The case for artificial black holes

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The event horizon is predicted to generate particles from the quantum vacuum, an effect that bridges three areas of physics—general relativity, quantum mechanics and thermodynamics. The quantum radiation of real black holes is too feeble to be detectable, but black-hole analogues may probe several aspects of quantum black holes. In this paper, we explain in simple terms some of the motivations behind the study of artificial black holes.

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1. Astrophysical black holes

It is quite remarkable that some laboratory experiments on Earth may test elusive aspects of astrophysics. Probably the best example is the quantum physics of the black hole. Classical black holes are strictly black: nothing, not even light, can escape. However, according to quantum physics, the black hole spontaneously emits particles (Hawking 1974, 1975; Birrell & Davies 1984; Brout et al. 1995a). These particles are virtual particles of the quantum vacuum that materialize at the horizon of the black hole: the vacuum resembles a sea of particle and antiparticle pairs that are continuously created and annihilated. When a particle has the misfortune of being created at one side of the horizon while leaving its partner on the other side, the pair can no longer annihilate each other and is forced to materialize. This explanation is of course only a cartoon picture and not a quantitative physical model: it explains why particle creation is possible at horizons, but it is not suitable to make a quantitative prediction of the particle flux. Calculations (Hawking 1974, 1975; Birrell & Davies 1984; Brout et al. 1995a) show that the black hole emits black-body radiation, in thermal equilibrium, with the temperature

\[ k_B T = \frac{\hbar \alpha}{2\pi}, \quad \alpha = \frac{c^3}{4GM} = \frac{c}{2R}, \]

where \( G \) is the gravitational constant; \( k_B \) is Boltzmann’s constant; \( c \) is the speed of light in vacuum; \( M \) is the mass of the black hole; and \( R \) is the Schwarzschild radius. Both the spectrum and the quantum statistics of the radiation are consistent with the thermal prediction (1.1) first made by Hawking (1974). This

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is a remarkable and mysterious result: it is simple and universal; it connects the physics of the very small quantum mechanics characterized by $\hbar$, to the physics of the very large general relativity with $G$ and thermodynamics with $k_B$. It implies that the black hole has an entropy (Bekenstein 1973), a celebrated relationship that has been frequently used as a benchmark for potential quantum theories of gravity, such as superstring theory (Green et al. 1987) and loop quantum gravity (Rovelli 1998).

The problem is that it appears to be extremely difficult to observe Hawking’s effect in astrophysics, as one can see from a simple estimation of the temperature (1.1). The characteristic frequency $\omega$ of the radiation is $c/(4\pi R)$; therefore, the characteristic wavelength $8\pi^2 R$ is approximately two orders of magnitude larger than the Schwarzschild radius. A solar-mass black hole with $R \approx 3 \times 10^3$ m emits an electromagnetic Planck spectrum peaked at wavelengths of hundreds of kilometres. It is very cold indeed, with a Hawking temperature of approximately $6 \times 10^{-8}$ K, much below the cosmic microwave background. Smaller black holes radiate stronger, for a simple reason: in our cartoon picture, the particles are separated by tidal forces. Smaller black holes need to reach the central singularity in a smaller distance and therefore have higher gravity gradients at their horizons.

2. Artificial black holes

In the laboratory we can make small things. However, with present technology, we cannot create real black holes. What we can make are artificial black holes (Novello et al. 2002; Volovik 2003; Unruh & Schützhold 2007), objects that are analogue models of black holes. Most proposals for artificial black holes are based on the simple idea (Unruh 1981; Visser 1998; Volovik 2003) illustrated in figure 1. Imagine a moving medium, say a river, with variable speed. This river represents gravity. Consider waves propagating in the moving medium. The waves are dragged by the medium; co-propagating waves move faster and counter-propagating waves are slowed down. Suppose that at some point the river exceeds the speed of the waves. Clearly, beyond this point no counter-propagating wave would be able to propagate upstream. The point where the moving medium matches the wave velocity is a horizon. In the first practical
demonstration of a horizon (Rousseaux et al. 2008), water waves in a narrow channel were sent against the current. The simple analogy between gravity and moving media is surprisingly accurate and universal; for example, the propagation of sound in moving fluids is exactly equivalent to the propagation of scalar waves in general relativity, as was first noted by Moncrief (1980) and shortly later by Unruh (1981). Most other candidates for artificial black holes are based on variations of this idea.

