Introduction. Cosmology meets condensed matter

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At first sight, low-temperature condensed-matter physics and early Universe cosmology seem worlds apart. Yet, in the last few years a remarkable synergy has developed between the two. It has emerged that, in terms of their mathematical description, there are surprisingly close parallels between them. This interplay has been the subject of a very successful European Science Foundation (ESF) programme entitled COSLAB (‘Cosmology in the Laboratory’) that ran from 2001 to 2006, itself built on an earlier ESF network called TOPDEF (‘Topological Defects: Non-equilibrium Field Theory in Particle Physics, Condensed Matter and Cosmology’). The articles presented in this issue of Philosophical Transactions A are based on talks given at the Royal Society Discussion Meeting ‘Cosmology meets condensed matter’, held on 28 and 29 January 2008. Many of the speakers had participated earlier in the COSLAB programme, but the strength of the field is illustrated by the presence also of quite a few new participants.

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1. Phase transitions and topological defects

One of the most fruitful of the various analogies between our two fields has been in the description of phase transitions and the topological defects that often form at these transitions.

Spontaneous symmetry breaking is very commonly associated with phase transitions. Typically, there is a high-temperature phase in which the symmetry, described by some group $G$, is manifest, and a low-temperature phase in which it is partially or completely hidden. This occurs when the ground or vacuum state of the system is not invariant under all the operations of $G$, but only under those of some subgroup $H$ of unbroken symmetries. There is then necessarily a degenerate ground or vacuum state; the remaining operations of $G$ transform one ground state into another.

This kind of pattern is seen in many condensed-matter systems—superfluids, superconductors, Bose–Einstein condensates (BECs) and others—as well as in particle physics models used to describe the physics of the early Universe. Just as
a superfluid or superconductor will go through a phase transition when it cools past the critical temperature $T_c$, so the early Universe, cooling after the big bang, may have undergone a sequence of symmetry-breaking phase transitions.

Topological defects are a common feature of these phase transitions. They may be of any dimension: point-, line- and surface-like—monopoles, string or vortices, and domain walls, respectively. When a system goes through a transition into a broken-symmetry phase, it has to make a spontaneous choice among the degenerate ground states. But this choice may vary from point to point, and there may be frustration, where the parameters describing the selected ground state cannot be chosen everywhere in a continuous way. The simplest example is that of a U(1) symmetry, where the relevant parameter is the phase of a complex order parameter $\psi$. If this changes by $2\pi$ around some loop in space, then by continuity somewhere within the loop $\psi$ must go to zero, rendering the phase undefined. There must be, passing through the loop, a linear defect, a vortex or string, the locus of points where $\psi = 0$. Well-known examples of this mechanism are vortices in superfluids and BECs, and magnetic flux tubes in superconductors. The case of a BEC in a diffuse atomic gas is particularly interesting; because the atoms interact only weakly, the physics is relatively uncomplicated. In the early Universe too, the breaking of a U(1) symmetry may have led to the formation of cosmic strings.

2. The rate of defect formation

One of the bonuses derived from the interaction between diverse fields is that it may suggest novel questions about familiar processes. For example, traditional studies of phase transitions in condensed-matter systems have tended to concentrate on going through the transition very slowly, maintaining near-equilibrium behaviour throughout. However, in cosmology, the transitions are driven by the Hubble expansion of the Universe, which proceeds at a finite rate outside our control. Causality arguments then require a certain minimum density of defects to be generated, the basis of what has come to be known as the Kibble–Zurek (KZ) mechanism (Kibble 1976; Zurek 1985). This naturally leads to the question of how the number of defects formed is related to the quench rate, the rate at which the system goes through the transition. Simple arguments suggest a power-law dependence and this prediction of the KZ mechanism has now been tested in a wide variety of systems (for a review, see Kibble 2002).

