Magnetic fields in the aftermath of phase transitions

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The COSLAB effort has focused on the formation of topological defects during phase transitions. Yet there is another potentially interesting signature of cosmological phase transitions, which also deserves study in the laboratory. This is the generation of magnetic fields during phase transitions. In particular, cosmological phase transitions that also lead to preferential production of matter over antimatter (‘baryogenesis’) are expected to produce helical (left-handed) magnetic fields. The study of analogous processes in the laboratory can yield important insight into the production of helical magnetic fields, and the observation of such fields in the Universe can be invaluable for both particle physics and cosmology.

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1. Introduction

Phase transitions are a common theme in condensed matter systems and cosmology. In the laboratory, the experimental condensed matter physicist studies the response of materials under changes in external parameters, such as temperature, pressure and magnetic field, often observing sharp changes in certain properties that signal a phase transition. In cosmology, we do not have the luxury of a controlled experiment. Yet it seems clear, based on the success of big bang nucleosynthesis, that the expansion of the Universe caused it to cool down and that matter within the early cosmos could also have gone through phase changes. We are now situated in a low-temperature environment and can only deduce the occurrence of a phase transition by studying its aftermath.

Luckily, a wide class of phase transitions leave a telltale remnant in the form of topological defects. They are ‘defects’ because they indicate regions of the system that were unable to complete the phase transition. They are ‘topological’ because the reason for the incomplete phase transition is due to the topology involved in the phase transition. Topological defects in the Universe could manifest themselves as magnetic monopoles, cosmic strings and domain walls, and there is an ongoing search for these structures in astronomical and cosmological surveys. As we heard at the meeting, similar searches in condensed...
matter have met with success in a variety of systems, and there is effort to quantify the production of topological defects at a phase transition.

This paper deals with phase transitions in which the requisite topology for defects is absent. Such phase transitions are certainly relevant in the context of cosmology because the standard model of the electroweak interactions has a phase transition without topological complications. Is there any hope of observing the aftermath of such phase transitions?

I shall argue that cosmic magnetic fields may be remnants from a phase transition. Further, in certain settings, the magnetic fields are helical, their non-trivial helicity having its origin in the charge–parity (CP) violation present in particle physics. So the detection of cosmic magnetic fields and their helicity would allow us to infer a cosmological phase transition and provide us with an alternative probe of fundamental CP violation. Just as for topological defects, the laboratory may be a convenient setting to study magnetic field generation during phase transitions. Whether this is feasible would depend on how far we can prepare analogies between the cosmological and condensed matter systems.

2. Magnetic fields from cosmic phase transition

There are several alternative paths to deduce that magnetic fields must be left over after a phase transition. The first is in analogy with the formation of topological defects. During the phase transition, the order parameter must take on a value independently in every correlation size domain. Denoting the order parameter by $\Phi$, this implies non-zero spatial gradients, i.e. $\nabla \Phi \neq 0$. The gradient energy density, however, can still vanish since it is given by $|D_\mu \Phi|^2$ and the gauge fields may cancel out the spatial gradients, giving $D_\mu \Phi = 0$. At a phase transition, though, the system is thermal and the energy density does not vanish. On dimensional grounds, we expect $D_\mu \Phi \sim T_{pt}^2$, where $T_{pt}$ is the temperature at which the phase transition occurs. Consequently, we also expect there to be a contribution to the energy density coming from the electric and magnetic field strengths. As a result, we expect some magnetic field to be left over after the phase transition, and since this field is embedded in a highly conducting plasma, some of it will remain frozen-in (Vachaspati 1991).

The same conclusion can be reached by realizing that, even if a model does not contain topological defects, it will almost certainly contain ‘embedded defects’, as is the case in the standard electroweak model (Achucarro & Vachaspati 2000). These are unstable solutions in the model and can be in the form of magnetic monopoles. Such unstable objects can be produced by the Kibble mechanism and will decay soon thereafter. However, in this process, the embedded magnetic monopoles will leave behind their magnetic field, which is frozen in the ambient plasma (Vachaspati 1994).

These considerations imply a weak magnetic field after a phase transition. Also, the coherence scale of the magnetic field is quite small. Yet the process may still be of relevance to the generation of galactic magnetic fields, provided we make rather optimistic assumptions about the galactic dynamo.

