Brane inflation and defect formation

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Brane inflation and the production of topological defects at the end of the inflationary
phase are discussed. After a description of the inflationary set-up, we discuss the
properties of the cosmic strings produced at the end of inflation. Specific examples of
brane inflation are described, such as $D-D$, $D3/D7$ and modular inflations.

Keywords: brane inflation; cosmic superstrings; moduli stabilization

1. Introduction

Superstring theory has so far been our best hope for a theory that unifies quantum
mechanics, gravity and the other known forces in nature (see, for example,
Polchinski (1998) and Becker et al. (2006) for an overview of string theory). Until
recently, it has been very difficult to conceive tests of stringy predictions, i.e.
departure from usual particle physics, which are not all at very high energy scales
close to the Planck scale. However, it has recently been realized that cosmological
considerations in string theory lead to verifiable predictions, testable with current
and future cosmological experiments such as WMAP and PLANCK. The reason
for this is twofold. On the one hand, theoretical developments have led to string
predictions at energy scales less than the Planck scale due to a method of
compactification with fluxes. On the other hand, cosmologists now believe that the
observed structures in the Universe, such as galaxies and clusters of galaxies, have
their origin in the very early Universe (e.g. Mukhanov (2006) for an overview).
Most cosmologists believe that the seeds of these structures were created during a
period of rapid exponential expansion, called inflation.

Inflation was proposed to solve a number of cosmological problems in the early
Universe, such as the flatness problem, the horizon problem and the over-
production of defects. It predicts an almost scale-invariant power spectrum
and temperature anisotropies in excellent agreement with the recent maps
from satellite experiments of COBE and WMAP (e.g. Liddle & Lyth 2000;

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A fundamental issue is to find realistic models of inflation within particle physics theories. On the other hand, if string theory really is the theory of everything, then it would be natural to search for a model of inflation within string theory. A number of recent developments within string theory have enabled one to start to build a model of inflation, called brane inflation (e.g. Dvali & Tye (1998), Kachru et al. (2003b), Burgess et al. (2004) and references therein). In the following section, we will describe the idea of brane inflation and discuss the formation of topological defects in these models.

The outline of this paper is as follows. We first give a general introduction to brane inflation in string theory. Brane inflation results in the formation of lower-dimensional branes and, in particular, $D$-strings. We discuss the properties of $D$-strings in the low-energy supergravity description of brane inflation. Finally, we discuss the possibility that cosmic string formation at the end of brane inflation may not be generic in the context of modular inflation.

2. Brane inflation

The number of string inflation models is rather large, hence we will focus on a limited number of examples. All these examples will have a supergravity counterpart and are describable by a field theory at low energy. There are two types of relevant models. The first is the $D3 - \bar{D}3$ system and the second is the $D3/D7$ system; both realize hybrid inflation in string theory.

Before delving into the presentation of these systems, let us recall some basic notions about branes in string theory (see Becker et al. 2006). String theory is formulated in 10 space–time dimensions, with nine spaces and one time. It deals with closed and open strings. The open strings are strands ending on extended objects called branes, i.e. their extremities are attached to submanifolds of the 10-dimensional space–time. Type IIB string theory admits branes of odd spatial dimensions. After reducing space–time on a six-dimensional manifold in order to retrieve our four dimensions of space–time, only $D3$ and $D7$ branes give supersymmetric (which is a symmetry between fermions and bosons) configurations. Anti-$D$ branes preserve opposite supersymmetries and therefore break all supersymmetries in the presence of branes. A $D3 - \bar{D}3$ system breaks all supersymmetries explicitly while a $D3/D7$ system preserves supersymmetry. The compactification process on a six-dimensional manifold introduces a host of massless fields called moduli. They parametrize all the possible deformations of the six-manifold. The existence of massless moduli would have a catastrophic effect in the low energy description of string theory: they would imply a strong modification of Newton’s law of gravity. To avoid this, moduli must become massive and therefore acquire a potential that stabilizes their value. The potential for the moduli has been obtained recently in a scenario called the KKLT approach (see Kachru et al. 2003a). It involves two new ingredients. The first one are fluxes that resemble constant magnetic fields created by wires. These fluxes stabilize half of the moduli, leaving only the moduli measuring the size of the six-manifold untouched. They also imply that the compactification manifold is warped, i.e. looks like an elongated throat attached to a region called the bulk, as shown in figure 1. When space–time is warped the invariant distance...
becomes
\[ ds^2 = e^{-A(y)}(dt^2 - dx^2) - dy^2, \tag{2.1} \]
where \( A(y) \) is the warp factor and \( y \) represents the extra dimensions. At the bottom of the throat, energy levels are redshifted and can be much lower than the string scale. The size moduli can be stabilized, thanks to non-perturbative effects on \( D7 \) branes. Unfortunately, the stabilization is realized with a negative vacuum energy, i.e. Anti-de Sitter (AdS) space–time. It is necessary to lift this up to Minkowski space–time with an ‘uplifting’ mechanism, which is the second new ingredient.