Several presentations at this meeting have dealt with aspects of this relationship. The paper of Zurek & Dorner (2008) discusses the extension of these ideas to cases where the transition occurs in space rather than in time, with some control parameter varying across the system, so that one part of it is in the symmetric high-temperature phase and another in the low-temperature broken symmetry phase. The authors show that the spatial extent of the region over which the corresponding order parameter changes from the symmetric value, usually zero, to a value appropriate to the broken-symmetry ground state is related to the spatial gradient of the control parameter by very similar arguments. The superconductivity transition is a particularly interesting one. In superfluids and BECs the symmetry that is broken is a global symmetry, where the group parameters are independent of space or time. But in a superconductor
it is a \textit{local} symmetry, the well-known gauge symmetry of electrodynamics, in which the group parameters are space–time dependent. The symmetries in particle physics models of relevance to the early Universe are also mostly local, gauge symmetries. In such cases, the theory of defect formation is more complicated owing to the effects of the gauge fields—the electromagnetic field in the case of a superconductor. It is not yet entirely clear how far the KZ mechanism applies here, or how it must be modified. There are two competing mechanisms of defect formation, based on fluctuations of the scalar field and the gauge field, leading to different correlation patterns between defects and antidefects, whose relative importance varies according to circumstances (Hindmarsh & Rajantie 2000).

At this meeting, Rivers (Rivers \textit{et al.} 2008) discussed the case of transitions in annular Josephson junctions, where the second defect formation mechanism should not be significant, and the KZ mechanism should apply. A long series of experiments on such junctions has been undertaken at the Technical University of Denmark, Lyngby, with collaborators from Salerno, London and Moscow, with some initially confusing results. In these experiments, the average number of defects formed is less than unity. Rivers showed that under these circumstances the derivation of the KZ causality bounds requires some modification, and argued that the origin of the bounds lies in the Gaussian nature of the order parameter, and that using this approach the results can be well understood.

Another long series of experiments, this time on thin superconducting films, has been conducted by Polturak’s group at Technion in Israel. They have verified the predictions of the KZ mechanism qualitatively, but interpretation of these experiments has been hampered by the fact that only the difference between the numbers of defects and antidefects (the net magnetic flux) was measurable, whereas one would really like to know the total number of defects and antidefects, and also their spatial correlations. This is particularly important in distinguishing the two defect formation mechanisms, which predict very different correlation patterns. But at this meeting, Polturak (Golubchik \textit{et al.} 2008) described a novel magneto-optical detection system, based on the Kerr effect in EuS\textsubscript{e}, that should allow this problem to be resolved. The results of experiments with this system are eagerly awaited.

A further interesting case where the KZ mechanism is relevant is discussed by Breid & Anglin (2008) in the context of a trapped ultracold gas of fermionic atoms where the atom–atom interaction can be tuned by slowly sweeping through a Feshbach resonance. Unlike the Bose-condensed gases, the cooled fermionic system can only condense via the creation of composite bosons. However, by playing with the Feshbach resonance the fermionic system can exhibit the whole range of behaviour from the tight binding to create diatomic molecules which can then Bose–Einstein condense, to the opposite end of the scale with weak binding which leads to the creation of Cooper pairs via a BCS process. Hitherto, this behaviour has largely been studied at fixed values of the induced effective interaction. However, under a very slow sweep of the Feshbach resonance it should be possible to take the system adiabatically from one extreme to the other (Breid & Anglin 2008). As the system is taken slowly from the BCS regime to the BEC regime, at some point the BEC should begin to appear. However, the slow sweep will inevitably lead to long wavelength excitations in
the system, which imply that the condensation will occur inhomogeneously by a mechanism similar to the KZ scenario. Here again, concepts developed in the cosmological context are being fed back into condensed-matter physics.

