The use of sonic analogues to black and white holes, called dumb or deaf holes, to understand the particle production by black holes is reviewed. The results suggest that the black hole particle production is a low-frequency and low-wavenumber process.

Keywords: black holes; analogues; Hawking radiation

1. Introduction

Black holes are one of the most surprising predictions of Einstein’s theory of gravity. In 1916, immediately after Einstein had published his theory of gravity, Karl Schwarzschild published a solution of the theory, a spherically symmetric solution, with a strange singularity at a radius of \(2GM/c^2\), where \(M\) is the mass; \(G\) is Newton’s constant; and \(c\) is the velocity of light. By 1920, Eddington had already shown that the apparent singularity seemed to be just a result of choosing bad coordinates. Despite that, in the late 1930s, Eddington rejected the possibility that such an object could actually occur in nature, casting scorn on Chandrasekhar’s arguments that at the end of the life of a sufficiently large star, that star would collapse leaving only a gravitational field behind. Even in the early 1970s, when I was a graduate student, black holes (as my adviser, John Wheeler, had labelled them in the late 1960s) were discounted by the astronomical community as something that could actually be observed.

In 1972, I was asked to give a colloquium to the physics department at Oxford. In trying to describe what a black hole was, I came up with an analogy. Imagine you are a blind fish, and are also a physicist, living in a river. At one place in the river, there is a particularly virulent waterfall, such that at some point in the waterfall, the velocity of the water over the waterfall exceeds the velocity of sound in the water. It is clear that if another fish, which has fallen over the falls, shouts after passing that point, that sound will never get out to someone on the other side of the river. The sound will still travel through the water with its same speed as always, but the flowing water will sweep that sound over the falls faster than that sound could hope to travel out.

Furthermore, if a fish screams as it falls through that surface, the parts of the sound in that scream emitted at points closer and closer to that surface will take longer and longer to get out to a point far from the falls, because the net velocity of the sound will be smaller and smaller if the sound emitted is closer to that
special surface. That last bit of sound emitted just before the fish goes through that surface will take an infinite time to get out, since the net outward velocity of the sound goes to zero as that surface is approached.

It is also clear that the fish itself, as it goes through that surface, will not note anything unusual—the velocity of the water will not cause anything unusual to happen in the local physics around that sonic horizon.

These phenomena are analogous to what happens in a black hole. If something falls through the horizon of a black hole, it can never again send out a signal to the outside world. As it falls into the black hole, the light takes longer and longer to get out, with the light emitted just as one falls through the horizon taking an infinite time. One can never see anything fall into the black hole, just as one can never hear anything fall into the sonic analogue.

But the analogy turned out to be better than just one for making such a hand-waving description of a black hole. In 1980, while teaching a course on fluid mechanics, I realized that the analogy was much better than I had thought (Moncrief 1980; Unruh 1981). If one looks at the equations of motion of sound waves in such a flowing background fluid, those equations are exactly those of a scalar field in a non-trivial metric:

\[
\frac{1}{\sqrt{-g}} \sum_{ij} \frac{\partial}{\partial x^i} \sqrt{-g} g^{ij} \frac{\partial}{\partial x^j} \phi = 0.
\]

In the fluid case, \( \phi \) is the potential for the small perturbations in the irrotational fluid flow, with \( \delta \mathbf{v} = \mathbf{v} \phi \), and the metric

\[
g_{tt} = \rho^2 (c^2 - |\mathbf{v}|^2),
\]

\[
g_{ti} = \rho^2 v^i,
\]

\[
g_{ij} = \rho^2 (\delta_{ij} - v^i v^j),
\]

where \( \rho \) is the density of the fluid in that background flow; \( v^i \) are the components of the velocity of the background fluid flow; and \( c \) is the velocity of sound.

