3He: cosmological and atomic physics experiments

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Because the superfluid 3He order parameter exhibits many similarities with that of our Universe, the superfluid condensate may be considered as a quantum vacuum that carries various types of quasiparticles and topological defects. The condensate thus provides a test system for the experimental investigation of many general physics problems in cosmology, atomic or nuclear physics that are otherwise difficult or even impossible to investigate experimentally.

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1. Introduction

In this article, I have space only to describe two types of experiment: the analogy between superfluid 3He and the Universe during fast symmetry-breaking transitions, and the Bose–Einstein condensation (BEC) of magnons.

The phase transitions in 3He are characterized by broken gauge, spin and orbital rotation symmetries, which have many similarities with unification and electroweak transitions in the Universe. Fast transitions can be initiated by the local heating of 3He by the nuclear reaction on neutron capture which deposits 764 keV of energy into the liquid. This energy overheats a micrometre-sized region of the superfluid 3He into the normal state. The region expands and then cools very rapidly through the transition in analogy with the early Universe after the big bang. It has been found experimentally (Ba¨uerle et al. 1996, 2000) that a tangle of vortices forms after the fast phase transition in quantitative agreement with Kibble–Zurek theory of cosmic string formation (Kibble 1976; Zurek 1985). The analogy between superfluid 3He and the Universe can be very deep. There are different superfluid states of 3He—A, B and A1. According to Linde (1990), various states of Universe vacuum may also exist. Studies of the transition between different states of superfluid 3He can throw light on these profound properties of our Universe.

The second problem that I would like to highlight here is the BEC of magnons in superfluid 3He. There are several different mechanisms of magnon coherence. First of all, a coherent NMR signal can be radiated by stationary spin waves (SSWs) trapped in a cavity, a spin wave resonator. It can radiate a long-lived

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induction decay signal (LLIDS) of small amplitude (Corruccini & Osheroff 1978; Giannetta et al. 1981). At very low temperatures when dissipation processes are virtually zero, a self-adjustment process of the spin–orbit interaction leads to the formation of a ‘Q’ ball which radiates a ‘persistent’ NMR signal (Bunkov et al. 1992; Bradley et al. 2004). This magnon condensation mechanism is analogous to the BEC of diluted atomic gases in a trap. The Q ball provides a formal analogy with the mechanism of elementary particle formation (Bunkov & Volovik 2007).

The other type of magnon BEC takes place in the presence of a large magnetic field gradient. In this case, the field gradient separates the deflected magnetization into two domains (Borovik-Romanov et al. 1984, 1989; Fomin 1984). In one domain the magnetization is stationary and in the other it is deflected just more than the magic 104° and precesses as the ‘homogeneously precessing domain’ (HPD). Inside the domain the dipole–dipole spin–orbit field compensates the magnetic field gradient. Unlike in the SSW, the HPD generates a LLIDS signal of large amplitude. However, the main difference is the frequency of the signal, which corresponds to the Larmor frequency at the domain boundary, and which falls with relaxation (see review of Bunkov 1995). In the case of a persistent signal the frequency increases with relaxation (Bunkov et al. 1994). The magnon condensation in the HPD has many similarities with the superfluid state and is named ‘spin superfluidity’ (Bunkov & Volovik 2008). Other new types of magnon BEC have been observed: coherent precession with fractional magnetization in 3He-B (Dmitriev et al. 1997), coherent precession of magnetization in 3He-A (Sato et al. submitted) and in 3He-B in a squeezed aerogel (Elbs et al. 2008).

2. The Universe and 3He

We believe that the Universe was created in the ‘big bang’, and that the early Universe cooled through a cascade of symmetry-breaking phase transitions and inflation. These transitions may have produced many types of topological defects: domain walls, cosmic strings (vortices) and monopoles (Zel’dovich et al. 1975; Kibble 1976). The latest supersymmetry scenarios suggest that the initial symmetry of the Universe was $SO(10)$. After a cascade of transition, we have now a lower symmetry state $SU(3) \times SU(2) \times U(1)$. A number of other low-symmetry states could have been produced during these transitions. Linde (1990) has shown that the ground state energy of these states lies between the energy of the $SU(3) \times SU(2) \times U(1)$ state and the $SU(4) \times U(1)$ state. From this point of view, the Universe is very similar to superfluid 3He, where different states can be found. The question of transitions between different states in 3He is thus relevant for the Universe as will be discussed later.

