_{\text{dry}} \), \( St_0 \), \( \delta_t/\delta_0 (\delta_t \approx \delta_c) \); and \( \eta \), the latter being newly introduced in the present model that requires \( St_c \).

\[ \text{(b) Results with the recovery factor } \eta \]

In figure 3, a series of \( e_m \) predictions from equation (3.4) is compared with both the \( e_c \) results and the experimental data to examine the effects of the newly introduced recovery factor. Collisions between surfaces of similar and dissimilar elastic properties result in distinctive \( e_{\text{dry}} \) in the experiments and thus are examined separately. In addition to \( e_{\text{dry}} \), different values of \( \delta_t/\delta_0 \) and \( St_c \) are also combined to calculate \( e_m \).
With $e^{\text{dry}} = 0.97$ and $\delta_c/\delta_0 \approx 10^{-3}$, values that were used by Joseph et al. (2001), the first prediction, $e_{m1}$, is calculated using the largest $St_c = 20$ observed in the experiments. This is to be compared with the $e_{c1}$ curve and the on-wall experimental data. These data are for immersed collisions between a steel or Nylon impact sphere and a Zerodur wall (Zerodur is a glass-like material with $E = 91$ GPa and $v = 0.24$). As may be seen, the mixed collision curve $e_{m1}$, illustrated by the boldest solid line, lies below $e_{c1}$ and better captures the on-wall data for $St_0 > 80$. Knowledge of the actual impact conditions allows the calculation of a minimum approach distance as $\delta_{r0}/\delta_0 = 2 \times 10^{-2}$, commensurate with the scaled asperity height of Delrin spheres, $\delta_c/\delta_0 = 2.5 \times 10^{-2}$. This value, together with the lower limit of $St_c = 5$, is used for a second EHL mixed model prediction, $e_{m2}$, which is plotted as the second boldest solid line. The second set of parameters results in an upper bound for $e_{wet}$, while $e_{m1}$ serves as a lower bound. The other two parameter combinations, $\delta_c/\delta_0 = 10^{-3}$ with $St_c = 5$ and $\delta_{1D}/\delta_0 = 2 \times 10^{-2}$ with $St_c = 20$, were also examined and the resulting $e_{wet}$ curves, not shown, fall between the bounding $e_{m1}$ and $e_{m2}$ curves. The first two choices of parameter combinations are also appropriate for wet collisions between identical spheres, presented with the solid squares.

When binary collisions take place between spheres of distinctive elastic properties, plastic deformation is more likely to occur than in collisions between identical particles, reducing the value of $e^{\text{dry}}$. Thus, to provide a comparison with the dissimilar binary collision experimental data, a value of $e^{\text{dry}} = 0.85$ is used when applying equation (3.4) to calculate $e_{m}$. This is an averaged value from
Yang & Hunt’s (2006) measurements on steel–Delrin and glass–Delrin sphere pairs for $U_{i0}=10$–500 mm s$^{-1}$. The critical Stokes number is also extrapolated from the experiments to be $5<St_{c}<15$. Again, the combinations of the upper and the lower bounds of $St_{c}$ with the small and large $\delta_{i}/\delta_{0}$ estimations result in the $e_{m3}$ and $e_{m4}$ curves that are plotted as bold dashed lines in figure 3. These curves are seen to lie on either side of $e_{c2}$ and capture the experimental results well at higher $St_{0}$. The general agreement between the $e_{m}$ curves and the experimental data supports the inclusion of $\eta$ in a more general collision model.

A hybrid collision model is thus proposed as the following. With the known impact conditions, the minimum approach distance can be calculated first and compared with the sphere surface roughness. If $\delta_{c}/\delta_{0}$ exceeds $\delta_{i}/\delta_{0}$ by at least one order of magnitude, an asperity contact occurs and the EHL asperity collision model is adequate for an $e_{wet}$ prediction using equation (2.3). If $\delta_{i}/\delta_{0}$ is larger than $\delta_{c}/\delta_{0}$, however, the liquid film covers the surface roughness, preventing mutual surface contact. Thus equation (2.4) should be applied. Finally, if $\delta_{i}/\delta_{0}$ is commensurate with $\delta_{c}/\delta_{0}$, a mixed contact should be considered and equation (3.4) can be used, together with a properly determined $St_{c}$ range, leading to a practical estimation of the upper and lower bounds of $e_{wet}$.