Turbulence in classical fluids has become a subject of great importance, and intense research. Superfluids also exhibit turbulence, called quantum turbulence, which is closely associated with the behaviour of vortex lines. One of the complicating factors in the interpretation of some of the early experiments on defect formation in superfluid $^4$He is that defects can also form as a result of turbulence induced by interactions with the walls. Quantum turbulence is a very interesting phenomenon in its own right, which formed the subject of the contribution here by Vinen (2008). He described the various forms of turbulence that can arise in a superfluid, showing that quantum turbulence is very similar to classical turbulence on a large scale, but on a small scale the two must be rather different. In both cases, there is a cascade of energy to progressively smaller scales since the energy is dissipated on small scales. In classical turbulence this is the well-known Richardson cascade. However, in quantum turbulence at small scales there are no viscous forces to dissipate the energy and a different mechanism comes into play with the generation by recombination processes of helical Kelvin waves on the vortices which then dissipate by radiation of phonons or quasiparticles leading to a further Kelvin cascade at small scales.

3. Helium-3

As we noted in §1, when a symmetry group $G$ is broken, there is always a degenerate ground or vacuum state. The unbroken symmetry subgroup $H$ is the group of operations of $G$ which leaves the chosen ground state invariant. The other operations of $G$ transform one ground state into another. In this case, the set of degenerate vacuum or ground states—the vacuum manifold $\mathcal{M}$—is labelled by the points of the quotient space $G/H$, namely the left cosets of $H$ in $G$.

In superconductors, BECs and superfluid $^4$He, the symmetry group $G$ is just a $U(1)$, while $H$ is trivial (containing only the identity). In other cases, however, the symmetry is more complicated. In particular, the lighter isotope of helium, $^3$He, which becomes superfluid only below a couple of millikelvin, has a much richer and more interesting symmetry group, remarkably similar to the symmetries that appear in the standard model of particle physics.

The standard model is described by the symmetry group $SU(3) \times SU(2) \times U(1)$, where $SU(3)$ represents the colour symmetry of the strong interactions, and $SU(2) \times U(1)$ the electroweak symmetry, which is broken down to the $U(1)$ gauge symmetry of electrodynamics below a temperature corresponding to an energy scale of the order of 100 GeV (although it is possible that this is not a genuine phase transition but only a rapid crossover). There may also be a much higher temperature transition, at approximately $10^{15}$ or $10^{16}$ GeV, above which we have a grand unified symmetry—about which more is discussed below.

Because a $^3$He atom is a fermion, the atoms themselves cannot undergo Bose condensation. The mechanism of superfluidity in $^3$He involves the formation of Cooper pairs of $^3$He atoms, similar to the Cooper pairs of electrons in the BCS model of superconductivity. However, in this case, the strongest binding occurs not in the $^1S$ state but rather in $^3P$, meaning that a Cooper pair has both orbital

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and spin angular momenta equal to one (in units of $\hbar$): $L=S=1$. The symmetry group, therefore, is not merely $\text{U}(1)$ but rather $\text{SO}(3)_{L} \times \text{SO}(3)_{S} \times \text{U}(1)$, where $\text{SO}(3)_{L}$ and $\text{SO}(3)_{S}$ represent separate rotations of the orbital and spin angular momenta, and $\text{U}(1)$ is the usual phase symmetry, as in $^4\text{He}$. In consequence, $^3\text{He}$ exhibits very unusual symmetry-breaking behaviour: there are two superfluid phases, $^3\text{He}$-$\text{A}$ and $^3\text{He}$-$\text{B}$, with different symmetries (and even a third phase in the presence of a magnetic field), and a very rich family of defects.

Not surprisingly, therefore, several contributions at this meeting were devoted to aspects of the superfluid transition in $^3\text{He}$. Bunkov (2008) discussed experiments on defect formation during fast symmetry-breaking transitions from the normal Fermi-liquid phase of $^3\text{He}$ to a superfluid phase, emphasizing in particular the importance of the existence of two competing superfluid phases, $\text{A}$ and $\text{B}$. In general, the phase transition will not just lead to a single superfluid phase, but to a patchwork of different phases, after which the regions of the more stable phase grow at the expense of the other—a phenomenon that may well have an analogue in the early Universe. He also reviewed the interesting and still controversial topic of the nucleation of the (stable) $\text{B}$ phase within the metastable $\text{A}$ phase.