Of course, this analogy is limited. Waves in moving media behave like waves in gravitational fields, in curved space–time, but the mechanism for creating this effective space–time geometry is different from gravity. In place of mass in general relativity, moving media generate effective geometries. Artificial black holes capture the kinematic aspects of black holes, not the dynamic ones. On the other hand, concerning the kinematic aspects, analogue models of gravity demonstrate not only the classical physics of black holes, showing, for example, the trapping of waves, but also the quantum physics, because the interaction between the moving medium and the wave is linear, regardless of how small the wave amplitudes are, even down to the quantum scale.

However, quantum effects such as Hawking radiation are subtle. In moving media, the Hawking temperature should be related to the velocity gradient at the horizon. The gradient of a velocity has the dimension of a frequency. Consequently, one might guess that the characteristic frequency $\alpha$ in Hawking’s formula (1.1) corresponds to the velocity gradient at the horizon, which turns out to be the correct result (Unruh 1981). The observation of the Hawking effect presumes that the moving fluid is colder than the Hawking radiation, so that the radiation is detectable, and usually requires cooling to ultra-low temperatures. Bose–Einstein condensates or quantum liquids like superfluid helium-3 (Volovik 2003) could still behave as fluids even at the extremely low temperatures needed, whereas the water wave analogue of the black hole (Rousseaux et al. 2008) is likely to remain a demonstration of the classical physics of the black hole. On the other hand, at the heart of the Hawking effect lies a classical process—the generation of negative frequency waves—that experiments with waves in water can demonstrate. Recently, we have seen the partial conversion of positive into negative frequency waves (Rousseaux et al. 2008), but the quantitative findings of this experiment significantly disagree with theory. Other processes, not only the Hawking effect, may enhance or facilitate the mode conversion, which is a point that needs further clarification.

Optical black holes, where media move faster than the speed of light in the medium, the speed of light in vacuum $c$ divided by the refractive index $n$, are excellent candidates to observe the quantum effects of black holes, because velocity gradients within a few wavelengths of light are conceivable. Such gradients would correspond to Hawking temperatures of the order of $10^3$ K. We found a method to create such a superluminally moving medium in the laboratory (Leonhardt & König 2005) and recently obtained the first experimental results on classical effects of horizons (Philbin et al. 2008).

To establish an analogue of the event horizon in optics, one has to move an optical medium at the speed of light in the medium. This is possible in principle, because media typically reduce the speed of light in vacuum $c$ by a refractive index $n$ that is larger than unity, but it seems very difficult in practice, because the refractive index normally does not exceed unity by a large factor; light is still...
fast in normal media. One could imagine using slow light in atomic media (Leonhardt & Piwnicki 2000; Leonhardt 2002), but typically here only the group velocity is reduced and not the phase velocity of light, and slow light is restricted to a small frequency window that is easily exceeded by the frequency shifts at horizons. However, it turns out (Leonhardt & König 2005; Philbin et al. 2008) that moving an optical medium at the speed of light is much easier than slowing down light. Moreover, this happens all the time in optical telecommunication.

