Introduction. Turbulence transition in pipe flow: 125th anniversary of the publication of Reynolds’ paper

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The 125th anniversary of Osborne Reynolds’ seminal publication on the transition to turbulence in pipe flow offers an opportunity to survey our understanding of the nature of the transition. Dynamical systems concepts, computational methods and dedicated experiments have helped to elucidate some of Reynolds’ observations and to extract new quantitative characteristics of the transition. This introduction summarizes some of the developments and indicates how the various papers in this volume contribute to an improved understanding of Reynolds’ observations.

Keywords: pipe flow; turbulence; travelling wave

On 15 March 1883, Osborne Reynolds presented the results of ‘An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous, and of the law of resistance in parallel channels’ to the Royal Society of London. It was first published in the Proceedings of the Royal Society (Reynolds 1883a), followed by a longer account in the Philosophical Transactions of the Royal Society (Reynolds 1883b). The longer paper incorporates the brief one as §I and contains comprehensive documentation of his experiments as well as the famous sketch of his experimental rig, which to this day is preserved at Manchester University. The historical probes into the archives of the Royal Society by Jackson & Launder (2007) showed that the referees were well aware of the significance of the contribution: Lord Rayleigh observes that ‘the paper records some well contrived experiments on a subject which has long needed investigation …’, and Sir George Stokes emphasizes that Osborne Reynolds ‘shows for the first time that the distinction between regular and eddying motion depends on a relationship between the dimensions of space and velocity’. That dimensionless relationship became later known as the Reynolds number (see von Kármán (1954) and Rott (1990) for the historical developments). For flow in a pipe, the Reynolds number is usually based on the mean flow speed $U$, the diameter of the pipe $d$ and the kinematic viscosity of the fluid $\nu$.

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Re in his fundamental contribution, Reynolds documents the key features of the turbulence transition in pipe flow as they have been reproduced in many experiments and student demonstrations since. From a modern perspective, his observations focus on three properties of the flow: there is a lower critical Reynolds number below which no turbulence is observed; an upper critical one above which the laminar flow can no longer be sustained; and a minimal amplitude which perturbations have to exceed in order to cause transition for intermediate Reynolds numbers.

The puzzling aspects of this transition emerge when one tries to assign numbers to these phenomena. For the upper critical Reynolds number, above which the flow becomes turbulent owing to ‘natural’ causes (i.e. by perturbations present in the system and not induced externally), he found values close to 13 000. Ekman (as quoted in Rott 1990) increased this to approximately 44 000 using Reynolds’ apparatus, and Pfenniger (1961) managed to keep the flow laminar up to 1 00 000. Together with the result from linear stability analysis that the parabolic profile remains linearly stable against infinitesimal perturbations for all Reynolds numbers (Salwen et al. 1980; Schmid & Henningson 2001; Meseguer & Trefethen 2003), these observations suggest that transition occurs only when the perturbation exceeds a critical amplitude, and that the different values for natural transition are linked to the level of background fluctuations and perturbations in the experiments. Some observations on this connection will be discussed below, but many aspects of it remain to be explored.

A second puzzle concerns the values for the lower critical Reynolds number below which no turbulence can be induced, no matter how large a perturbation is added. The data presented in his 1883 paper suggest 2020 (Rott 1990), but in his later paper (Reynolds 1895) he quotes the range of 1900–2000. Over the years, the uncertainty and variability in this number have grown larger. For instance, Prandtl & Tietjens (1931) wrote that ‘no turbulence can be expected to occur below 1000’, and many textbooks quote values of approximately 2000. Very revealing are the entries to the Wikipedia online encyclopaedia in its different languages. As of 28 February 2008, the quote for the critical Reynolds number is 2300 in the English, French and Swedish versions, and 2320 in the German version. In other languages, intervals for the transition are given in the form of lower values below which the flow is laminar and upper values above which it is turbulent. For instance, the Spanish version gives the range of 2000–4000, the Polish one gives 2300–10 000, the Dutch version has 2300–3500, and the Portuguese one 2000–3000. Of course, the quoted values depend on the criteria used for their definition, and Rotta (1956) or Wygnanski & Champagne (1973) and Wygnanski et al. (1975) carefully describe their criteria.