I would now like to discuss a somewhat different approach to the generation of primordial magnetic fields, one which I believe is more promising as it leads to stronger, more coherent fields. Also, the mechanism is richer as it shows that

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there may be a remarkable connection between baryogenesis at phase transitions
similar to the electroweak transition and the helicity of cosmic magnetic fields
(Cornwall 1997; Vachaspati 2001). Hence, in the tradition of bringing together
particle physics and astrophysics, witness dark matter and dark energy,
astronomical observables may provide yet another tool to study baryogenesis
and CP violation in particle processes.

3. Elements of baryogenesis

Particle physics, as we know it today, is almost completely symmetric in matter
and antimatter. Thus, we would expect that the Universe should be composed of
equal amounts of hydrogen and antihydrogen. However, this is not the case.
Galaxies are seen to collide but never annihilate. Cosmic rays arriving to us from
cosmological distances are mostly matter and only approximately 0.01%
antimatter, consistent with what is expected due to secondary production.
There are also strong constraints on scenarios that assume a domain structure for
the distribution of matter and antimatter, since there would be $\gamma$-ray production
due to annihilation at the domain boundaries (Cohen et al. 1998).

The overwhelming preponderance of matter in the Universe was addressed by
Sakharov and can be understood if three conditions, now known as the ‘Sakharov
conditions’, are met. These are that fundamental particle physics should contain
violations of charge (C) conjugation and CP conjugation, it should allow for the
conversion of antimatter to matter, and there should be a period in cosmology
where thermal equilibrium is not maintained. Interestingly, all three of the
Sakharov conditions are met in the standard model of particle physics within
standard cosmology (for a review, see Riotto & Trodden 1999).

From the viewpoint of generating magnetic fields, the most important
ingredient of the standard model is that it allows for transitions between matter
and antimatter via a particular mechanism that is suppressed at low energies,
since it proceeds by quantum tunnelling, but can become important at high
temperature (Kuzmin et al. 1985), where it is known as a ‘sphaleron transition’
(Manton 1983; Klinkhamer & Manton 1984).

Assuming the standard model of electroweak interactions, violations of CP
and departures from thermal equilibrium at the electroweak phase transition fall
short of what is needed to explain the amount of matter–antimatter asymmetry
required for the synthesis of light elements (‘big bang nucleosynthesis’). The
required antimatter to matter ratio is approximately 1 part in $10^9$ whereas the
standard electroweak model leads to a ratio more like 1 part in $10^{20}$. This
suggests that particle physics needs to be extended beyond the standard model.
It is quite possible that baryogenesis in the correct particle physics model will
continue to be via sphaleron or sphaleron-like configurations. We will proceed
under this assumption.

4. More on the sphaleron

Baryon number is classically conserved in the electroweak model, but quantum
anomalies spoil this conservation. One way to understand this situation is that
the electroweak theory contains an infinite set of gauge vacua, each labelled by a
topological index (figure 1) called the ‘Chern–Simons’ number, and defined by

\[ N_{\text{CS}} = \frac{N_F}{32\pi^2} \epsilon_{ijk} \int d^3x \left[ -g^2 B_{ij} B_k + g' \left( W_i^a W_k^a - \frac{g}{3} \epsilon_{abc} W_i^a W_j^b W_k^c \right) \right], \tag{4.1} \]

where \( N_F = 3 \) is the number of particle families, and \( W_\mu \) and \( B_\mu \) are the \( SU(2) \times U(1) \) gauge fields with coupling constants \( g \) and \( g' \), respectively. Low-energy classical dynamics occurs within one of these topological sectors; quantum dynamics allows for tunnelling between different vacua. In the tunnelling process from one vacuum to another, the fermions respond by producing baryons or antibaryons. Since this is a tunnelling process, the rate of baryon number violation is exponentially suppressed and makes it irrelevant for the generation of matter in cosmology.

At high energies, the transition from one vacuum to another may proceed without the need for tunnelling, as depicted in figure 1. Now the path goes over the barrier separating the two vacua. The top of the barrier corresponds to a solution of the field equations that has precisely one unstable decay mode that causes it to ‘fall’ into one or the other vacuum. This solution is called a ‘sphaleron’ from the Greek roots meaning ‘to fall’. Baryons are produced/annihilated as the sphaleron decays and the Chern–Simons number changes.