In fibre-optical telecommunication, the information carriers are light pulses confined to the core of optical fibres. The propagation of these pulses depends on the effective refractive index of the fibre, but they also modify the refractive index due to the Kerr effect (Agrawal 2001): the effective refractive index of the fibre gains an additional small contribution $\delta n$ that is proportional to the instantaneous pulse intensity. This contribution to the refractive index $n$ moves with each pulse. The pulse thus establishes a moving medium, although nothing material is moving. This effective medium naturally moves at the group velocity of light in the fibre, because it is made by light itself, and it easily exceeds the phase velocity for light modes in the near ultraviolet. Each pulse establishes two artificial event horizons for ultraviolet light modes, a black-hole horizon at the front where the Kerr effect of the pulse enhances the refractive index, reducing the phase velocity below the pulse speed, and a white-hole horizon at the trailing end of the pulse. These horizons would also exist and have a physical effect if no ultraviolet light is present, which is normally the case. They should still spontaneously emit a Hawking radiation of ultraviolet light quanta. So, whenever people communicate via fibre optics, they create numerous artificial event horizons as a side effect without noticing it. However, the effective Hawking temperature of such telecommunication horizons is extremely low and therefore such effects have remained unnoticed and are completely negligible in practice. The reason is that the Hawking temperature is given by the logarithmic derivative of $\delta n$ at the horizon (Philbin et al. 2008) such that substantial temperatures can only be achieved for intensity gradients smaller than the relevant optical wavelengths. It takes both the latest advances in the generation of few-cycle optical pulses (Brabec & Krausz 2000; Kärtner 2004) and some of the most advanced optical fibres, photonic crystal fibres (Russell 2003), in order to enhance an effect that in principle exists in ordinary fibre-optical telecommunication to the level where it becomes observable in the laboratory.

### 3. Probing trans-Planckian physics

What would we learn from artificial black holes? In general relativity, the physics at horizons appears to tend to extremes. Although the gravitational field is regular near the horizon (unless expressed in irregular coordinates such as Schwarzschild coordinates), waves freeze here, because they are trapped.

In the vicinity of a horizon, waves are forced to oscillate with ever decreasing wavelengths the closer they are to the horizon (figure 2). The wavelength decreases below all scales, including the Planck scale where one commonly is in doubt whether the presently known physics is applicable. This issue is known as the trans-Planckian problem (t’Hooft 1985; Jacobson 1991). Some unknown mechanism must regularize the horizon of the astrophysical black hole.

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Artificial black holes have natural and known mechanisms to regularize the extremes of waves at horizons. Understanding them in detail may illuminate some of the trans-Planckian physics of gravity. The simplest mechanism is dispersion: decreasing wavelengths in the laboratory frame correspond, via the Doppler effect, to increasing local frequencies in the medium. In our optical model (Leonhardt & König 2005; Philbin et al. 2008), the refractive index will change with frequency due to optical dispersion (Born & Wolf 1999) and the waves are able to escape from the horizon. Consider the ray tracing at the black-hole horizon backwards in time as shown in figure 3, remembering that backwards in time the flow changes direction. For normal dispersion (Born & Wolf 1999) the speed of light is reduced and the rays are dragged with the moving medium to the right. For anomalous dispersion, the speed of light increases with increasing frequency and the rays are dragged to the left. Seen forwards in time, the rays stem from the subluminal region for normal dispersion and from the superluminal region for anomalous dispersion.

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Strictly speaking, genuine horizons do not exist due to dispersion. However, the Hawking effect is remarkably robust against moderate dispersion (Unruh 1995; Brout et al. 1995b; Balbinot et al. 2005). Realistic experiments, however, tend to operate in a strongly dispersive regime. Here several natural questions
arise. The horizon is the place where the speed of the medium is equal to the speed of the wave. Which wave speed matters, the group or the phase velocity or something in between? Both the group and the phase velocity depend on frequency. Which frequency matters? What is indispensable for the Hawking effect and what is not? What really is a horizon? Real experiments pose questions that otherwise one would have the liberty to avoid. They pose a challenge and they will give real data to support our understanding. It is highly probable that analysing laboratory analogues of black holes will shed new light on one of the most fascinating effects in physics, the quantum creation at horizons.

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