The 125th anniversary of the publication of Reynolds’ seminal paper is a welcome opportunity to take stock of where we are in our understanding of these puzzling observations. The contributions to this volume highlight several recent developments in pipe flow and in the closely related duct and channel flow. They are arranged so that we can begin with a discussion of the lowest critical Reynolds number, turn to a discussion of critical perturbations and close with the unexpected phenomenon of transient turbulence.
The difficulties in defining a critical Reynolds number arise from the absence of a linear instability: an investigation of the Navier–Stokes equation linearized around the parabolic profile shows that it is linearly stable for all Reynolds numbers (Salwen et al. 1980; Schmid & Henningson 2001; Meseguer & Trefethen 2003). Perturbations that are sufficiently small that they can be described by the linearized equations of motion will therefore decay, and the nonlinear terms are essential for maintaining the turbulent dynamics (Gebhardt & Grossmann 1994). But with the nonlinear terms included, other persistent flow structures around which the turbulent flow organizes itself can exist, very much as in other systems where periodic orbits carry chaotic dynamics (Cvitanovic & Eckhardt 1991). Such structures will have to be three dimensional, and have been found by Faisst & Eckhardt (2003), Wedin & Kerswell (2004), Pringle & Kerswell (2007) and Eckhardt et al. (2008). A family of highly symmetric states is discussed by Pringle et al. (2009). Since turbulence can arise only once such structures exist, the current lowest value of $Re_{TW} = 773$ for the existence of travelling waves defines a minimal requirement for turbulence.

It turns out that all these coherent states are unstable and can appear transiently in the dynamics only. Nevertheless, they have been identified in experimental and numerical simulations (Hof et al. 2004; Kerswell & Tutty 2007; Schneider et al. 2007a). Proper orthogonal decomposition is another means of extracting large-scale coherent structures, and is used by Duggleby et al. (2009) to identify these structures. They have related these structures to turbulent drag and drag reduction by wall manipulations (Duggleby et al. 2007). Some of these structures might be related to the coherent travelling waves. For instance, proper orthogonal decomposition gave first evidence for helical travelling waves, which also exist as exact coherent states (Duguet et al. 2008).

The linear stability of the laminar profile implies that the exact coherent structures have to arise in finite-amplitude saddle-node bifurcations at some finite distance from the laminar profile. A sufficiently large perturbation will hence be able to trigger turbulence, but not all perturbations are equally effective. The studies of Boberg & Brosa (1988) and Trefethen et al. (1993) have highlighted the fact that, even when all eigenvalues of the linearized problem indicate decay, there can be transient amplification and growth in energy owing to the non-normality of the linearized operator. A link between these structures, their amplification and nonlinear instabilities as seen in experiments on turbulent puffs is discussed by van Doorne & Westerweel (2009). Theoretical approaches to extract perturbations that are efficient in triggering turbulence are presented by Biau & Bottaro (2009) and Cohen et al. (2009), respectively.

