Annihilation of an AB/BA interface pair in superfluid helium-3 as a simulation of cosmological brane interaction

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This study presents measurements of the transport of quasiparticle excitations in the B phase of superfluid ³He at temperatures below 0.2Tc. We find that creating and then removing a layer of A-phase superfluid leads to a measurable increase in the thermal impedance of the background B phase. This increase must be due to the survival of defects created as the AB and BA interfaces on either side of the A-phase layer annihilate. We speculate that a new type of defect may have been formed. The highly ordered A–B interface may be a good analogy for branes discussed in current cosmology. If so, these experiments may provide insight into how the annihilation of branes can lead to the formation of topological defects such as cosmic strings.

Keywords: superfluid; helium-3; interface; brane; topological defects

1. Introduction

We have performed series of investigations on the interface between the A and B phases of superfluid ³He at the lowest achievable temperatures (see Bartkowiak et al. 2004; Bradley et al. 2006a, 2008). At low temperatures and magnetic fields, the B phase is the stable superfluid phase. A transition to the A phase can be induced by a magnetic field of the order of 0.5 T, so that an A–B interface can be stabilized in a field gradient. These two phases are coherent condensates with different broken symmetries, and the transition between them is first order. The A–B interface itself is a coherent structure with the components of the superfluid order parameter transforming smoothly from A phase to B phase via the planar phase. This phase boundary is arguably the best-characterized and most ordered interface available for experiment (Bartkowiak et al. 1999, 2000, 2004).

It has long been argued that the broken symmetries in superfluid ³He may provide a laboratory analogue for the SU(3) × SU(2) × U(1) symmetry-breaking phase transitions undergone by the Universe after the big bang (Bauerle et al. 1996;...
Runtuu et al. 1996; Volovik 2003). In superfluid $^3$He, in the most general case, the superfluid acquires a phase and must choose directions for the orbital angular momentum $\mathbf{L}$-vector and spin vector $\mathbf{s}$. This breaks the $\text{SO}(3) \times \text{SO}(3) \times \text{U}(1)$ symmetry, similar enough to the broken symmetries of the Universe to warrant exploration of potential analogies. In the Universe, the broken symmetries yield a subtle structure to space–time, whereas in superfluid $^3$He they are manifested by real physical directionalities in the liquid that we are able to detect. The broken symmetries in $^3$He are sufficiently complex that the superfluid may exist in several phases, of which two, the A phase and the B phase, are stabilized under current experimental conditions. We can view the A and B phases as two different quantum vacuum states that are separated by the A–B phase boundary. At present, there is interest in whether this interface could be used to simulate a two-dimensional brane embedded in a three-dimensional matrix of bulk superfluid. A layer of A phase in a B-phase background is bounded by an AB and BA interface, an analogue brane and anti-brane. In some brane-world scenarios, colliding branes leave behind topological defects of lower dimension such as cosmic strings.

Our most recent measurements have focused on the transport of quasiparticle excitations. These quasiparticles are broken Cooper pairs thermally excited out of the condensate. At temperatures far below $T_c$, this excitation ‘gas’ is so rarefied that the mean free path greatly exceeds the experimental volume and the transport is in the ballistic limit. Furthermore, at such low temperatures the quasiparticles are excited just above the superfluid energy gap so that the transport is extremely sensitive to any variation in the gap structure of the condensate. The A–B interface presents a very strong variation in the gap and this has a large effect on ballistic quasiparticle transmission.

Here, we review our experimental techniques and discuss recent results that may have cosmological implications. We use a shaped magnetic field to probe the field-induced distortion of the B-phase order parameter gap structure. On increasing the field further, we stabilize a layer of A phase between the two volumes of B phase, and measure the impedance to quasiparticle transmission introduced by the mismatch in the A- and B-phase gaps. Our measurements are also sensitive to smooth changes in the order parameter caused by liquid crystal-like textures in the superfluid, and to topological defects within the texture that further distort the gap structure. By reducing the field we remove the A-phase layer and return to B phase. This brings the AB/BA interfaces together and their disappearance provides a simulated brane/anti-brane annihilation. We observe that this process does indeed leave behind textural defects that impede the quasiparticles, and we discuss the creation of zero-dimensional ‘boojums’ and one-dimensional line defects.