It is interesting to note that a smaller rebound distance results in more severe viscous dissipation by $F_{lub}$, and that a higher $St_{c}$ is indicative of effective hydrodynamic damping that dissipates more particle inertia. Thus, with identical values of $e_{dry}$, the combination of a small rebound distance and a large critical $St_{c}$, e.g. $e_{m1}$ and $e_{m3}$, generates a lower bound for $e_{m}$ from the EHL mixed collision model. Similarly, impact conditions that lead to a larger rebound distance and a smaller $St_{c}$ should result in an upper bound for $e_{m}$, as illustrated by the $e_{m2}$ and $e_{m4}$ curves in figure 3.

(c) A new parameter for the minimum approach distance $\delta_{i}$

Since the minimum approach distance, $\delta_{i}$, plays a crucial role in determining the rebound scheme, this subsection examines in detail the dependency of $\delta_{i}$ on the impact conditions. During a wet collision, the incoming particle momentum is transmitted through the interstitial layer in the form of a hydrodynamic pressure rise. The lubrication pressure deforms the particle, transforming a portion of the particle kinetic energy into strain energy, a process that is characterized by the elasticity parameter $\varepsilon$ introduced by Davis et al. (1986). In all EHL theories, the minimum film thickness is estimated as a function of $\delta_{0}$ and $\varepsilon$. The particle momentum that initiates the deformation process, however, is commonly neglected. To correlate the particle momentum, $m_{p}U_{i0}$, with the deformation process, an elasto-inertial parameter is proposed as

$$\xi \equiv \frac{m_{p}U_{i0}}{PT_{D}}.$$  

The denominator in equation (3.5) is a measure of the elastic deformation impulse, resulting from the action of a deformation force $P$ over duration $T_{D}$. Hertz contact theory gives $P = 2/3P_{0}\pi r^{2}$, where $P_{0}$ and $r \approx 40\pi E^{*}Y a^{2}$ denote the maximum stress and the radius of the contact area, respectively, of the deforming particle. Since EHL theory considers only elastic deformation, the lower yield stress $Y$ of the two colliding spheres is employed to estimate $P_{0}$. To characterize the duration of the impacting–deforming process involving an
interstitial liquid at low \(Re\) and quasi-steady flow conditions, a diffusion time, \(T_D = a^2/\nu_f\), is adopted, with a liquid viscosity \(\nu_f = \mu/\rho_i\). Using these expressions for \(P\) and \(T_D\), the elasto-inertial parameter can be manipulated into 
\[
\zeta = \frac{\delta c/0 = 10^{-3}}{\gamma/(800\pi)E^2 Y^{-3} (\mu U_{i0}/a)}.
\]
The minimum approach distance, normalized by \(\delta_0\), can then be expressed as 
\[
\frac{\delta f/0}{\delta 0} = 1/3\varepsilon^{2/5} \approx 1330(E^2 Y)^{6/5} \gamma^{2/5} \zeta^{-2/5}, \tag{3.6}
\]
illustrating the dependence of the minimum approach distance on the liquid properties through \(\zeta\) and \(\gamma\). In deriving equation (3.6), \(\varepsilon = 4E^* \mu U_{i0} a^{3/2}/\delta 0^{5/2}\) and \(\delta 0 \approx 0.01 a\) are used. This new parameter, \(\zeta\), for a particle–wall impact in liquids plays a role similar to the Weber number for a deforming droplet in motion, a number that characterizes the competing droplet inertia and deformation impulse due to the surface tension force.

(d) Results with the new parameter \(\zeta\)