Anyway, today we have our Universe, which is presented to us for investigation. There remain many paradoxes and questions to be answered. What we have now is a quantum vacuum populated with particles (bosons and fermions) combined to form atoms and molecules. Recently, humans have succeeded in observing the long predicted BEC of atomic gases. Earlier, unpredicted coherent quantum states of matter had been discovered; superfluidity, and its electromagnetic analogue superconductivity and spin superfluidity. In the Universe, we also believe that we have non-baryonic ‘dark matter’ which does not interact with baryonic matter.
by electromagnetic forces, although we can infer its existence through the gravitational interaction. There are currently many suggestions concerning the nature of dark matter. The most popular candidates are a new class of supersymmetric particles known as weakly interacting massive particles and light particles known as axions. The latter can exist in the form of a superfluid coherent quantum state. Finally, we have the further strange ‘dark energy’ that accelerates the expansion of our Universe.

The Universe is a very complicated system with many unknown parameters. One could not imagine that analogous properties could be observed and studied just in the droplets of superfluid $^3$He. Well, one man could, Volovik (2003), who wrote the beautiful book *The Universe in a helium droplet*. How about experimentalists? Yes, in Europe we are studying piece by piece the various analogies with cosmological phenomena in experiments with superfluid $^3$He and other ordered materials. For a decade the European Science Foundation has supported a fruitful collaboration in this field between condensed matter, cosmology and particle physicists.

3. $^3$He as a test tube for the Universe

Superfluid $^3$He can play the role of a ‘test tube’ for the Universe since both are macroscopic quantum systems with rather complicated order parameters for the appropriate quantum vacua and various excitations and topological defects. The broken symmetries of both systems have many similarities. Of the few superfluid systems we know, superfluid $^4$He is too simple, only one gauge symmetry is broken there. However, in superfluid $^3$He, gauge, spin rotation and orbital rotation symmetries are all broken simultaneously. As a result, various superfluid states with different broken symmetries can exist below the transition, similar to the hypothetical states of the Universe. In $^3$He we can observe states with (i) combined spin–orbit rotation symmetry ($^3$He-B), (ii) two gauge symmetries ($^3$He-A), and (iii) one gauge symmetry ($^3$He-A1). After the transition, superfluid $^3$He can support many types of topological defect: monopoles (named boojums); various vortices (analogues of cosmic strings); and two-dimensional defects (analogues of branes). All these defects are shown schematically in figure 1. Various types of quasiparticle are also shown, the relics of normal $^3$He (fermions); magnetic excitations from the ordered magnetic ground state, magnons (bosons) and various excitations of the orbital fields (analogous to the interaction Higgs bosons), and the analogues of photons and gravitons. I would like to emphasize here for the first time that the quasiparticles and magnons do not interact directly with each other but only through the orbital momentum! This is a very interesting new analogy. They behave like luminous matter and dark matter, which interact only through gravity! Furthermore, magnons can form a BEC state, which we will discuss in detail later. One of the hypothetical dark matter candidates, the axion, can also possibly exist in a BEC state, which increases the difficulty of observing them directly in terrestrial experiments. The many examples of similarities make superfluid $^3$He a very important material for ‘experimental’ studies of cosmology!

There is one additional very important similarity. A current hot topic in cosmology concerns the recently discovered acceleration of the expansion of the

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Universe with an all-pervading dark energy supposedly responsible. Is there a dark energy analogue in $^3$He? Yes! At relatively high temperatures and pressures, the superfluid A-phase is stable. However, the liquid in the A-phase can be cooled to lower temperatures where the B-phase is stable with the A-phase in a metastable state. When the transition is finally triggered at some point, the B-phase expands to the region of the A-phase. The difference between the chemical potentials of the A and B phases is the dark energy which drives the expansion of the B-phase!