Let us first discuss inflation in the \( D3 - \overline{D3} \) system (see Kachru et al. 2003b). First of all, the \( D3 \) has a very specific role in this model. It gives the positive energy that is necessary to lift up the negative minimum of the moduli potential. Inflation is obtained after introducing a \( D3 \) brane that is attracted towards the \( \overline{D3} \) brane. The inflaton field is the distance between the branes. As the branes get closer, an instability develops in a way similar to hybrid inflation, with potential as in figure 2. This model is a nice description of hybrid inflation in string theory and one of the main achievements of string inflation, providing a fundamental description of the inflaton field. However, there are problems with this. As a paradigm, the \( D3 - \overline{D3} \) system has a few unambiguous consequences: the level of primordial gravitational waves is very small and cosmic strings with a very low tension may be produced,
\[ \mu = e^{-A(y)} \mu_0, \tag{2.2} \]
where \( \mu_0 \) is the fundamental string tension. Nevertheless, the lack of a proper supersymmetric setting for the \( D3 - \overline{D3} \) renders it less appealing than the \( D3/D7 \) system that we are about to discuss.
The $D3/D7$ system has two big advantages over the $D3 - \overline{D3}$ systems: it contains supersymmetry and a shift symmetry, which eliminates the so-called $\eta$ problem (see Dasgupta et al. 2004). In the $D3/D7$ system, the inflaton is the inter-brane distance, while there are waterfall fields corresponding to open string joining the branes. It is a clean stringy realization of $D$-term inflation. In this context, the existence of Fayet–Iliopoulos terms is crucial. They are needed to lift the moduli potential and provide the non-trivial value of the waterfall fields at the end of inflation.

They can be obtained using magnetic fields on $D7$ branes. The condensation of the waterfall fields is the equivalent of the stringy phenomenon whereby a $D3$ brane dissolves itself into a $D7$ brane, leaving an embedded $D1$ brane as remnants. These branes are one-dimensional objects differing from fundamental strings. It has been conjectured and there is evidence in favour of the fact that these $D$-strings correspond to the Bogomol’nyi–Prasad–Sommerfield (BPS) $D$-term strings of the low-energy supergravity description. Unfortunately, it turns out that the tension of these strings is quite high, implying that they would lead to strong features in the cosmic microwave background (CMB) spectra. At the moment, the problem of both generating a low tension $D$-string and inflation at the relevant scale, given by the COBE normalization, has not been solved. Including more $D7$ branes gives semi-local strings (see Urrestilla et al. 2004) that alleviate the problem. On the positive side, these models have the nice feature of giving $n_s \approx 0.98$, which could be compatible with the data. On the other hand, the spectrum of gravitational waves is undetectable.

The $D3 - \overline{D3}$ and $D3/D7$ systems are promising avenues to obtain realistic string inflation models. At the moment, they are no more than plausible scenarios. Inflation being a very high scale phenomenon, it is clear that the study of early Universe phenomena could be a crucial testing ground for string theory.
3. Cosmic $D$-strings

The formation of cosmic strings appears to be a generic feature of recent models of brane inflation arising from fundamental string theory (e.g. Sarangi & Tye (2002) and Copeland et al. (2004) and for reviews see Polchinski (2004) and Davis & Kibble (2005)). Indeed, lower-dimensional branes are formed when a brane and anti-brane annihilate, with the production of $D3$ and $D1$ branes, or $D$-strings, favoured (see Majumdar & Davis 2002). It has been argued that $D$-strings have much in common with cosmic strings in supergravity theories and that they could be identified with $D$-term strings (see Dvali et al. 2004). This is because $D$-strings are minimum energy solutions known as BPS solutions. Hence, the $D$-term strings in supergravity are such BPS solutions. Not only are BPS strings of minimum energy, but the BPS property means that the equations of motion, which should be second-order differential equations, reduce to first-order equations, making them simpler to solve.

While cosmic strings in global supersymmetric theories have been analysed before (see Davis et al. 1997), the study of such strings in supergravity is at an early stage since there are added complications. In Brax et al. (2006), an exhaustive study of fermion zero modes was performed. In supersymmetry, there are fermions as well as bosons in the action. The fermion partners of the Bose string fields can be zero modes. These are zero-energy solutions of the Dirac equation. It was found that, owing to the presence of the gravitino, the number of zero-mode solutions in supergravity is reduced in some cases. For BPS, $D$-strings with winding number $n$, the number of chiral zero modes is $2(n-1)$, rather than $2n$ in the global case. When there are spectator fermions present, as might be expected if the underlying theory gives rise to the standard model of particle physics, then there are $n$ chiral zero modes per spectator fermion.