Bunkov also discussed the intriguing phenomenon of BEC of magnons in superfluid $^3\text{He}$. The concept of BECs normally applies to systems with a conserved particle number, but can also apply to sufficiently long-lived quasiparticles, such as magnons. In fact, this intriguing phenomenon is closely analogous to BEC in diffuse atomic gases, but in some respects richer: especially noteworthy is the existence of several distinct condensed states of magnons, including stationary spin waves and homogeneously precessing domains.

In particle physics models of the early Universe, based on fundamental (super)string theory, an important role is played by ‘branes’, effectively higher dimensional defects. One popular idea, discussed further below, is that of a brane world in which our observable Universe is confined to a 3-brane, a three-dimensional surface in the higher dimensional space required by string theory. Here, inflation, the period of accelerating expansion that is believed to have occurred shortly after the big bang (which is required to explain the near-flatness and homogeneity of the Universe), is triggered by a collision between branes, possibly involving annihilation of a colliding brane and antibrane.

Interestingly, there is a possible analogue of this process in $^3\text{He}$. Here, the colliding branes are interfaces between the $\text{A}$ and $\text{B}$ phases which may be thought of as 2-branes. At this meeting, Haley (Bradley et al. 2008) described experiments on an analogue of a brane–antibrane collision, namely the collision of a $\text{BA}$ and an $\text{AB}$ interface. At low pressure and temperature, it is the $\text{B}$ phase that is stable, but an applied magnetic field can induce a transition to the $\text{A}$ phase. Applying a field, the group could create a slab of $\text{A}$ phase sandwiched between regions of the $\text{B}$ phase, and then by reducing the field observe the collision and annihilation of the two branes. In particular, they found an increase in the thermal impedance, most naturally interpreted as due to the generation of lower dimensional defects, analogous to the strings that are predicted to be formed in a brane–antibrane collision in the early Universe.

An even more ambitious idea, also based on an analogy with $^3\text{He}$, is a theory of emergent gravity, presented here by Volovik (2008). Owing to its special symmetry properties, the $^3\text{He}$-$\text{A}$ phase has an anisotropic Fermi surface; the energy gap vanishes along one special direction, at the Fermi point. In the
neighbourhood of this point, the quasiparticle spectrum has the typical form of a relativistic particle, with energy proportional to momentum. It exhibits an approximate Lorentz symmetry that is exact at the precise Fermi point. The idea of emergent gravity is that the gravitational interaction is not fundamental but only emerges as a low-energy effective interaction (below the Planck scale) in some as yet unknown more fundamental theory that possesses a Fermi point. Volovik has shown that, independently of the detailed properties of that underlying theory, the emergent fields describing the collective excitations would comprise chiral (Weyl) fermions, gauge fields and also gravity. However, to recover general relativity as an effective theory, one must ensure that the approximate Lorentz invariance extends to energies well above the Planck scale. The theory can reproduce many of the results of conventional general relativity, and serves to explain a number of otherwise puzzling features, including the observed flatness of the Universe and the mass hierarchy problem. It also serves to explain the observed very small magnitude of the cosmological constant: a very significant result is that if the emergent vacuum state is self-sustaining, then the cosmological constant is necessarily zero.

4. Strings in the early Universe

Cosmic strings became very popular in the 1980s because they seemed to offer a possible explanation for the origin of the small primordial density perturbations from which stars, galaxies and clusters eventually grew. However, observations of the small anisotropies in the cosmic microwave background (CMB) by the COBE and WMAP satellites demonstrated conclusively that this explanation in not viable. It is generally believed that the origin of the density perturbations can be traced to quantum fluctuations during inflation—the very early period of accelerating expansion postulated to explain various features of the Universe, in particular its near flatness and homogeneity. Inflation predicts an anisotropy pattern in good agreement with observation. Since cosmic strings can contribute at most approximately 11% to the perturbations (Bevis et al. 2008), they have dropped out of favour.

However, recent years have seen a revival of the idea, primarily because it has emerged that they are very natural consequences of fundamental string theory. Various aspects of this idea were discussed at this meeting by Davis (Davis et al. 2008), Polchinski (2008) and Sakellariadou (2008).

