‘Exotic’ quantum effects in the laboratory?

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This article provides a brief (non-exhaustive) review of some recent developments regarding the theoretical and possibly experimental study of ‘exotic’ quantum effects in the laboratory with special emphasis on cosmological particle creation, Hawking radiation and the Unruh effect.

**Keywords:** analogue gravity; Hawking radiation; Unruh effect

1. Introduction

The combination of quantum field theory with non-trivial classical background configurations—such as a gravitational field—yields many fascinating effects (see Birrell & Davies 1982; Fulling 1989). A striking example is the amplification of omnipresent quantum vacuum fluctuations by the influence of the classical background configuration and thereby their conversion into real particles. This phenomenon may occur in an expanding universe (cosmological particle creation) and lies at the heart of Hawking radiation (Hawking 1974, 1975), which is related to the Unruh effect (Unruh 1976) via the principle of equivalence.

In order to discuss these phenomena, it is useful to recall some of the basic properties of the quantum effects under consideration. Because the created particles stem from quantum (vacuum) fluctuations, they vanish in classical limit \( \hbar \rightarrow 0 \). The particles are always created in pairs (squeezed vacuum state) in an entangled state (which is an inherently non-classical feature). In cosmological particle creation, the two particles of each pair have opposite spins and momenta, whereas for the Hawking and Unruh effects, the two partners occur on either side of the horizon. The two-particle states are pure quantum states while averaging over one partner yields the thermal density matrix (entanglement entropy).

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1 According to our understanding of the evolution of our Universe, traces of such an effect—i.e. the amplification of quantum vacuum fluctuations during the epoch of cosmic inflation—can still be observed today in the anisotropies of the cosmic microwave background radiation.

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2. The underlying analogy

Apart from their quantum nature, which implies a suppression of these effects in a typical laboratory scenario due to the smallness of $\hbar$, they are intrinsically relativistic phenomena and hence additionally hard to observe in view of the large value of the speed of light $c$. Therefore, it can be advantageous to consider the analogues of these effects in appropriate condensed-matter systems, where the speed of light $c$ should be replaced by the propagation velocity of the quasi-particles under consideration (e.g. phonons that travel with the speed of sound; see Unruh 1981). The analogy can be made explicit by writing down the most general linearized $O(\phi^3)$ low-energy $O(\partial^3)$ effective action for scalar Goldstone-mode quasi-particles $\phi$, which can be cast into the following form:

$$L_{\text{eff}} = \frac{1}{2} (\partial_{\mu}\phi)(\partial_{\nu}\phi) G_{\mu\nu}(x) + O(\phi^3) + O(\partial^3),$$

where $G_{\mu\nu}(x)$ denotes some tensor that encodes the details of the underlying condensed-matter system. Through the replacement $G_{\mu\nu} \to g_{\text{eff}}^{\mu\nu} \sqrt{-g_{\text{eff}}}$, we see that the kinematic aspects (i.e. the equations of motion and the commutation relations, etc.) of the quasi-particles $\phi$ are completely equivalent to a scalar field in a curved space-time described by the effective metric $g_{\text{eff}}^{\mu\nu}$.

Now, how can we exploit this analogy and what can we learn by using it? Of course, the first answer could be that the aforementioned quantum effects have merely been predicted, but not directly observed yet. Hence, the analogy (2.1) facilitates an experimental verification of these phenomena (by means of their laboratory analogues) and a test of the assumptions/approximations entering their derivation. Furthermore, it allows us to study the corrections imposed on these effects due to the impact of interactions, the back-reaction of the quantum fluctuations onto the classical background, the influence of a finite temperature bath and decoherence, etc. Finally, apart from the purely experimental point of view, we may use the condensed-matter analogues as theoretical toy models for quantum gravity, which inspire new ideas for studying the influence of the microscopic structure on macroscopic phenomena (for example, see §3).

The other way around, the analogy (2.1) may help us to understand non-equilibrium (quantum) phenomena in general condensed-matter systems in terms of the vast amount of geometrical tools and concepts developed within general relativity and to extract universal features (for the low-energy quasi-particle kinematics, we may forget about the microscopic details of the underlying condensed-matter system and derive everything from the effective metric $g_{\text{eff}}^{\mu\nu}$, see Schützhold & Uhlmann (2005); Schützhold & Unruh (2007); Schützhold (2008)).