4. Fast transitions in $^3$He and the Universe

The conventional discussions of cosmological transitions in the Universe do not go beyond the simple model of a gauge transition with an energy profile in the shape of a ‘Mexican hat’. In this case, the order parameter is described by a single complex number or two Higgs fields with a single minimum and a degenerate phase. In condensed matter this transition corresponds to the $^4$He superfluid transition or to the superconducting transition in conventional superconductors. The order parameter in $^3$He, however, is described by nine complex numbers, giving a manifold of 18 dimensions. The hypothetical Universe $SU(5)$ symmetry can be represented by a 25-dimensional manifold. The multidimensional manifolds usually show an energy profile with a form much more complicated than the simple Mexican hat profile. There can be many local minima and saddle points, each corresponding to different states. In order to illustrate this, let us present graphically the potential as a deformed two-dimensional Mexican hat, as shown in figure 2.

At the transition, the vacuum state moves downhill from the central maximum (false vacuum). The direction of this movement is determined by some small initial perturbation, which has an equal probability for any of the quantities in the order parameter matrix. However, because the energy potential profile is not symmetric, the probabilities of dropping into state A or B are different and furthermore not determined by the ground state energy but by the asymmetry of the profile near the high-symmetry state (false vacuum).
We know practically nothing about the energy profile for corresponding cosmological transitions. However, for the superfluid transition in $^3$He we know the energy profile very well, at least in the Ginzburg–Landau approximation near $T_c$. On the basis of this energy profile, Bunkov & Timofeevskaya (1998) calculated the probability of nucleation of different states in $^3$He near $T_c$ by Monte Carlo methods. It was found that the probability strongly depends on the pressure, as shown in figure 3. In the same figure, the difference in energies between the A and B states is also shown. It was found that the pressure at which the two energies are equivalent is significantly different from that for the same nucleation probability. This observation allows us to suggest a new scenario for the A–B phase transition relevant for the Universe.

First of all, let us discuss the application of the Kibble–Zurek scenario to the superfluid $^3$He transition. During a fast transition, superfluid regions nucleate independently in causally independent regions. During subsequent cooling, these regions grow and touch each other but the phase of the wave function in these regions can match each other with a probability of only approximately 50%. If there is no match, then a topological defect appears. Using an equation suggested by Zurek (1985), one can calculate the density of the vortex tangle.

Figure 2. Schematic two-dimensional representation of the energy asymmetry of the 18-dimensional manifold of the superfluid $^3$He order parameter.

Figure 3. The probability of the A-state nucleation as a function of pressure for temperatures near $T_c$ and the energy difference ($\Delta F$) between the A and B states.
created for a given cooling rate. Bäuerle et al. (1996, 2000) measured the balance of energy after a neutron capture reaction in $^3$He and found that the missing deposited energy corresponds quantitatively to that retained in the vortex tangle. This experiment showed that the Kibble–Zurek theory works well, at least for a fast transition in superfluid $^3$He at low pressure.

The Kibble–Zurek scenario was developed on the basis of the simple Mexican hat potential. What should be changed in the case of a manifold order parameter? G. E. Volovik (1997, personal communications) and Bunkov & Timofeevskaya (1998) suggested that the causally independent regions can drop into different states! The consequences of these modifications for cosmology have not yet been discussed. For example, we do not know the creation probability of the vacuum of our Universe compared with that for the creation probability of the possible alternative vacua. We are not even certain that our vacuum is not a metastable state! And if it is, how long might other states have survived after the transition? Can the expansion of the $SU(3) \times SU(2) \times U(1)$ vacuum to the regions, initially occupied by other vacua, be considered as inflation? What about the dark energy which is observed in the Universe? Might it not just be the difference in energy between two low-lying states of the vacuum of the Universe, as it is in superfluid $^3$He? While we cannot give an answer, not knowing the energy profile for the multidimensional vacuum manifold of the Universe, we can apply these questions to superfluid $^3$He itself.