While this analogy is classical, a direct result of the Euler–Lagrangian equations for hydrodynamic flow, it should also extend to the quantum regime. That is, if we are doing quantum fields on a classical background space–time, one should be able to model that by looking at the quantum fluctuations of sound waves (phonons) in a background fluid flow. The best known of the results of quantum field theory on a background space–time was Hawking’s (1974, 1975) discovery that a quantum instability near the black hole horizon resulted in the black hole evaporating—emitting a constant stream of radiation with a temperature of

\[
T = \frac{\hbar c^3}{8\pi kGM},
\]

where \( k \) is Boltzmann’s constant. The same derivation showed that a sonic horizon should emit a thermal spectrum of phonons, quantized sound waves.

There were, however, problems with his derivation. The first is that in calculating the temperature, one finds that the particles emitted at time \( t \) after the black hole forms are the result of the amplification of vacuum fluctuations of frequency around \( \frac{c^3}{GM} e^{tc^3/GM} \). This means that the radiation emitted a second after the formation of a solar mass black hole arises from vacuum fluctuations

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of order $e^{105}$, a frequency for which the concomitant energy $h\omega$ is $e^{105}$ times the mass of the universe. It is clear that at these frequencies the assumption that these fluctuations have no effect on the background space–time is a very bad approximation.

A second problem is that if we take seriously the suggested thermodynamics of a black hole, since the temperature is a function of the energy $Mc^2$, the black hole should have an entropy equal to $k\pi((2GM/c^2)^2/(Ghc))$ or $k/4$ times the surface area of the black hole measured in the so-called Planck units. A wide variety of arguments showed that, given the temperature of the black hole, this entropy was real, in that it was absolutely critical to take it into account if one wanted to make sure that second law of thermodynamics was valid even when black holes were taken into account (Unruh & Wald 1982).

But what is this entropy? In the second half of the nineteenth century, people like Maxwell, Gibbs and Boltzmann argued that entropy was not a fundamental concept, but was an approximation based on the extremely complicated nature of the fundamental dynamics of any macroscopic object. It was a count of the complexity of the microscopic states consistent with any macroscopic specification of an object. The key question is whether the entropy of a black hole is the same, a result of the complexity of some unknown microscopic internal states of a black hole, or whether it was a fundamental entropy unrelated to any internal states of the black hole.

On the one hand, people argued that the unitarity of quantum mechanics would imply that on a microscopic level, the entropy, measured by $\rho \ln(\rho)$ must be a constant of the motion of any complete system. If black hole evolution was unitary, then the entropy of a black hole must be of the same form as that of any other physical system.

On the other hand, in the derivation of the temperature and of the energy of a black hole, there is absolutely no hint of any internal degrees of freedom. Furthermore, owing to the internal singularity of the black hole, ‘information’ could flow into the singularity, be lost to the outside world, and the overall evolution of the quantum state would not be unitary. Just as the flow of information to infinity, for example, by the emission of radiation by an object, leads to a non-unitary evolution for the system itself, the flow of information into the singularity of a black hole would lead to a loss of unitarity, and an increase in entropy, of the world outside the black hole.

The black hole sonic analogue, which I have called dumb holes (from deaf and dumb), can cast at least some light onto both of these questions. Does the black hole temperature require the absurdly high frequencies for its derivation? Is the entropy of a black hole due to microscopic complexity or is it fundamental? As with all arguments from analogy, the arguments are not foolproof and in general are indirect.

2. Planck’s scale physics and black hole temperature

The question of whether or not the ultra-high-frequency behaviour of the theory is important to the question of the temperature of a black hole has become known as the Planck scale issue in black hole evaporation. The Planck scale is the scale
at which the wavelength of the radiation,
\[ \lambda = \frac{c^2 \pi \hbar}{E}, \quad (2.1) \]
is equal to the black hole radius of that amount of energy
\[ R = \frac{2GE}{c^4}. \quad (2.2) \]
This gives
\[ E_{\text{Planck}} = \sqrt{\frac{\pi \hbar c^5}{G}}, \quad (2.3) \]
or rather since that factor of \( \pi \) is usually neglected
\[ E_{\text{Planck}} = \sqrt{\frac{\hbar c^5}{G}}. \quad (2.4) \]
This gives a length scale
\[ R_{\text{Planck}} = \sqrt{\frac{G\hbar}{c^3}}. \quad (2.5) \]

For any mode with an energy higher than this, one would expect the gravitational effects of the mode to drastically alter the behaviour of the mode. In particular, one would expect the naive calculations carried out in deriving the thermal emission from a black hole to be inapplicable for modes of higher energy than this.