Superstring theory (or its modern version, M-theory) is currently the most promising attempt to unify all the particle interactions, including gravity. It is based on the notion that electrons, quarks and the rest are not elementary point particles but different vibration states of tiny strings. The theory is only consistent in 10 space–time dimensions (or 11 for M-theory). It was initially supposed that the six or seven extra dimensions must be curled up on the scale of the Planck length.

The idea of cosmic superstrings, fundamental strings extending over cosmic distances, was first suggested by Witten (1985), but he pointed out that there were very serious objections to the idea. The string tension would naturally be of the order of the square of the Planck energy, much too large to be acceptable observationally. Moreover, long strings would probably be unstable,
breaking up into small pieces. But, recent developments have shown that these objections can be overcome; indeed, this happens naturally (for a review, see Polchinski (2005) and references therein). In particular, the compact dimensions need not be so very small, so the effective energy scale can be much lower.

A key role in string theory is played by D-branes: a D$p$-brane is a surface of spatial dimension $p$ (space–time dimension $p+1$), on which fundamental (F-) strings can terminate (D stands for Dirichlet boundary conditions). A very nice idea is the brane-world scenario—the idea that our visible Universe is confined to a D3-brane in the higher dimensional space. Branes can also furnish an explanation for inflation. In the model of brane inflation, discussed in the talks by Davis and Sakellariadou, the big bang is triggered by a collision between two branes, a process that releases a lot of energy and creates lower dimensional defects, in particular cosmic strings. This is the process for which the experiments described by Haley (Bradley et al. 2008) provide an analogue.

The result is that string theory provides a very rich family of potential cosmic strings. There are the F-strings themselves, D1-branes or D-strings, D$p$-branes with $p > 1$ with all but one dimension wrapped around some compact cycle, and even combinations of several F- and D-strings bound together. As explained in particular by Polchinski (2008) there are important differences between the dynamics of cosmic superstrings and that of the ordinary (‘solitonic’) strings that emerge as topological defects in field theories. In particular, when ordinary cosmic strings intersect, they almost always exchange partners, but for the cosmic superstrings the probability of this happening may be much less than unity. Moreover, the superstrings may have junctions where three strings meet. Consequently, the evolution of a network of cosmic superstrings may be substantially different—although, as Polchinski points out, even for ordinary cosmic strings there are aspects of the evolution of which we are very unsure.

5. Cosmic magnetic fields

Cosmological phase transitions may have other interesting effects, in particular the possible generation of the observed matter–antimatter asymmetry. At this meeting, Vachaspati (2008) suggested that the same mechanism might also generate the primordial magnetic fields. Even if there are no stable defects arising from a cosmic phase transition, the spatial variation in the order parameter may well induce small-scale magnetic fields, but to create stronger, more coherent fields, requires some additional effect.

One of the big challenges for cosmology is to explain baryogenesis, the origin of the excess of baryons over antibaryons. Although the standard model does incorporate CP and baryon-number violation, it seems that one needs to go beyond it to find a viable explanation. One popular idea is that it involves quantum tunnelling between vacua with different baryon numbers, proceeding through the production and decay of unstable sphalerons. The sphaleron decays to a configuration with linked magnetic loops, with a definite helicity. Studies described by Vachaspati suggest a relation between the residual magnetic helicity and baryon number. Then there is a possibility of an inverse cascade in

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which the magnetic energy flows from small to large scales. One of the exciting things about this idea is that it makes observable predictions. The observation of magnetic helicity is certainly challenging but may become possible.

A very interesting question is whether an analogous mechanism exists in condensed matter systems. The transitions in $^3$He are in many ways similar, and it seems quite possible that they should also generate magnetic fields with non-zero helicity, which could perhaps be observed.

6. Dumb holes

Another area where very useful analogues in condensed-matter systems have been found is the physics of black holes. A black hole occurs of course when a massive object is compressed to such small size that the effect of gravity prevents the escape of even light signals. It is a region bounded by an event horizon, a surface across which particle and light signals can pass inwards but not outwards.