5. The A–B phase transition in $^3$He

The nucleation of the B-phase from the overcooled A-phase remains an unsolved problem. All the obvious mechanisms give nucleation rates that are literally hundreds of orders of magnitude below those observed experimentally. Leggett (1984) suggested that the transition might be triggered by particle absorption that would overheat the $^3$He locally. He also developed a theoretical scenario, the ‘baked Alaska’ model in which he supposed that the B-phase seed expands to the critical dimension $R_c$ while shielded from the surrounded A-phase by a region of normal $^3$He (see Leggett & Yip 1990). From our point of view, this mechanism is very artificial.

Experimentally, the A–B phase transition has been studied systematically in Helsinki (Hakonen et al. 1985). Their results showed a ‘catastrophic line’ at approximately $0.67 T_c$: a narrow range of temperatures in which the B-phase is always nucleated. They interpreted their data as being in conflict with the baked Alaska model. Systematic Stanford experiments (see review by Schiffer et al. 1995) demonstrated the importance of radiations for the A–B transition. The experimental probability of nucleating the A–B transition as a function of neutron or gamma radiation and temperature has been fitted by the theoretical expression below, suggested for the baked Alaska model,

$$\tau = C \exp[a(R_c/R_0)^n], \quad (5.1)$$

where $R_0$ is the value of $R_c$ at low temperature and magnetic field. The value for $n$ was estimated from the theory to lie between 3 and 5. However, the fitting gives a value for $n$ equal to 1.5! This difference of a factor of two or more in the exponents was claimed by Schiffer et al. (1995) to be in experimental agreement.

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with the theory! From our point of view, it shows only a lack of theory to explain the experiment. Furthermore, the baked Alaska scenario has an internal problem. It does not take into account the balance of energies of the whole ensemble of excited quasiparticles inside the hot spot. There is *not* enough phase space to distribute all the quasiparticles within the region of a shielding shell as was shown by Bunkov & Timofeevskaya (1999).

Importantly, we do not need a normal shell to protect the interior of the hot bubble from the influence of the outside state during the fast cooling. It has been shown by Kibble & Volovik (1997) that the diffusion temperature front moves so fast that the A-phase lags behind and seeds of new states appear independently of the surrounding $^3$He state, as shown in figure 4. After the neutron capture reaction, the dimensions of the hot spot ($R_h$) are considerably larger than the critical radius for B-phase nucleation ($R_c$). Thus, there will be a huge number of seeds of both A and B phases nucleated inside the hot spot. On our estimations, at 29 bar, $R_h \approx 30 \mu m$ and $R_0 \approx 0.5 \mu m$, while the radius of independently created seeds $R_s$ is approximately $0.07 \mu m$.

We can suggest that the subsequent evolution of the very complicated mixture of A and B phases is determined, first, by the relative densities of the seeds of the two phases, second by the surface energy of the seeds’ domain boundaries, and only third by the energy balance between the phases. Computer calculations by Bunkov & Timofeevskaya (1998) play an important role in the understanding of the A–B transition. They show that the probability equilibrium does not correspond to the equilibrium of the energy between the two states, but is significantly shifted to lower pressures, as shown in figure 3. This is the reason for the asymmetry in the A–B transition. Bunkov (2000) explained that when the B-state is energetically preferable, but the A-state has higher probability of nucleation, then the A-state seeds percolate inside the hot spot. Consequently, B-state seeds disappear owing to the A–B surface tension. In order to complete the transition, the B-phase seeds must percolate up to the critical cluster dimensions. This is possible when the number of B seeds in a region of dimensions

![Figure 4. A schematic of the A- and B-phase seeds that appear after diffusion cooling of the hot spot.](image-url)
$R_c$ is greater than the number of A seeds. There are two parameters that play an important role in these conditions: the probability of B-seed creation and the dimensions of $R_c$.