Following Dvali et al. (2004), we use the supergravity description of $D$-strings by $D$-term strings. Consider a supergravity theory with fields $\phi^\pm$, charged under an Abelian gauge group. The $D$-term bosonic potential

$$V = \frac{g^2}{2} (|\phi_+|^2 - |\phi_-|^2 - \xi)^2$$  \hfill (3.1)

includes a non-trivial Fayet–Iliopoulos term $\xi$. Such a term is compatible with supergravity provided the superpotential has charge $-\xi$. But here the superpotential vanishes. The minimum of the potential is $|\phi_+|^2 = \xi$. It is consistent to take the cosmic string solution to be

$$\phi_+ = f(r)e^{in\theta},$$ \hfill (3.2)

where $n$ is the winding number and $\phi_- = 0$. The function $f(r)$ interpolates between 0 and $\sqrt{\xi}$. The presence of a cosmic string bends space–time. Its gravitational effects lead to a deficit angle in the far-away metric of space–time. In the following, we consider the metric

$$ds^2 = e^{2B}(-dt^2 + dz^2) + dr^2 + C^2 d\theta^2$$ \hfill (3.3)

for a cosmic string configuration. This is the most general cylindrically symmetric metric. Far away from the string, the energy–momentum tensor is
approximately zero and therefore $B$ is zero. Now we also find that

$$C = C_1 r + C_0 + O(r^{-1}). \quad (3.4)$$

When $C_1 \neq 0$, the solution is a cosmic string solution with a deficit angle $\Delta = 2\pi(1 - C_1)$.

In supersymmetric theories, a cosmic string generally breaks all supersymmetries in its core. BPS objects are an exception to this rule, as they leave one-half of the original supersymmetry unbroken. $D$-strings, in which we will be interested in this paper, are an example of this. These strings have vanishing $T_{rr}$ and the conformal factor $B$ is identically zero. Moreover, for $D$-strings one finds

$$\Delta = 2\pi|n|\xi. \quad (3.5)$$

The supersymmetry algebra in four dimensions allows for one-half of BPS configurations that saturate a BPS bound, giving an equality between the mass, i.e. the tension, and a central charge. Other cosmic strings have higher tension and are not BPS, i.e. they break all supersymmetries. This implies that $C_1$ is less than the BPS case, giving

$$\Delta \geq 2\pi|n|\xi. \quad (3.6)$$

Let us characterize the BPS cosmic strings. We are considering a $U(1)$ symmetry breaking, so we take its generator $T_s \phi^i = nQ_i \phi^i$ and $A_\mu = \delta_\mu^a n a(r)$. The bosonic fields satisfy the first-order equations

$$\partial_r \phi^i = \mp n \frac{1}{C} Q_i \phi^i \quad (3.7)$$

and

$$\mp n \frac{\partial_r a}{C} = D = \xi - \sum_i Q_i K_i \phi^i, \quad (3.8)$$

where $\xi$ is the Fayet–Iliopoulos term that triggers the breaking of the $U(1)$ gauge symmetry and $K$ is the Kahler potential. The Einstein equations reduce to

$$C' = 1 \pm A_\theta^B, \quad (3.9)$$

where

$$A_\theta^B = \frac{i}{2} \left( \bar{K}_j D_\theta \bar{\phi}^j - K_j D_\theta \phi^j \right) + \xi A_\theta. \quad (3.10)$$

The simplest BPS configuration will just have one $\phi$, with unit charge and $K = \phi^+ \phi^-$. Note that the BPS cosmic strings are solutions of the first-order differential equations. These equations are consequences of the Killing spinor equations when requiring the existence of one-half of supersymmetry.

### 4. A specific realization of brane inflation

As mentioned previously, a specific realization of brane inflation is obtained in the system with $D3$ and $D7$ branes, giving rise to the formation of $D$-strings at the end of inflation. Here we use the KKLT model with superpotential

$$W = W_0 + Ae^{-aT}, \quad (4.1)$$
where $W_0$ arises after integrating out the other moduli and the exponential term arises from non-perturbative effects. In order to keep the supersymmetry, we can uplift with a so-called Fayet–Iliopoulos term. This is more complicated in supergravity than in ordinary supersymmetry. However, it can be achieved using a method to ensure that any anomalies are cancelled, such as the so-called Green–Schwarz mechanism (see Achucarro et al. 2006). This gives us a more complicated superpotential,

$$W_{\text{mod}}(T, \chi) = W_0 + \frac{A e^{-aT}}{\chi^b},$$

(4.2)

where $\chi$ is a field living on $D7$ branes. Following Brax et al. (2007a), we use a no-scale Kahler potential and include the inflationary terms arising from the superstrings between the branes,

$$W_{\text{inf}} = \lambda \phi^+ \phi^-.$$

(4.3)