Since the early Universe was very hot, sphaleron transitions between vacua containing different numbers of baryons are not suppressed and can proceed fast enough to be relevant cosmologically. If there were no CP violation, \( N \) sphaleron transitions would cause an excess of baryons or antibaryons simply due to \( \sqrt{N} \) fluctuations. This would imply a domain structure where some regions of space are dominated by baryons and others by antibaryons. Such a domain structure is not observed and it is necessary that CP violation be present to ensure that baryons dominate over antibaryons throughout the Universe. With CP violation present in the model, sphaleron transitions proceed more efficiently in one direction than the other and produce more baryons than antibaryons.

5. Decay of the sphaleron

We now examine the decay of a sphaleron more closely. As we will see, this not only changes the baryon number but also produces helical magnetic fields. This observation allows us to connect the observed baryon number with the helicity of a magnetic field.
A heuristic way to connect the sphaleron to magnetic fields is to use the relationship between the sphaleron and magnetic monopoles and electroweak strings (Nambu 1977; Vachaspati 1992; Achúcarro & Vachaspati 2000). Then, the sphaleron may be thought of as two loops of electroweak string that are linked together, as in figure 2 (Hindmarsh & James 1994; Vachaspati & Field 1994; Garriga & Vachaspati 1995). The magnetic flux in the electroweak strings is $\mathcal{Z}$-magnetic flux, where $\mathcal{Z}$ is the electroweak gauge field, and is not related to the electromagnetic gauge field, which we shall denote by $A$. The decay of the sphaleron now corresponds to a decay of the strings. This can proceed in many ways but one channel is where the strings break by the formation of (electromagnetic) magnetic monopoles and then shrink. If the magnetic fields are frozen in an ambient plasma, what remain from this process are two linked loops of electromagnetic magnetic field. These carry magnetic helicity, defined as

$$\mathcal{H} = \int d^3x \mathbf{A} \cdot \mathbf{B},$$

where the integral is over all space.

The heuristic picture of the decaying sphaleron can be confirmed by evolving the electroweak equations of motion with a sphaleron as initial condition. Since the sphaleron is a static solution of the electroweak equations of motion, it is necessary to perturb it so that it then decays. It is quite possible that different initial perturbations will lead to different outcomes. However, what is important is that the decay due to some large class of perturbations ends up by producing helical magnetic fields. In C. Copi et al. (2008, unpublished data), we have studied the decay of a sphaleron by numerically evolving the lattice electroweak equations by building on earlier work by Ambjørn et al. (1991), Moore (1996), Tranberg & Smit (2003), García-Bellido et al. (2004) and Graham (2007). (A similar analysis has also been done by Diaz-Gil et al. (2007).) In the numerical analysis, we have implemented absorbing boundary conditions by extending the scheme of Olum & Blanco-Pillado (2000) to non-Abelian gauge systems. The initial configuration is taken to be an approximate sphaleron following Klinkhamer & Manton (1984). This is not a static solution and decays. The results (figure 3) show that the Chern–Simons number decays as the sphaleron decays, while the magnetic helicity grows. This numerical study confirms the heuristic picture that sphaleron decay produces helical magnetic fields.

Figure 2. (a–c) A possible decay mechanism for two linked loops of electroweak $\mathcal{Z}$-string (a, thick curves). (b) The $\mathcal{Z}$-strings can break by the formation of magnetic monopoles and an electromagnetic magnetic field connects the monopole–antimonopole pairs. (c) The $\mathcal{Z}$-strings can shrink and disappear, leaving behind two linked loops of electromagnetic magnetic field depicted by thin curves. The initial linked loops of $\mathcal{Z}$-string are related to the sphaleron and the final state consists of helical magnetic fields.
6. Magnetic fields and baryogenesis

By counting the number of baryons produced in the decay of the linked electroweak strings, and evaluating the helicity of the final linked loops of magnetic field, we obtain the relation (Cornwall 1997; Vachaspati 2001)

$$h = \frac{1}{V} \int_V \mathbf{d}^3 x \mathbf{A} \cdot \mathbf{\nabla} \times \mathbf{A} \sim -\frac{n_b}{\alpha}. \quad (6.1)$$

Once magnetic fields are produced, their evolution follows the magnetohydrodynamic (MHD) equations. One well-known result is that magnetic helicity is conserved if the plasma has high electrical conductivity. The evolution of the helical magnetic field produced by sphaleron decay has been discussed by Vachaspati (2001). Of particular interest is the growth of the coherence scale of the magnetic field. Since the magnetic field carries helicity, there is a possibility for the ‘inverse cascade’ that makes magnetic energy flow from small to large length scales. The inverse cascade makes the magnetic field smoother and more coherent. However, there are also some unresolved issues in the evolution of the magnetic field, especially at the cosmological epoch when electrons and positrons annihilate and the electrical conductivity drops quite rapidly. These are issues that are important in the cosmological context since we are observing the aftermath of a phase transition at very late times. In the laboratory, however, these issues are not important because we can examine the system soon after a phase transition.