3. UV catastrophe

Before turning to experiments, let us discuss an example where condensed matter serves as a toy model for theoretical studies. The Hawking effect describes the evaporation of black holes and results in the emission of thermal radiation with the temperature (Hawking 1974, 1975)

$$T_{\text{Hawking}} = \frac{1}{8\pi M} \frac{hc^3}{G_N k_B},$$

(3.1)
where $M$ is the mass of the black hole. Hawking’s derivation of this effect is based on the semiclassical approach of quantum fields propagating in a classical background space-time and the assumption that this treatment is valid up to arbitrarily large energies. However, there are many reasons to expect that this semiclassical approach fails at high energies, such as the Planck scale. For example, one could argue that it should be impossible to resolve spatial details smaller than the Planck length $\ell_p = \sqrt{\hbar G_N/c^3} \approx 1.6 \times 10^{-35}$ m, because the necessary (Planckian) momentum and hence energy concentrated in such a small volume would collapse into a black hole.

Inspired by the analogy to condensed matter (cf. Jacobson 1991), a simple model for ultra-high energy deviations from classical general relativity is a modification of the dispersion relation $\omega = ck \to \omega = \omega(k)$ of the propagating degrees of freedom (e.g. photons, gravitons). For example, in analogy to special relativity where the acceleration of a massive particle up to the speed of light is prohibited by a diverging energy $E = mc^2/\sqrt{1 - v^2/c^2}$, one could speculate that quantum gravity impedes a sub-Planckian spatial resolution in a similar manner via $\omega = ck/\sqrt{1 - \ell_p^2 k^2}$. However, calculating the Hawking radiation for such a dispersion relation (Schützhold & Unruh 2008; see also Brout et al. 1995; Corley 1998; Unruh & Schützhold 2005), we find that we reproduce the thermal emission at low energies $k_\ell_p k \ll 1$, but, in addition, we obtain a large amount of radiation (UV catastrophe) at ultra-high energies $k = O(1/\ell_p)$. Since black holes are supposed to exist for a macroscopic time, the dispersion relation $\omega = ck/\sqrt{1 - \ell_p^2 k^2}$ should be excluded as a realistic model for ultra-high energy deviations from classical general relativity. One way to avoid this UV catastrophe is a sub-luminal dispersion relation $d\omega/dk < c$.

4. Trapped ions

For an experimental verification, there are several possible systems. Let us start with trapped ions (see also Schützhold et al. 2008b). In a strongly elongated trap, the (quantized) positions $Q_i(t)$ of the ions satisfy the equation of motion

$$\ddot{Q}_i + \omega_{ax}^2(t) Q_i = \Gamma \sum_{j \neq i} \frac{\text{sign}(i-j)}{(Q_i - Q_j)^2},$$

(4.1)

where $\Gamma$ encodes the strength of the Coulomb repulsion. If the axial trapping frequency $\omega_{ax}(t)$ is time dependent, the chain of ions expands or contracts linearly, which can be described in terms of the scale parameter $b(t)$. Linearization $\dot{Q}_i(t) = b(t) Q_i^0 + \delta \dot{Q}_i$ around the classical solution $b(t) Q_i^0$ determined by the initial positions $Q_i^0$ yields the wave equation of the (longitudinal) phonon modes $\delta \dot{Q}_i$.

$$\left( \frac{\partial^2}{\partial t^2} + \omega_{ax}^2(t) \right) \delta \dot{Q}_i = \frac{1}{b^3(t)} \sum_j M_{ij} \delta \dot{Q}_j - \frac{1}{b^3(t)} \nabla^2 \delta \dot{Q}. \quad (4.2)$$

The time-independent matrix $M_{ij}$ is strongly localized around the diagonal and approaches the second spatial derivative $\nabla^2$ in the continuum limit. Hence, the
above equation is analogous to a scalar field within an expanding/contracting universe with \( b(t) \) corresponding to the scale parameter of the universe and \( \Omega_{\text{ax}}^2(t) \) being roughly analogous to the Ricci curvature scalar.

Consequently, we may simulate cosmological particle creation in an ion trap: starting in the ground state of the ion chain at rest and subsequently varying the axial trapping frequency \( \Omega_{\text{ax}}(t) \) will entail the creation of pairs of phonons (where both partner particles occupy the same vibrational mode). The advantage of this set-up lies in the well-developed pumping, cooling and detection techniques, which are quite advanced in view of the potential of ion traps for quantum computing. As a result, the proposed experiment should be realizable with present-day technology and is presently in preparation. Through the comparison of first and second side-band transitions, it is even possible to test the two-phonon nature of this quantum effect and thereby to distinguish it from classical effects such as heating. As a major drawback, one should mention the limited size (number of ions), for which a coherent control can be maintained. Therefore, the analogue of cosmological particle creation is most probably feasible with this set-up, but Hawking radiation is not so yet.