In the case of a fluid, one knows that the fluid equation of motion is inapplicable at high frequencies and short wavelengths. At wavelengths shorter than the interatomic spacing, sound waves do not exist and thus the naive derivation of the temperature of dumb holes will fail. But unlike for black holes, for dumb holes, the theory of physics at short wavelengths, the atomic theory of matter, is well established. For black holes, a quantum theory of gravity is still a dream. Thus, if one could show that for dumb holes the existence of the changes in the theory at short wavelengths did not destroy the existence of thermal radiation from a dumb hole, one would have far more faith that whatever changes in the theory quantum gravity created, whatever nonlinearities quantum gravity introduced into the theory, the prediction of the thermal radiation from black holes was robust. On the other hand, if the introduction of the atomicity of matter invariably destroyed the thermal radiation for dumb holes, one would strongly suspect that the thermal nature of black holes would not survive the complications introduced by quantum gravity.

It was Jacobson (1991, 1993), infected by a talk I had given at the University of Texas in 1980 on the sonic analogue, who suggested that one of the key attributes of an atomic fluid (a fluid made of atoms) was the altered dispersion relation. Relativistic systems have the feature that although the group and phase velocities of modes of a field can vary, the group velocity is always less than the velocity of light and the products of the phase and group velocities (of importance later in this paper) are \( c^2 \). But this is not true of the modes of an
atomic fluid. We have all done the calculation in our introductory solid-state course of calculating the relation between the temporal frequency $\omega$ and the wavenumber $k$ for a set of beads connected by springs. If all beads are equivalent, the relation is

$$\omega^2 = \omega_0^2 \sin^2 \left( \frac{k}{k_0} \right),$$

(2.6)

where the group velocity at low frequencies or wavenumbers is the constant $\omega_0/k_0$, but this drops (to zero) as $k$ approaches $\pi k_0/2$.

This implies that if one had a dumb hole, such that the velocity of the fluid approached the group velocity of the waves at low wavenumbers and at high wavenumber (short wavelength) the fluid would be travelling much faster than the waves. Thus, while at low wavenumbers, one would have the expected exponential relation between the time and the wavenumber into the past for a packet of thermal wavenumber escaping the dumb hole horizon in the future, eventually the group velocity of this blue shifted wave packet would drop and the wave packet would have had to have come far from the horizon.

In other words, the expected exponential relation between the outgoing wave at time $t$ after the formation of the horizon, and the wavenumber of the wave packet, would fail. Instead, the packets would be blue shifted (going into the past) until the wavenumber approached $k_0$, before which time they must have come from outside the horizon, swept towards the horizon by the fluid flow. Thus, the early time quantum fluctuations that caused the late-time radiation would always have a wavenumber of about $k_0$, and not exponentially large.

Detailed numerical work by myself (Unruh 1995), then by Corley & Jacobson (1996, 1998) and analytically by Corley (1997, 1998) and by Brout et al. (1995) showed that this picture is true, but despite the absence of any exponential relation, the horizon still emitted radiation with the same horizon temperature proportional to $\left( \frac{\hbar}{2\pi K} \right) \left( \frac{c^2 - v^2}{c^2 \, dr} \right)$ predicted by the naive, Hawking-type, calculation. Testing many different types of dispersion relation demonstrated the robustness of this result. Furthermore, analytic, saddle point type, arguments (Corley 1998; Unruh & Schützhold 2005) also showed that this was the expected temperature of the horizon, independent of the form of the dispersion relation—subluminal (group velocity at high wavenumber less than that at lower) as in the naive ‘beads on a spring’, superluminal (group velocity at high wavenumber greater than that at lower as in a Bose–Einstein condensate) or variable (as in liquid He).