We get the potential $V = V_F + V_D$, where

$$V_D = \frac{g^2}{2} (|\phi^+|^2 - |\phi^-|^2 - \xi)^2$$

(4.4)

for the $D$-term, and the $F$-term is calculated from the superpotential. The Fayet–Iliopoulos term, $\xi$, is obtained from anomaly cancellation and depends on the fields, $\chi$ and $T$. The scalar potential is quite complicated, consisting of the part from the superpotential, which is called the $F$-term part, and the $D$-term part. We have plotted them in figure 3, where we can see that the original $F$-term part has a minimum that is less than zero, but the combined potential has a minimum at zero when one adds in the contribution of $V_D$.

Inflation proceeds as in figure 2 with the charged $\phi^\pm$ fields being zero, but the $\phi$ field being non-zero until the $D3$ brane gets close to the $D7$ brane. The charged fields become tachyonic, acquiring a vacuum expectation value. They are called ‘waterfall’ fields since the fields fall to their minimum values. At this point
cosmic superstrings, or $D$-strings, are formed. We have analysed the inflation in this theory and found that it gives the density perturbations and spectral index in agreement with the data from WMAP3. We have also analysed the string solutions formed at the end of inflation. In our model of $D$-term inflation coupled to moduli, cosmic strings form during the breaking of the Abelian gauge symmetry $U(1)$ by the waterfall field $\phi^+$ at the end of inflation. The cosmic strings in our model are not BPS as in usual $D$-term inflation. This is due to the non-vanishing of the gravitino mass and the coupling of the string fields to the moduli sector. We find that the cosmic string solution has the form

$$\phi^+ = \xi^{1/2} e^{in\theta} f(r), \quad \phi^- = \phi = 0, \quad A_\theta = na(r),$$

and the moduli sector fields depend only on $r$. The functions $f(r)$ and $a(r)$ differ from the standard ones appearing in the Abelian string model (e.g. Vilenkin & Shellard 2000) as they include the effect of contributions from the moduli field and $\chi$ arising from the full potential. The Higgs field tends to its vacuum value at infinity, and regularity implies that it is zero at the string core. Without loss of generality, we can take the winding number as positive, $n>0$. If the variation of the moduli fields inside the string is small, the string solution will be similar to a BPS $D$-string, with a similar string tension, which (for $n=1$) is

$$\mu \approx 2\pi \xi.$$

The above result will receive corrections from the one-loop contribution to the potential. We also expect that the variation of the moduli fields inside the string will reduce to $\mu$.

### 5. Is cosmic string formation generic?

There have been recent attempts to use the string moduli fields themselves as the inflaton field. This type of model is known as racetrack inflation (see Blanco-Pillado et al. 2004, 2006; Brax et al. 2007). One can use either a $\tilde{D}$ brane or a $D$-term for uplifting, as in §4. In this class of theory there are no matter fields since the open string modes between branes are not included. Consequently, there are no cosmic strings produced at the end of the inflation. However, this is a rather artificial situation. Here we will describe a preliminary attempt to include the matter fields.

The superpotential for racetrack inflation is

$$W = W_0 + Ae^{-aT} + Be^{-bT},$$

where $a$ and $b$ are constants, and $A$ and $B$ are constants in the original racetrack model, but can be functions of fields in the case of $D$-term uplifting as in §4. To include the open string modes then one adds the additional term

$$W = W_{RT} + \lambda \phi \phi^+ \phi^-.$$

Preliminary results suggest that cosmic strings are produced only during inflation and are inflated away (P. Brax et al. 2008, unpublished data). It is, therefore, unlikely that cosmic strings are produced in the racetrack models discussed here.
6. Conclusions

Recent developments in string theory have resulted in the possibility of testing string theory with cosmology. Brane inflation is the most promising string-motivated model of cosmological inflation. This predicts cosmic $D$-string formation at the end of inflation, except for the possible case of moduli inflation. It is thus possible that cosmic $D$-strings could provide a window into physics at very high energy and very early times. This exciting possibility has even been investigated in the laboratory (see Bradley et al. 2008), where the analogue of branes in superfluid helium-3 have been observed to annihilate and leave remnant strings behind. Much work is still required before realizing the possibility of testing string theory with cosmology. A better characterization of the properties of $D$-strings would be necessary and a clear understanding of how their experimental signals might differ from ordinary cosmic strings would be required. Similarly, more theoretical work is necessary on string-motivated inflation models. However, the progress so far has been fruitful.

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