5. Bose–Einstein condensates

One option to overcome the size limitation of ion traps—while maintaining low temperatures and coherent control—is a Bose–Einstein condensate (see Dalfovo et al. 1999). At length scales much larger than the healing length \( \xi \) (typically of the order of micrometres), its dynamics can well be described by the Bernoulli equation. Thus the analogue of a black hole can be realized by a stationary flow that exceeds the speed of sound (of the order of millimetres per second) at some point. This point corresponds to the black-hole horizon and thus emits Hawking radiation with the temperature being determined by the velocity gradient at that position (Unruh 1981)

\[
T_{\text{Hawking}} = \frac{\hbar}{2\pi k_B} \left| \frac{\partial}{\partial r} (v_0 - c_s) \right|.
\]

Inserting the above values, we obtain typical temperatures of several nano-kelvin, which could also be within the reach of present-day technology (Garay et al. 2000; Wüster & Savage 2007).

Apart from the possibility of achieving very low temperatures, Bose–Einstein condensates have some main advantages. Nowadays, we can handle relatively large samples containing many healing lengths \( \xi \approx \text{O}(\mu m) \) whose shape can be influenced by laser fields, which represent external potentials for the Bernoulli flow. This allows us to generate various velocity profiles for the condensate flow—including quasi-one-dimensional flow—without the usual problems of supersonic flow velocities (exceeding the Landau critical velocity; cf. Unruh 2002) induced by the friction with walls, etc. The detection of single (or pairs of) phonons is not as highly advanced as for trapped ions, but could also be done in principle via similar optical techniques (see Schüttzhold 2006).

Unfortunately, there is also a major drawback: all the gaseous Bose–Einstein condensates realized in the laboratory are only metastable states—the true ground state is a solid. The main decay channels of the gaseous state are inelastic three-body collisions. Such an event transforms three indistinguishable bosons
from the cloud into a molecule plus a remaining boson which carries the excess energy/momentum, and thereby all three of them are effectively extracted from the condensate. Now, if all three of these bosons stem from the same macroscopically occupied single-particle wave function of the condensate ($k=0$), three-body collisions would just slowly diminish the number of condensed bosons. However, in the presence of interparticle interactions (which are necessary for the propagation of sound), the many-particle ground state does also contain a small population of the higher single-particle states $k>0$. This small fraction is called quantum depletion because it is generated by the quantum fluctuations (plus the interaction). Thus, if one of the three bosons involved in the inelastic collision stems from the quantum depletion and is removed, this event causes a deviation from the many-particle ground state, i.e. an excitation (Dziarmaga & Sacha 2003). Ergo, three-body collisions do also heat up the condensate, which might swamp the Hawking signal to be detected. For example, considering a Bose–Einstein condensate containing $10^7$ particles and 1% quantum depletion with 1% three-body losses (in total) during the experiment (which are already quite optimistic values), there would be $O(10^3)$ noise phonons in addition to the weak Hawking signal. In order to avoid this problem (and other issues, such as the black hole laser instability; see Corley & Jacobson 1999), it is probably desirable to employ a very fast detection method.

6. Electrons in laser fields

As the final example, let us consider the Unruh effect (Unruh 1976). This effect is related (though not equivalent) to Hawking radiation and states that an observer undergoing a uniform acceleration $a$ experiences the Minkowski vacuum as a thermal state with the Unruh temperature

$$T_{\text{Unruh}} = \frac{\hbar}{2\pi k_B c} a. \quad (6.1)$$

The most direct way of measuring this effect would be to accelerate a detector strong enough and to measure its excitations, which is, however, very hard to do. Thus, we focus on an indirect signature: an accelerated electron would also ‘see’ a thermal bath of photons and hence it could scatter a photon out of this thermal bath into another mode. Translation of this scattering event into the inertial frame corresponds to the emission of two entangled photons by the accelerated electron (Unruh & Wald 1984), which is a pure quantum effect. In order to achieve a significant signal, the acceleration $a$ and thus the electric field $E$ accelerating the electron should be very large. Hence, we propose sending an ultra-relativistic electron beam into a counter-propagating laser pulse of high intensity, such that the electric field felt by the electrons is boosted and thus strongly amplified. The probability that one of these electrons emits an Unruh pair of two entangled photons can be estimated via (Schützhold et al. 2006, 2008a)

$$P_{\text{Unruh}} = \frac{\alpha_{\text{QED}}^2}{(4\pi)^2} \left[ \frac{E}{E_S} \right]^2 \times O\left( \frac{\omega T}{30} \right) \ll 1, \quad (6.2)$$
where $\alpha_{\text{QED}} \approx 1/137$ is the QED fine-structure constant and $E_S = m^2 / q$ denotes the Schwinger limit (Schwinger 1951). The electric field $E$, frequency $\omega$ and length $T$ of the laser pulse are measured in the rest frame of the electrons. If $N_e = 10^9$ electrons with a boost factor of $\gamma = 300$ hit an optical laser pulse with 100 cycles, whose (laboratory frame) intensity is of the order of $10^{18} \text{ W cm}^{-2}$, we obtain around one Unruh event in 100 shots. Because the above values are well within the reach of present-day technology, the generation of photon pairs should be feasible. Their detection and the elimination of the background noise are a bit involved, but the unique features of this quantum effect (correlated photon pairs, distinguished phase-space region, etc.) should facilitate a measurement.