One of the aspects of these dumb holes, which fascinates me most, is the possibility of using them to understand where and how the particles in the black hole evaporation process are created. While the lack of dependence of the temperature on the form of the high-frequency dispersion relation suggests that those high frequencies are irrelevant to the formation of the particles, it still leaves open the possibility that it is the behaviour of the fields at the highest wavenumbers where the group velocity of the waves equals the low-frequency speed of sound. That is, the particles are created at those highest frequencies and then are red shifted in the evolution very near the horizon. This has led Ralf Schützhold and me (Schützhold & Unruh submitted) to examine a more interesting scenario. Let us assume that the dispersion relation is such that the group and phase velocities are equal at low frequencies, and again at

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some intermediate frequencies, but that those two velocities are different. Let us also set up the fluid flow such that the rate of change of the fluid velocity, at the points where the fluid velocity equals each of those different phase and group velocities, is different, i.e.

\[
\frac{d(c_1^2 - v^2)}{c_1 \, dr} \bigg|_{v=c_1} \neq \frac{d(c_2^2 - v^2)}{c_2 \, dr} \bigg|_{v=c_2}
\]

(2.7)

and thus the temperatures associated with the horizons of those two wave velocities are also different. Which temperature will determine the distribution of particles far from the dumb hole? The answer is surprising. If we assume that the velocity profile is of the form \(1/r\), our analytic results suggest the following.

Define the geometric average velocity

\[
\bar{c}^2 = \left( \frac{d\omega^2}{dk^2} \right) = \frac{\omega}{k} \frac{d\omega}{dk}
\]

(2.8)

which is the geometric mean of the group and phase velocities of the wave. Calculate these velocities when the wave is far from the horizon and define the horizon for that wavenumber as the place where the velocity of the fluid equals these velocities. Calculate the derivative of the fluid flow at that point. This will define the temperature of the particles emitted at that wavenumber and frequency. Note that near the horizon, this is not the geometric average of the phase and group velocities since the closer to the horizon, the more blue shifted (shorter wavenumber) the wave will be. But it seems to be this velocity and this horizon, which define the temperature, i.e. the particles appear to be created not near the horizon but far away. The particle creation seems to be a phenomenon not of the horizon, but of the space–time, a wavelength away from the horizon.

This also suggests in the black hole case that physics within the horizon cannot be important for the emission of radiation and thus also cannot be the locus of memory of the black hole of the radiation that has escaped. This lends support to the idea that black hole evaporation is not a unitary process, rather the thermal stream is fundamental, and not simply the result of averaging over unobserved degrees of freedom.

If the black hole evaporation is to be unitary, then the black hole must remember both the initial state that formed the black hole and the state of all of the radiation that has escaped thus far. This memory must furthermore be such that it can affect the subsequent radiation that escapes, so as to create the correlations needed to uniquely correlate the final state of the system with the initial state. (That unique correlation is what unitarity means.) If the radiation is created only at low energies, then at those low energies the black hole horizon is an excellent description of the causal physics near the black hole. Thus, that escaping radiation can be affected only by things outside the black hole. If that radiation is to be affected by the memory the black hole has for the initial state, then the physical embodiment of that memory must be outside the horizon to enable it to affect the radiation. (This of course neglects the possibility of large scale non-locality. If such large scale non-locality were a feature of the world, physics would essentially become impossible, since one would have to know everything about the whole universe in order to be able to do
anything. The arguments between Edmund and Edgar in *King Lear* as to whether it is the stars or our own actions that are responsible for our fate would swing over to Edgar.)

It is of course possible that the black hole carries its memory outside the horizon, where the low-frequency radiation would have access to it. This would mean that the black hole, rather than having no hair, has a large amount of ‘hair’, differences in the exterior structure beyond the mass, angular momentum and charge.

### 3. Experiment

One of the most exiting possibilities for dumb holes is the possibility of experimental observation. Experiments with application to the classical black hole are easy. Using the observation by Schützhold & Unruh (2002) that black hole analogues could be formed where the field is the shallow water gravity waves on water, one can easily create classical analogues to black holes. In figure 1, there is an example of a white hole horizon in the flow of water in a flume at the University of British Columbia laboratory of Greg Lawrence that Schützhold, Lawrence and I set up approximately 5 years ago. The water flows into the flume at speeds higher than the speed of the shallow water waves ($\sqrt{gh}$). At the line, the velocity of the flow decreases to less than the shallow water wave velocity. The small jump in the water level can be regarded as a wave of infinite wavenumber impinging on the horizon from the left.