2. Experiment and results

The experiment is performed in a tailpiece attached to the inner cell of a Lancaster-style nested nuclear demagnetization stage (Bradley et al. 2006c; figure 1). Figure 2 shows the experimental volume, consisting of a vertical cylinder of epoxy-impregnated paper that is 8 mm in diameter and 45 mm long. The cylinder is closed at the bottom but open at the top through a 3 mm diameter
Figure 1. Schematic of the nuclear demagnetization stage, the tailpiece where the experiments are performed and the superconducting solenoid assembly used to control the A–B interfaces in the tailpiece.

(a) Schematic of the experimental cell and (b) a typical magnetic field profile used to stabilize a layer of A phase. The cylindrical volume is 8 mm in diameter with an active length of 45 mm below the orifice.
orifice providing a thermal path to the bulk superfluid $^3$He in contact with the copper refrigerant. The $^3$He inside this volume cools to approximately $0.15T_c$. At the top and bottom of the cylinder are mounted pairs of vibrating wire resonators (VWRs; Fisher et al. 1992). These are small loops of superconducting NbTi wire that are in a vertical magnetic field and so can be driven to oscillate at resonance by an AC current from a signal generator, owing to the Lorentz force. The generated Faraday voltage is measured with a lock-in amplifier. Each pair of VWRs has one wire of 13.5 μm diameter to act as a heater (source of quasiparticles); and the other of 4.5 μm diameter to act as a thermometer (a measure of quasiparticle density). To apply heat to the cell, one of the 13.5 μm diameter VWRs is driven above the critical velocity for breaking Cooper pairs. This provides a source of quasiparticle excitations just above the energy gap that are injected directly into the superfluid. Because the VWRs are superconductors, the heater power is given directly by the product of drive current and measured in-phase voltage.

The phase diagram of superfluid $^3$He is shown in figure 3. At zero pressure, the normal fluid enters the superfluid B phase at $T_c=0.929$ mK. In low magnetic fields, the B-phase order parameter has an energy gap that is isotropic in momentum space. As the field is increased, the gap becomes distorted; it is enhanced perpendicular to the field but suppressed along it, defining an orbital angular momentum $l$-vector. When a large enough field is applied, there is a first-order transition to the A phase. The A-phase order parameter has an axis in momentum space where the gap goes to zero. Its $l$-vector is defined as the direction of this axis. The gap perpendicular to this axis is larger than the isotropic B-phase gap. The reader is directed to the book of Vollhardt & Wölfle (1988) for more details.

The magnetic field required to produce and stabilize an A-phase layer across the middle of the cylinder is generated by a set of superconducting solenoids, shown in figure 1, thermally anchored to the still of the dilution refrigerator. These solenoids provide the much localized 340 mT field required to form A phase at low temperature and zero pressure. A typical field profile is shown in figure 2. Compensation coils maintain the field below 50 mT at the sites of the wire resonators to ensure that they remain in low-field B phase where their behaviour is well characterized. In this case, the frequency width or damping of their resonance, $\Delta f_2$, is proportional to the quasiparticle density, varying as the Boltzmann factor $\exp(-\Delta B/k_B T)$, where $\Delta B$ is the isotropic B-phase energy gap (Bäuerle et al. 1998).

The measurement protocol is as follows. We cool the superfluid at zero pressure to the zero-temperature limit, in practice to approximately 150 μK or $T_c/T \sim 8$, making sure that the VWRs are all in regions of low magnetic field. We measure the equilibrium background quasiparticle density from the frequency widths of the two thermometer VWRs at the top and bottom of the radiator cylinder. Then we inject power of the order of a few tens of picowatts to the lower heater VWR and observe the change in widths of the top and bottom thermometers. For a given applied power, the extra damping of the top thermometer is always the same, regardless of the field profile. This is because all the power applied leaves the cylinder as a beam of quasiparticle excitations and the beam power is determined solely by the density of excitations near the orifice at the top. However, the extra VWR damping of the bottom thermometer depends on the thermal resistance presented by the $^3$He along the length of the cylinder.
We measure the increases in quasiparticle density at the top and bottom of the cell, $n_t$ and $n_b$, respectively, and we use these changes in the excitation density along the cylinder to infer the transport of quasiparticles. Extracting an absolute excitation flux from the data is non-trivial; the excitations are in the extreme ballistic limit, scatter only from the walls of the cylinder and the details of the wall reflection process are a subject of current study (Bradley et al. 2006b). Consequently, we quote the data simply in terms of a notional resistive thermal impedance, which we express as the ratio $n_b/n_t$. This ratio provides a measure of the resistance presented by the superfluid to the flow of quasiparticle excitations from the bottom to the top of the cylinder. It is this quantity that we use to probe the response of our system to the introduction and annihilation of our phase-interface branes.