The observation that the upper critical Reynolds number can be increased by carefully controlling perturbations in the system shows that the amplitude of tolerable perturbations decreases as the Reynolds number increases. Determining just how this amplitude scales with Reynolds number is not easy. Scaling arguments based on the nonlinear interactions suggest a $1/Re$ behaviour, as was also seen in some experiments (Hof et al. 2003). Refined experiments (Peixinho & Mullin 2007) and also the numerical simulations by Mellibovsky & Meseguer (2009) and Viswanath (2009), respectively, show a dependence on the type of perturbation, suggesting that at least for some perturbations the critical amplitude falls off as $Re^{-\alpha}$ with $\alpha$ close to 1.5.
Guided by the observation that between an initial condition that decays and one that becomes turbulent there should be one that does neither one nor the other, a numerical algorithm was developed which allows a trajectory intermediate between laminar and turbulent flow to be tracked (Skufca et al. 2006). When applied to pipe flow, this algorithm converges to a coherent structure consisting of a pair of vortices in the downstream direction (Schneider & Eckhardt 2006, 2009; Schneider et al. 2007b; Mellibovsky & Meseguer 2009). The high Reynolds number behaviour discussed by Viswanath (2009) suggests that these structures maintain their finite distance from the laminar profile: this implies that the scaling in critical Reynolds number then has to come from their stable and unstable manifolds.

The most unexpected observation of recent years in pipe flow is that of the transient nature of the turbulence (Brosa 1989; Hof et al. 2006). This transience does not arise from the advection of the turbulence by the mean flow, but is observed in a frame of reference co-moving with the turbulent puff: following the perturbation downstream, one notes a sudden relaminarization without any indication of the imminent decay (Peixinho & Mullin 2007). For these transitional Reynolds numbers, the distribution of lifetimes is exponential (Faisst & Eckhardt 2004; Peixinho & Mullin 2006, 2007; Willis & Kerswell 2007), and can be described by a characteristic time scale. Turbulence is persistent only if this time diverges at some critical Reynolds number. The current empirical evidence is discussed by deLozar & Hof (2009).

It seems that 125 years after Reynolds’ paper, we still do not have a full understanding of the nature of the turbulence transition in pipe flow. But the recent advances in numerical and experimental studies combined with the input from dynamical systems theory have helped to sharpen the questions that we would like to answer (Eckhardt et al. 2007; Eckhardt 2008).

As regards the upper critical Reynolds numbers we now know that the flow remains stable for all Reynolds numbers and that ‘natural’ transition at large Reynolds numbers comes about from the interplay between background fluctuations and increased sensitivity of the flow. Thus, the upper critical Reynolds number depends on the kind and amplitude of residual perturbations in the systems, and increases when the perturbations are reduced. The exact nature of the relationship between critical Reynolds number and perturbation remains unknown.

As regards the lower critical Reynolds we know that there are coherent structures that exist for Reynolds numbers as low as 773, but do not know whether this is the lowest $Re$ possible. Since there are no experimental observations of any motion resembling turbulence at these low Reynolds numbers, these structures do not affect an appreciable part of the state space of the system. Increasing the Reynolds number above approximately 1650, turbulent puffs are seen experimentally. There are various possibilities for what could happen in the state space of pipe flow between 773 and 1650, but they remain to be investigated.

The turbulent dynamics that can be observed at approximately 1650 is transient, i.e. when followed for a sufficiently long time, the flow spontaneously relaminarizes. The observations are consistent with an exponential distribution of lifetimes. The characteristic times in the exponential increase rapidly with Reynolds number, and become larger than approximately 2000 time units near approximately $Re=1950$. Owing to the rapid increase with $Re$, this number varies little even when the lifetimes are doubled, and hence may serve as a

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practical Reynolds number for pipe flow in a probabilistic sense. It remains an important theoretical question whether the turbulence remains transient for higher Reynolds numbers or whether there is a transition to a turbulent attractor.

Reynolds’ studies were motivated by their practical and philosophical aspects. For the practical aspect, the law of turbulent friction, some consensus was achieved long ago. The philosophical aspects, about the nature of the transition to turbulence in shear flows, remain attractive to this date. The progress of late builds on our progress in dynamical systems theory as well as the availability of large-scale numerical computations without which the detailed investigations reported here would not have been possible. As a consequence, we now have a theoretical setting in which to approach the problem with very focused questions. While it has taken 125 years to extract from Reynolds’ observations the questions paraphrased above, it will hopefully not take another century to solve them.

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