In order to compare the predictions of the new scenario with experimental results, we need to know the probability of A- and B-phase creation at low temperature. Unfortunately, we only know the probability of the A- and B-seed formations near $T_c$. However, we can make an estimate for lower temperatures from the $^3$He phase diagram. Were we to draw a line of equal probability parallel to the equilibrium line of the A–B transition in the phase diagram, we would delineate precisely the region where the A–B transition is triggered according to the measurements of Hakonen et al. (1985). In order to get a quantitative agreement with the probability of the A–B transition measured experimentally at Stanford (Schiffer et al. 1995), we should suggest that the probability of the A–B transition after a neutron impact corresponds to the probability of the correlated formation of approximately 50 seeds of the B-phase (Bunkov 2000). This means that the transition will trigger when approximately 50 B-phase seeds are created within a short distance of each other. It is difficult to envisage the dynamics of the interaction between the various seeds after the transition. Nevertheless, the correlation of 50 seeds looks like a reasonable result, because it makes up a big fraction of B seeds inside an $R_c$ volume, allowing the percolation of the B-state inside $R_c$ to trigger the transition.

Finally, this scenario, as suggested by Bunkov & Timofeevskaya (1998), is a natural explanation of the A–B transition in superfluid $^3$He on the basis of the well-established Kibble–Zurek scenario, without the need for any further artificial assumptions. Some criticisms of this theory are based mainly on a lack of understanding of the basic arguments.

6. The BEC of magnons

The theory of superfluidity and BEC is applicable to systems with conserved particle number. However, it can be extended to systems where the conservation law is only weakly violated. This means that it can be applied to systems with sufficiently long-lived quasiparticles, the discrete quanta of energy of which can be treated as the condensed matter counterparts of elementary particles. In magnetically ordered materials, the corresponding propagating excitations are magnons, the quanta of spin waves. Under stationary conditions the density of thermal magnons is small, but they can be generated by a resonant radio frequency (RF) field (magnetic resonance). One may expect that at very low temperatures this non-equilibrium gas of magnons could live a time long enough for the formation of a coherent magnon condensate.

In contrast to the static equilibrium magnetic state with a broken symmetry, a phase-coherent precession state is the dynamical state,

$$\langle \hat{S}_+ \rangle = S_+ = S \sin \beta e^{i\omega t+ia}. \quad (6.1)$$

Here $\hat{S}_+$ is the spin creation operator; $S_+ = S_x + i S_y$; $S = (S_x, S_y, S_z = S \cos \beta)$ is the vector of spin density precessing in the applied magnetic field $\mathbf{H} = H \hat{z}$; and $\beta$, $\omega$ and $\alpha$ represent the tipping angle, angular frequency and the phase of precession.
respectively. For the modes under discussion, the magnitude of the precessing spin $S$ is equal to the equilibrium spin density value $S = \chi H/\gamma$ in the applied field, where $\chi$ is the $^3$He-B spin susceptibility and $\gamma$ is the gyromagnetic ratio of the $^3$He atom. Similarly, to an atomic gas BEC, the spin precession in equation (6.1) can be rewritten in terms of a complex scalar-order parameter,

$$\langle \hat{\Psi} \rangle = \Psi = \sqrt{2S/\hbar} \sin \frac{\beta}{2} e^{i\omega t + ia}.$$

(6.2)

If the spin–orbit interaction is small (as is typical in $^3$He) its contribution to the magnon spectrum can be neglected in the general approximation. In this case, $\Psi$ coincides with the magnon annihilation operator, with the magnon number density being equal to the condensate density,

$$n_M = \langle \hat{\Psi}^\dagger \hat{\Psi} \rangle = |\Psi|^2 = \frac{S - S_z}{\hbar}.$$

(6.3)

This implies that the precession in superfluid $^3$He realizes an almost complete BEC of magnons. The small spin–orbit coupling produces a weak interaction between the magnons and leads to the interaction term in the corresponding Gross–Pitaevskii equation for the magnon BEC (below we use units with $\hbar = 1$),

$$\frac{\delta F}{\delta \Psi^*} = 0,$$

(6.4)

$$F = \int d^3r \left( \frac{\mathbf{\nabla} \Psi^2}{2m_M} - \mu |\Psi|^2 + E_D(|\Psi|^2) \right).$$

(6.5)