To investigate how \(\zeta\) modifies an EHL collision model prediction, equation (3.6) is incorporated into equation (3.4) for \(e_{\text{wet}}\). Since \(\zeta\) is closely related to the elastic properties of the colliding surfaces, the data for on-wall and binary collisions are examined separately for impact particles of different materials. In figure 4, the collision data using a steel sphere and a Zerodur wall are presented, with the impact conditions summarized in the caption. Following Davis et al. (1986), the minimum approach distance is estimated to be \(\delta f/0 = 1.15 \times 10^{-3}\) in this case. With \(\zeta\) ranging between \(4.3 \times 10^{-9}\) and \(\approx 4.2 \times 10^{-7}\), however, equation (3.5) predicts a range of minimum approach distances, from \(\delta f/0 = 9 \times 10^{-4}\) to \(6 \times 10^{-3}\). These values are comparable to the scaled asperity height, \(\delta c/\delta 0 \approx 10^{-3}\),
as the $\delta_D/\delta_0$ estimation. The EHL mixed collision model is thus applied, using the newly derived $\delta_D/\delta_0$ limits and the highest and the lowest values of $St_c$ typically accepted in the solid–liquid flow literature, namely 20 and 5. For $St_0 > 70$, the $e_m$ prediction using $St_c = 20$ agrees well with the experimental data, as shown by the solid lines, and outperforms the EHL asperity collision model. Most of the experimental data fall between the pair of limiting $e_m$ curves—the lower bound uses a combination of the highest $St_c$ and the minimum rebound distance; the upper bound (approaching the $e_c$ curve) corresponds to a combination of the lowest $St_c$ and the maximum rebound distance. When immersed collisions take place between two identical steel spheres, the lower $E^*/C_3$, compared with the on-wall case, gives a larger $\zeta$, lying between $2.3 \times 10^{-5}$ and $1.1 \times 10^{-3}$. The resulting minimum approach distance, $\delta_D/\delta_0 = 4 \times 10^{-3}$ to $2.5 \times 10^{-2}$ from equation (3.6), greatly exceeds $\delta_D/\delta_0 \approx 6.2 \times 9.5 \times 10^{-4}$. In figure 5, the $e_m$ curves employing combinations of the largest and the smallest rebound distance with either $St_c = 3$ or 15 are plotted together with the experimental binary collision data. At low values of $St_0$, there are points falling outside of the $e_m$ bounding curves, which may result from uncertain surface conditions in these collisions. In contrast to a stationary wall, the target sphere is now suspended by a fine string and free to move and rotate before contact. Thus, the target surface conditions may vary from collision to collision, generating variations in the rebound motion. For example, if the two colliding spheres interact at their combined asperity height, the rebound distance may be twice as large as the height of a single asperity. Furthermore, if the filling liquid properties change due to liquid hardening (Joseph & Hunt 2004) or the occurrence of cavitation (Ashmore et al. 2005), this may result in dramatic variations in the rebound motion.
distance and further scattering in the data. If a somewhat larger rebound distance, \(d_f/d_0 \approx 10^{-1}\), is used, a larger \(e_m\) is found as shown by the dash-dotted line in figure 5, especially notable at low \(St_0\).

For collisions of a ductile Delrin or Nylon sphere on a Zerodur wall in a liquid of \(\mu = 1–10\) cP at \(U_{i0} = 50–120\) mm s\(^{-1}\), a minimum approach distance is found in the range of \(\delta_f/\delta_0 = 5.1 \times 10^{-4}\) to \(1.8 \times 10^{-3}\). Under these impact conditions, equation (3.6) gives a thicker liquid film than previously, \(\delta_f/\delta_0 \approx 1.1–2.8 \times 10^{-2}\), a range whose value encompasses the scaled asperity height, \(\delta_c/\delta_0 = 1.25 \times 10^{-2}\), of a Delrin sphere, suggesting a mixed contact. In figure 6, the two bounding \(e_m\) curves are plotted, using the value \(e_{\text{dry}} = 0.90\) measured by Joseph et al. (2001). This lower value of \(e_{\text{dry}}\) as compared with previous cases results from the mismatching elastic properties of the colliding surfaces. The limiting critical Stokes numbers, 15 and 5, are extrapolated from the Nylon sphere data, shown as solid squares. The resulting \(e_m\) curves both lie below the EHL asperity model prediction, with the upper bound curve better following the experimental data. The discrepancy between the \(e_m\) and the Nylon sphere data may be attributed to the much greater Nylon surface roughness, \(\delta_c = 2\) \(\mu\)m, than the Delrin value, \(\delta_c = 0.796\) \(\mu\)m, considered in the model. A large \(\delta_c\) permits an asperity contact and thus the experimental data are better captured by \(e_c\), shown by the thin solid curve, rather than the two \(e_m\) curves.