The horizon now causes this low-wavenumber wave to ‘blue shift’, to increase its wavenumber. At shorter wavelengths, when the wavelength approaches the depth of the fluid, the velocity of the waves goes from the constant $\sqrt{gh}$ to the deep water wave speed $\sqrt{g/k}$, i.e. the group and phase velocities fall and those velocities become less than the velocity of the fluid.

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Because the flow is stationary, the frequency of the wave is a conserved quantity. The incoming infinite wavelength wave has zero frequency, so the outgoing wave also has zero frequency as seen in the laboratory frame. The phase velocity is zero and one sees only a set pattern of wave. However, the group velocity of this wave \( \frac{d\nu}{dk} \) is not zero, and is directed, in the laboratory frame, away from the horizon. It carries away energy from the horizon (and is the key way in which the energy loss of the flowing fluid due to its slowing down at the horizon is carried away from the horizon).\(^1\) The wavelength of the wave, which is due to the blue shifted low-frequency incoming wave, is just what one would expect from the ‘white hole’ analysis. The dispersion relation of gravity waves in a stationary fluid is

\[
\omega^2 = gk \tanh(kh), \quad (3.1)
\]

where \( k \) is the wavenumber; \( h \) is the depth of the fluid; and \( g \) is the acceleration due to gravity. In the flowing fluid, the wavenumber of the escaping ‘blue-shifted’ zero-wavenumber incoming wave is the solution to the equation \( vk = \sqrt{gh \tanh(kh)} \), where \( v \) is the local velocity of the fluid.

See also the paper in this issue by Leonhardt & Philbin (2008) and Philbin et al. (2008) where the equivalent phenomenon is demonstrated for a white hole horizon formed in an optical fibre due the nonlinearities in the refractive index of the fibre caused by an intense pulse of light in the fibre. Again, a lower frequency wave incident on the white hole horizon is blue shifted until the group velocity of the wave is less than the velocity of the pulse. See also the recent paper by Rosseaux et al. (2008) where the effect of the horizon in a water flow tank similar to that of figure 1 is discussed.

Of course, the ultimate goal is to examine not only such classical effects but also the quantum radiation. The temperature is given by

\[
T = \frac{\hbar}{k} \frac{1}{4\pi c} \frac{d(e^2 - v^2)}{dr}, \quad (3.2)
\]

where \( c \) is the velocity of the wave in the fluid at rest; \( v \) is the velocity of the fluid; and both can be functions of \( r \), the distance along the flow. If \( c \) is of the order of 300 m s\(^{-1}\) (the velocity of sound in liquid He) and the scale length of the flow \( 1/(d \ln(v)/dr) \) is 1 \( \mu \)m, this temperature is approximately 0.1 K. This quantum radiation will never be observed in water. The temperature scales roughly as \( c \) the velocity of the waves, which means that for a given scale, the temperature drops as the velocity of the waves drops. The energy emitted, which goes as \( T^4 \), scales as the square of the velocity, and is roughly \( 10^{-23} \) watts for a 1 \( \mu \)m sized dumb hole in liquid He and \( 10^{-33} \) watts for the mm s\(^{-1}\) velocities in Bose-Einstein condensates (Unruh 2002). It is the measurement of such incredibly small energies radiated which is the chief impediment to the measurement of the thermal effect of dumb holes. If one could make a light-based dumb hole, the energy radiated goes up to roughly \( 10^{-15} \) watts, still a small energy flux, but perhaps not completely absurd.

\(^1\) Note that this phenomenon is well known in the hydrodynamic literature, under the name of ‘hydraulic jump’ in which only very little of the energy of the transition from supersonic to subsonic flow is dissipated in turbulence of the jump but rather in that wave carrying away energy, even though it is completely stationary. See for example the Wikipedia article ‘Hydraulic jump’ and references therein (http://en.wikipedia.org/wiki/Hydraulic_jump).
References