In order to point out the main advantage of this proposal, one should realize that it is (in contrast to the previous examples in §§4 and 5) a real relativistic effect, i.e. the speed of light has not been replaced by the speed of sound. Again the generation of the photon pairs is most probably feasible with present-day technology and their detection should be within reach. The major drawback lies in the fact that it is not a direct measurement of the (original) Unruh effect: the acceleration is non-uniform and we did not consider the internal excitations of an accelerated detector, but an accelerated scatterer (i.e. electron).

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Appendix A. Classical analogues

It might be illuminating to compare the proposals discussed here with some recent developments regarding laboratory analogues of classical effects in curved space-times. Besides sound waves in Bose–Einstein condensates as discussed in §5, these analogues were based on surface waves (cf. Schützhold & Unruh 2002) in water (Rousseaux et al. 2008) or helium (Volovik 2003, 2005) and on light pulses in an optical fibre (Philbin et al. 2008).

First, it should be mentioned that the effects measured in these experiments were not generated by (the analogues of) black-hole horizons, but by other phenomena such as white-hole horizons and/or ergo regions. While the black-hole horizon marks the border (‘point of no return’) beyond which nothing can escape (to asymptotic infinity), the white-hole horizon just represents the time-inverse phenomenon, i.e. a surface that cannot be penetrated. As a result, everything piles up at the white-hole horizon (note that the gravitational attraction is the same as for a black hole) until the dispersion relation changes (see §3) or other nonlinear effects set in. This entails classical and quantum instabilities (cf. Eardley 1974), which are quite different from Hawking radiation. For example, the radiation emitted from a white hole occurs at ultra-short wavelengths and large amplitudes, whereas Hawking radiation has a small amplitude and contains long wavelengths. In terms of a fluid analogue based on one-dimensional flow, the black-hole horizon corresponds to the point where the flow is accelerated such that it exceeds the propagation speed of the waves. Consequently, the white-hole horizon is analogous to the place where

2 The signal could even be much more pronounced in undulators due to the coherent amplification (constructive interference of many electrons; Schützhold et al. 2008a).
the fluid is decelerated and its velocity drops below the wave speed. For surface waves, this point is also known as hydraulic jump (cf. Volovik 2005), which marks the transition from quasi-laminar to turbulent flow (demonstrating an instability).

In more than one dimension, it is also possible to model phenomena associated with rotating black holes by means of suitable fluid analogues: a radial inward flow accompanied by a rotating component is analogous to a Kerr black hole (cf. Schützhold & Unruh 2002). The horizon then corresponds to the point where the radial flow velocity exceeds the propagation speed of the waves. However, in the additional presence of a rotational flow component, there is a finite region outside the horizon, where the total flow velocity exceeds the wave speed, which is analogous to an ergo region. Inside this region, the waves can have a negative effective energy (measured with respect to the laboratory frame), but they might still be able to escape. As a consequence, a wave can be scattered off the rotating black hole such that the energy of the outgoing wave is larger than that of the ingoing wave (because the part of the wave falling through the horizon has a negative energy). The energy gain is then extracted from the rotational energy of the black hole. For point particles, this striking phenomenon is called the Penrose process (Penrose 1969) and in field theory language it is known as superradiant scattering (Press & Teukolsky 1972) or the Zel’dovich–Starobinsky effect (Zel’dovich 1971; Starobinsky 1973). Reflecting the outgoing amplified wave back into the black hole may then yield a blow-up instability (Press & Teukolsky 1972), which is of course much easier to observe in laboratory analogues (see Takeuchi et al. 2008) than in real black holes.

As a final remark, it should be stressed again that the aforementioned experiments were devoted to the detection of properties of classical wave propagation, i.e. not to the quantum features mentioned in §1 (e.g. entangled pairs). Clearly, water waves are most certainly not appropriate for observing quantum phenomena—for superfluid helium and optical fibres, the observability of genuine quantum phenomena remains an open question.

References


3Here the phrase ‘quantum phenomena’ is meant to denote the consequences of quantum fluctuations associated with a many-particle Hamiltonian (second quantization) rather than classical wave propagation such as scattering or tunnelling (as in first quantization).