Figure 4 shows typical sections of warm-up data for the measured ‘resistance’ $n_b/n_t$ under four different conditions of magnetic field and history. Each set of data is taken on a separate cooldown that takes one to two weeks to prepare and measure. In all cases we see that $n_b/n_t$ is greater than 1. Even for dataset A, where we measure the B phase in a field low enough that there is no distortion of the gap structure, the quasiparticle density is larger at the bottom than the top. This is a clear indication that there is a thermal resistance along the cylinder. Because the ballistic excitations interact only with the cell walls, this means that they must scatter diffusely. The degree of specularity of these collisions is a subject of ongoing investigation.

For dataset B the field is high enough to stabilize a layer of A phase across the cell, giving distorted B phase plus a slab of A phase and two phase boundaries AB and BA. The measured resistance does not depend on the thickness of the A layer, indicating that it is a property of the interface rather...

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than the bulk. We have previously reported on the Andreev reflection of quasiparticles owing to the gap structure at the A–B interface (Bradley et al. 2006a).

In datasets C and D, we monitor $n_b/n_t$ in a configuration where the field at the centre of the cylinder is just below that required to stabilize a slab of A phase across the cell, giving distorted B phase in the centre of the cell. The extra impedance is due to the distortion of the B-phase order parameter and texture in the high-field region. However, what is important in the present context is that datasets C and D are measured under the same applied conditions, but with different histories. Dataset D was taken after a layer of A phase had been created across the cell, and then removed, annihilating the AB and BA interfaces. The increased thermal resistance was an unexpected and startling result. While datasets C (virgin distorted B phase) are reproducible from run to run, datasets D (post-annihilation distorted B) are not. However, the impedance of post-annihilation B phase is always higher than that of the virgin distorted B phase. Therefore, removing the A-phase layer to annihilate the AB and BA interfaces must leave behind the defects in the distorted B-phase texture that impede the transmission of quasiparticles. But what are they, and do they bear any relation to defects spawned by collisions and annihilations in cosmological brane-world scenarios?

Before discussing the results, we should address one further issue. Do the textural defects arise from the annihilation of the AB and BA interfaces or simply from the fact that at some stage A phase has been present in the cell? We have checked this by adjusting the field profile to create a partial A-phase block across the cell, in the form of an annulus at the cylinder wall with a small hole at the centre connecting the upper and lower B-phase volumes. Removing this

Figure 4. The measured resistive impedance, $n_b/n_t$ (see text) for A, pseudo-isotropic B phase in low magnetic field; B, for a thin slab of A phase; C, high-field distorted B phase before A phase has been created; D, high-field distorted B phase, after an A-phase slab has been created and then removed. The increased resistance after the A phase has been present shows that defects have been left in the texture by the annihilation of the AB and BA interfaces.
annulus returns the measured impedance to that of virgin distorted B phase. For
defects to form and impede quasiparticle transmission, the phase boundaries
must have filled the whole cell cross section and split the B phase into two
volumes separated by A phase.

3. Discussion

The change in impedance clearly indicates that introducing and removing a slab
of A phase causes a very significant change in the texture of the cylinder. It is
tempting to imagine the simplest scenario that the phase-boundary annihilation
process leaves a two-dimensional soliton (or domain wall) in the B-phase texture
extending over the full cross section of the cylinder. However, it is probable that
we can discount this. The cross section of the cylinder where the A-phase slab
forms and where any soliton would remain, even after post-annihilation is at
a field maximum just below the AB transition field, rapidly falling off to the low-
field conditions at the top and bottom VWRs. Planar defects such as ‘n-solitons’
can exist in the B phase (Maki & Kumar 1977), but would be swept out of the
measuring region by the large field energies unless very strongly pinned at
the walls of the cylinder. Furthermore, a uniform two-dimensional defect would
not give the irreproducibility that we see in the post-annihilation impedance.
Thus, we believe that solitons do not give rise to the observed behaviour and we
must be seeing more local zero- and one-dimensional defects.