The last group of data presented in figure 7 is for binary collisions between a Delrin, steel or glass impact sphere and a Delrin target sphere. When applying the collision models for Delrin-on-Delrin impacts, a value of \(e_{\text{dry}} = 0.97\) is used; for elastically dissimilar steel-on-Delrin and glass-on-Delrin impacts, a lower value of \(e_{\text{dry}} = 0.85\) is used (Yang & Hunt 2006). The impact conditions,
$U_{i0} = 20–50 \text{ mm s}^{-1}$ and $\mu = 1–57 \text{ cP}$, give $\delta_{iD}/\delta_0 = 1.5–5.1 \times 10^{-3}$ and $\delta_{i}/\delta_0 = 1.2–3.0 \times 10^{-2}$ from equation (3.6), with $\zeta$ lying between $6.7 \times 10^{-7}$ and $1.2 \times 10^{-4}$. In figure 7, the bounding $e_m$ curves for the two values of $e_{\text{dry}}$ are shown together with the $e_c$ curves and the experimental results. The Delrin-on-Delrin collision data over the range $20 < St_0 < 80$ are well captured by the two $e_m$ curves. No experimental data are available for collisions between elastically dissimilar spheres in this range of impact Stokes numbers. For $St_{0} > 300$, the EHL mixed collision model is able to reproduce the measured $e_{\text{wet}}$ provided that a realistic value of $e_{\text{dry}}$ is used. For each value of $e_{\text{dry}}$, each of the $e_m$ curves approaches the respective $e_c$ curve as $St_0$ increases, since the liquid effects are diminished by large particle inertia. These phenomena can be observed in all the examined EHL collisions models, as expected.

4. Conclusions and discussion

This paper presents a collision model with a new mixed contact mechanism that considers the interaction between the surface asperities and the interstitial viscous liquid. In addition to a dry coefficient of restitution that characterizes energy loss in a mutual asperity contact, a recovery factor is proposed to account for the dissipation due to the inter-asperity liquid motion. Multiplication of the two dimensionless numbers, $e_{\text{dry}}$ and $\eta$, with the impact velocity determines the rebound velocity after a mixed contact. The recovery factor, a quantity that depends on the impact and critical Stokes numbers, approaches unity as the particle inertia becomes sufficient to overcome the inter-asperity viscous

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dissipation. Compared with an EHL collision model with asperity or film contact, the new mixed collision model predicts a lower $e_{\text{wet}}$ and better captures the experimental data under certain impact conditions. For a wet collision between rough and stiff particles, the asperity height often exceeds the predicted film thickness from EHL theory. In such cases, a pure asperity contact is sufficient to characterize the energy loss upon impact. However, when collisions take place between smooth and ductile surfaces, especially at low $St_0$, the inter-asperity liquid motion is no longer negligible. A mixed contact is observed to better estimate the kinetic energy loss than a pure asperity or a film contact.

The success of a hybrid collision model requires a feasible estimation of the rebound distance. A new elasto-inertial parameter, $\zeta$, has been proposed to correlate the impact particle momentum to the surface deformation impulse under the action of lubrication pressure. With identical impact velocities and material properties, $\zeta$ introduces the particle momentum into the dynamic process, giving rise to a $\delta_i/\delta_0$ estimation that further depends on the solid-to-liquid density ratio. The resulting $\delta_i/\delta_0$ generally covers a wider range of values than that derived from EHL theory. Consequently, an EHL mixed collision model with a $\zeta$ rebound distance and limiting values of $St_c$ determines upper and lower bounds for $e_{\text{wet}}$ which envelop the experimental data examined in this study over a wide range of $St_0$. This elasto-inertial parameter can further be interpreted as a solid-to-liquid density ratio multiplied by a deformation strain factor, $E^{-2}Y^{-3}\mu U_{i(0)}/a$, introducing a time-scale ratio into the quasi-steady EHL analysis. A similar dimensionless number, $\mu U_T/E^*a$, was used by Joseph & Hunt (2004) to characterize the effective shear strain between two immersed solid surfaces with non-zero tangential relative velocity, $U_T$. These authors measured the wet friction coefficient, $f$, for an immersed oblique collision and observed a strong correlation with $\mu U_T/E^*a$.

To summarize, the characterization of $e_{\text{wet}}$ for an immersed collision requires at least six dimensionless numbers: the particle Stokes number $St_0$, the solid-to-liquid density ratio $\gamma$, the dry coefficient of restitution $e_{\text{dry}}$, the recovery factor $\eta$ (or the critical Stokes number $St_c$), the scaled surface roughness and a scaled EHL film thickness that incorporates $\zeta$ (or a deformation strain factor).

References


*Phil. Trans. R. Soc. A* (2008)