Here the role of the chemical potential $\mu = \omega - \omega_L$ is played by the shift of the precession frequency $\omega$ from the Larmor value $\omega_L = \gamma H$; the latter may slightly depend on coordinates if a field gradient is applied. In coherent states, the precession frequency $\omega$ is the same throughout the whole sample even in a non-uniform field; it is determined by the number of magnons in the BEC, $N_M = \int d^3r n_M$, which is a conserved quantity if the dipole interaction is neglected. In the continuous NMR regime, $\omega$ is the frequency of the applied RF field $\omega = \omega_{RF}$, and the chemical potential $\mu = \omega_{RF} - \omega_L$ determines the magnon density. Finally, $m_M$ is the magnon mass and $E_D$ is the dipole interaction averaged over the fast precession. The general form of $E_D(|\Psi|^2)$ depends on the orientation of the orbital degrees of freedom described by the unit vector $l$ of the orbital momentum. The details can be found in the article of Bunkov & Volovik (2008).

Given the complex properties of the dipole–dipole potential, the various possible experimental conditions of the magnetic field and field gradient, and the actual geometry of the sample, different types of magnon BEC can realized in superfluid phases of $^3$He. First of all, the first term in the gradient energy can compensate a small inhomogeneity of the magnetic field (second term) or the texture (third term) and lead to the formation of a coherent signal from a SSW. CW-NMR investigations of the formation of an SSW in a magnetic field gradient were made by Masuhara et al. (1984). The SSW spectrum in the ‘flare out’ texture was used for the first observation of quantum vortices in superfluid $^3$He-B (Ikkala et al. 1982). In the conditions of pulsed NMR in a homogeneous magnetic
field the SSWs were studied by Corruccini & Osheroff (1978) and Giannetta et al. (1981). It was found that the SSW radiates a long-lived induction signal of relatively small amplitude, the coherence of which is due to cavity resonance.

A very unique coherent quantum state was found by Borovik-Romanov et al. (1984) in superfluid $^3$He. When a large gradient of magnetic field is applied, the deflected magnetization can reorganize itself into two domains. In the first, the magnetization is stationary and in the second it is deflected just beyond the magic $104^\circ$ angle and precesses spatially homogeneously (HPD). The additional dipole–dipole energy term compensates the large magnetic field gradient. The cell must be closed in order to prevent the magnons from leaving the cell. A detailed explanation of an approximate 100% BEC of magnons under these conditions can be found in a recent article by Bunkov & Volovik (2008). Figure 5 shows the instantaneous spectrum of the HPD induction signal. Immediately after the exciting pulse, the spins rotate in the local field, which causes a width of 600 Hz. After a delay of approximately 30 ms the magnons redistribute themselves under the influence of the magnetic field gradient and the HPD forms. The domain radiates a very monochromatic RF signal that shows a nearly complete BEC with a chemical potential corresponding to the Larmor frequency at the position of the domain boundary. With time, the number of magnons decreases, the domain boundary moves to a region of lower Larmor frequency and the chemical potential and frequency decrease correspondingly, as shown in figure 5. If the SSW magnon BEC corresponds to a BEC in a weakly interacting gas, then the HPD is characterized by an additional mean field, which compensates the magnetic field gradient. This effect is the magnetic analogy of the Meissner effect in superconductors. The HPD shows all the properties of a condensed matter superfluid state and can be considered a ‘spin superfluid’ state. Long ago long-range spin supercurrent transport of magnetization was observed in this state. It is important to note that the spin supercurrent is independent of any mass supercurrent in this state. Related phenomena have also been observed: spin current Josephson effect; phase-slip processes at the critical current; and a

Figure 5. The instantaneous spectrum of the HPD induction signal.
spin current vortex—a topological defect that is the analogue of a quantized vortex in superfluids and of an Abrikosov vortex in superconductors, etc. (see review of Bunkov 1995 and references therein).

In conclusion, the BEC of magnons in superfluid $^3$He has been observed under many different conditions, as described previously. It seems that this coherent magnon condensate in superfluid $^3$He will prove even more rich than the BEC of dilute gases.

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