Experiments on black hole formation are obviously extremely difficult, though it has been suggested that very small black holes may be formed in the high-energy particle collisions to be studied at the Large Hadron Collider at CERN.

However, there is a possible acoustic analogue, a dumb hole, a region surrounded by an acoustic event horizon across which sound waves can travel in one direction only. This can be achieved in a fluid where the fluid velocity is supersonic. This idea, and the question of what can be learnt from experiments on dumb holes, was discussed here by Unruh (2008).

The analogy is not perfect, but it extends well beyond the classical realm. One of the most interesting things to look for would be the acoustic analogue of Hawking radiation—the thermal radiation predicted by quantum theory to emerge from immediately outside the event horizon, leading to the shrinkage and ultimate complete evaporation of the black hole. The theory applies equally to dumb holes and suggests that there must be the thermal radiation of phonons from the vicinity of the acoustic event horizon. There are many puzzling questions about black holes, in particular that of just where the Hawking radiation comes from (and hence what high-frequency corrections one might expect), and, even more fundamentally, whether the process involves a loss of information, as is suggested by the apparent evolution from an initial pure quantum state to a thermal mixture. These questions may be easier to answer in the context of a dumb hole, where we have a complete underlying microscopic theory—whereas a complete theory of quantum gravity is lacking.

The observation of the analogue of Hawking radiation is experimentally very challenging, because the Hawking temperature is always extremely low, but significant progress towards it is being made. At this meeting Leonhardt (Leonhardt & Philbin 2008) described a very promising optical analogue system, where the medium is moving faster than the speed of light in it. Experiments have confirmed the classical aspects of the black hole phenomenon, and it is hoped that the observation of the Hawking radiation will follow.

Schützhold (2008) also discussed aspects of this same idea. He pointed out in particular that in a system of trapped ions one could simulate cosmological particle creation in an expanding (or contracting) Universe, although the observation of Hawking radiation would be difficult. Another promising idea is to
use BECs (Garay et al. 2001), but again the Hawking radiation might be swamped by the heating effect of collisions.

Somewhat similar to Hawking radiation is the thermal Unruh radiation that would be seen by an accelerated observer. Although subjecting a detector to the required acceleration does not appear feasible, it may be possible to observe the obverse of this phenomenon: an accelerated electron can scatter photons from the thermal bath in which it finds itself, a process that would appear in an inertial frame as the production of entangled photons. Schützhold suggested that this process should be observable; experiments are now underway to this end.

7. Conclusion

The interface between condensed-matter physics and early Universe cosmology is a very lively field. As described in the various contributions to this collection, it now covers a very wide field; many different phenomena have counterparts in the other discipline. This interaction has greatly enriched both fields.

It is of course important not to overstate the analogies. Ultimately, it is not possible to obtain information about the early Universe merely by doing experiments in the laboratory, nor indeed to discover how condensed-matter systems behave purely by observing the heavens. What the interaction can do is to suggest new ways of looking at familiar phenomena, and new questions to ask. It has, for example, led to a new interest among condensed-matter physicists in rapid phase transitions, and new questions about the influence of the quench rate. For cosmologists, it has provided a new way of validating some of their theoretical ideas. When we cannot yet test them directly by astronomical observation, we can at least show that the analogous ideas make sense when applied to systems on which we can do laboratory experiments—or sometimes discover that things are not as simple as we had imagined. For example, we may learn which of the possible resolutions of the black hole information-loss problem applies in the case of dumb holes, and although that will not decide the issue for black holes it will certainly give us valuable clues.

The set of talks presented at the ‘Cosmology meets condensed matter’ discussion meeting, on which the articles here are based, certainly demonstrates that this is a vibrant and lively interaction, which is attracting an increasing number of participants.

We are greatly indebted to the Royal Society for the efficient way in which they organized this meeting, and the publication of this collection. We are also grateful to the Institute of Physics for providing financial support for the attendance of young physicists. Not least, we wish to thank all the participants who made this a most successful meeting.

References