7. Detection in cosmology

Already we have several tools to look for magnetic fields in cosmology and we can constrain the cosmic magnetic field strength at various coherence scales using different observables. Big bang nucleosynthesis constrains the magnetic field at
the shortest distance scales (Kernan et al. 1996). Other measures, such as Faraday rotation, are more sensitive to magnetic fields with large coherence scales. Possibly there are even stronger constraints arising from gravitational wave production by non-helical magnetic fields (Caprini & Durrer 2002).

The future holds great promise for hunting primordial magnetic fields. The present experiments have a snapshot of the Universe when it was 300 000 years old and what we see is the cosmic microwave background (CMB). The CMB propagates to us from a distance of several gigaparsecs ($1\ Gpc \approx 10^{27}\ cm$). If a cosmic magnetic field is present, the polarization of the CMB will undergo Faraday rotation. Ongoing experiments are attempting to map out the CMB polarization at several different wavelengths, and we should have a detection of a cosmic magnetic field or be able to place constraints on its strength at a level comparable with 1 nG.

Ultra-high-energy cosmic rays (UHECRs) provide another probe of the cosmic magnetic field. Recently, the AUGER experiment has identified UHECRs as protons, also attempting to correlate the cosmic ray events with active galactic nuclei. Since the UHECRs are charged, they would bend in a cosmic magnetic field and the differences in angular position between the expected source of the UHECR and the arrival position may be used as a measure of the intervening magnetic field. This effort is still in its infancy but there is hope for the future.

While it is possible to think of many ways to detect primordial magnetic fields, it is harder to come up with ways to detect magnetic field helicity. Faraday rotation is only sensitive to the line-of-sight component of the magnetic field but helicity involves all components. So Faraday rotation by itself cannot be used to detect helicity. The bending of charged particles, as in UHECRs, is a different matter but requires proper identification of the sources as discussed by Kahniashvili & Vachaspati (2006). As the cosmic ray statistics builds up, it may eventually become possible to say something about the helicity of the cosmic magnetic field.

8. Elements in condensed matter systems

The production of magnetic fields during phase transitions is in the same general class of problems as the production of topological defects. Yet the analogy is a little harder as we now discuss.

In condensed matter systems, there is only one dynamical gauge field and that is electromagnetism. This is Abelian and not derived from non-Abelian fields above the phase transition. So the analogy with the electroweak model will necessarily be incomplete in this respect. On the other hand, rather complex symmetry breakings occur in condensed matter systems and the symmetry breaking structure in helium-3 is very similar to that in the electroweak model (Volovik & Vachaspati 1996). There are also close analogues between (global) vortices in helium-3 and $Z$-strings in the electroweak model. The production of magnetic fields in cosmology is similar to the production of gradients in one of the Nambu–Goldstone degrees of freedom. (Such gradients are called ‘texture’ in the condensed matter literature.) It should be possible to check experimentally if textures are an aftermath of phase transition and to quantify them.
The production of helical texture due to quantum anomalies may be harder to test experimentally. Yet we know that anomalous interactions analogous to baryon number violation are present in helium-3 (Bevan et al. 1997). Do these interactions produce helical texture?

9. Conclusions

The COSLAB effort has mostly focused on topological defects in the aftermath of phase transitions. This has been a heroic effort, not least because it involved bringing together cosmologists, condensed matter theorists and experimentalists to the same table. It is a tribute to the COSLAB group that several difficult and unconventional experiments were performed. Results that were based on dimensional analysis and computer simulations were successfully tested in real systems. The ultimate theoretical problem of describing topological defect production at a phase transition may require deep understanding of solitons in terms of particles, an unsolved problem as of now (Vachaspati 2006).

The production of magnetic fields during phase transitions is in the topological defects class of problems and has immediate application to cosmology. The puzzling aspects of the Kibble mechanism for defect formation in gauge systems become even more puzzling with its generalization to magnetic fields and would be worth testing experimentally. However, the cosmology and laboratory analogies for magnetic field production have not yet been established and will require further thought.

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References