To examine the defects that might remain after annihilation, we first look at
the textural configuration of the system in the two phases before annihilation.
We consider the textures defined by the orbital angular momentum vector $l$
inside the cylinder, subject to various boundary conditions. This is shown
(in very simplified form) in figure 5. The texture in the A-phase slab is governed
by the constraint that the $l$-vector must lie in the plane of the A–B interface but
intersect normally with the solid boundaries of the cell. We assume a flare-out
texture in the B-phase background before the introduction of the A-phase layer.
Because this texture must survive globally in the B phase after the arrival of the
A phase locally, then the simplest texture we can envisage in the A phase is that
shown. (In the figure we have ignored the fact that the $l$-vectors on the two sides
of the interfaces must lie perpendicular to each other. This simply adds a twist to
the pattern that does not change the argument.) To accommodate the above two
constraints, the $l$-vector in the A phase must have zero-dimensional defects on
the A–B interface: a source radial boojum on the upper interface and a
hyperbolic boojum on the lower providing a sink for vertical $l$ while acting as a
source for horizontal $l$. These two defects provide compliance with the $l$-vector
boundary conditions. Within the A phase the two boojums must be connected by
a one-dimensional defect, a vortex that carries a quantum of circulation.

This configuration is not confined to the A phase. The vortex cannot terminate
at the interface but must continue into the B phase as a vortex terminating on
the cell boundary, with two further boojums, one of each type, on the B-phase
sides to provide matching across the interface (G. E. Volovik 2007, personal
communication). This symmetrical picture is no doubt distorted by gradient
energies and the boojums may find themselves drawn to and subsequently pinned
at the cell walls. We can also envisage more complex arrangements with multiple

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structures existing simultaneously. This structure is already reminiscent of a brane and anti-brane linked by a defect (or defects) of lower dimension (see Majumdar & Davis 2002; Sarangi & Tye 2002).

The critical question here is what may happen to these structures when the brane/anti-brane annihilate. The annihilation process is rapid; the interfaces first meet on the axis of the cylinder, the B phase breaks through and the sharp edges of the hole fly apart under the surface tension force. This leads to extremely fast conversion of A to B phase, creating oscillations in the phase boundary and possibly resulting in droplets of A phase separating from the slab and then shrinking away. We expect this process to create large numbers of zero- and one-dimensional defects. The point defects have to end up as boojums on the cell walls, whereas the line defects would form a tangle across the middle of the cell.

What form do these defects take? The simplest scenario would be a tangle of hard core vortices whose ends are pinned at the cell walls. These do not need to mould to textural constraints, and it is known that quasiparticles are Andreev reflected from the flow field around such vortices (Bradley et al. 2004). A reasonable number of these defects, of the order of $10^2$, would be compatible with the extra impedance to quasiparticle transmission that we measure.

Turning back to the boojums at the cell walls, these must be connected in pairs by linear defects. One might expect these linear defects to shrink to the walls and pull the end boojums closer together. Here, we postulate a new type of linear defect that could stretch across the cell, linking a hyperbolic and radial boojum. As shown in figure 6, this defect may survive if the end boojums are pinned. The $l$-vector structure of the boojums is reproduced along the defect axis but with one $90^\circ$ rotation near each end to match the $l$-vector at the defect surface to the surrounding flare-out.

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Figure 5. The texture in the A-phase slab before annihilation. The $l$-vector is constrained to lie parallel to the A–B interface but perpendicular to the cell walls. This can be satisfied by two boojums, zero-dimensional defects, one on each phase boundary. This then implies that a vortex must continue into the B phase (see text).

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texture. This rotation of the distorted B-phase $\mathbf{l}$-vector impedes the transport of ballistic quasiparticles up the cell by increasing the average energy gap that they experience; the order parameter is rotated so that the enhanced (i.e. increased) gap is aligned along the vertical cell axis rather than the suppressed gap one has in defect-free high-field B phase. This tube-like defect would distort the texture over its diameter of approximately 100 $\mu$m, implying that many are needed to explain the observed increase in the quasiparticle impedance. It is worth pointing out that observing this defect would be a new discovery.

4. Conclusions

We have performed a series of experiments that measure the transport of ballistic quasiparticle excitations at the lowest achievable temperatures in superfluid $^3$He. During these investigations, we made the unexpected discovery that creating and then annihilating a layer of A phase produces linear defects in the background B phase that are capable of impeding the transmission of quasiparticles. We speculate that these may include a new form of linear defect that has not been observed previously. The phase interface between the A and B phases in the low temperature limit may provide a model for a cosmological brane that can be accessed experimentally. We have shown in this analogue system that the annihilation of an AB/BA ‘brane/anti-brane’ leads to the production of topological defects, whose effect can be measured. This confirms that the concept of defect formation associated with brane annihilation in the early Universe can be reproduced in an analogous system in the laboratory.

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