Unscented Kalman filter approach to tracking a moving interfacial boundary in sedimentation processes using three-dimensional electrical impedance tomography

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The monitoring of solid–fluid suspensions under the influence of gravity is widely used in industrial processes. By considering sedimentation layers with different electrical properties, non-invasive methods such as electrical impedance tomography (EIT) can be used to estimate the settling curves and velocities. In recent EIT studies, the problem of estimating the locations of phase interfaces and phase conductivities has been treated as a nonlinear state estimation problem and the extended Kalman filter (EKF) has been successfully applied. However, the EKF is based on a Gaussian assumption and requires a linearized measurement model. The linearization (or derivation of the Jacobian) is possible when there are no discontinuities in the system. Furthermore, having a complex phase interface representation makes derivation of the Jacobian a tedious task. Therefore, in this paper, we explore the unscented Kalman filter (UKF) as an alternative approach for estimating phase interfaces and conductivities in sedimentation processes. The UKF uses a nonlinear measurement model and is therefore more accurate. In order to justify the proposed approach, extensive numerical experiments have been performed and a comparative analysis with the EKF is provided.

Keywords: electrical impedance tomography; two-phase flow; settling curves; extended Kalman filter; unscented Kalman filter

1. Introduction

The sedimentation or settling of suspensions is of great interest in many industrial applications such as mining, waste water treatment, and the pulp and paper industry (White & Verdone 2002; Yoshida et al. 2005). In a sedimentation...
process, a solid–fluid suspension is separated into different layers under the influence of gravity. For example, solid particles (such as minerals, polymer, radioactive drugs and solid wastes) are stored and transported by mixing with a liquid medium to a place where they can be processed. Sedimentation monitoring provides information about the properties of sediments and thus can be used to control and optimize industrial processes. In the modelling of the sedimentation process, it is generally assumed that the fluid settles into three different layers separated by sharp interfaces. The top layer is a clear liquid, the middle layer is a diluted slurry, where the actual sedimentation takes place, and the bottom layer is a compact layer, where hindered settling occurs. These layers settle with a velocity that is assumed to be a function of the solid composition of the layers. During the sedimentation process the phase interfaces between the three layers change with respect to time and are commonly referred to as settling curves. The rate of change of these settling curves gives rise to settling velocities. These settling curves and velocities provide necessary information about the sedimentation process (Diehl 1997; Burger & Concha 1998; Burger et al. 1999; Coronel et al. 2003; Garrido et al. 2003).

Several measurement techniques have been employed to obtain settling curves and velocities. Some of these consider optical techniques that use a laser as a source and a photodiode as a receiver (Pinelli et al. 2004). Another method given in Zhu et al. (2000) uses digital image processing techniques, where a series of digital photographs of the sedimentation tank are taken and digital image processing techniques such as edge detection are used to process the images to obtain the settling curves and velocities. Different phase layers have different electrical properties and therefore electrical methods can also be applied to monitor the sedimentation process (Blewett et al. 2001; Bolton et al. 2002; Cullivan et al. 2005; Murphy et al. 2006). Moreover, such methods can implicitly estimate parameters such as the underlying conductivity distribution. Also, transparent sedimentation tanks are not needed unlike for the other measurement techniques. Considering the advantages of the electrical methods, electrical impedance tomography (EIT) has been successfully used for sedimentation monitoring (Bolton et al. 2002; Ricard et al. 2005; Murphy et al. 2006; Tossavainen et al. 2007). In EIT, a set of electric currents is passed through electrodes mounted on the surface of the object and the voltages are measured. Based on the current–voltage relationship, the internal conductivity distribution is reconstructed. Reconstruction algorithms for EIT are generally classified into static and dynamic algorithms. The static algorithms are usually employed for visualization of the time-invariant internal conductivity distribution. For those cases which involve fast changes, dynamic algorithms are used (Kim et al. 2001, 2007a,b). One of the most commonly used dynamic algorithms for nonlinear systems is the extended Kalman filter (EKF).

In the EKF, the state distribution is approximated by a Gaussian random variable (GRV) and is propagated through a linear approximation of the system around the operating point at each time instant. Linearization can only be applied if the Jacobian matrices can be formulated. However, some systems contain discontinuities and representation of a phase interface can be complex. In such cases, derivation of Jacobian matrices is often difficult (Julier & Uhlmann 2004). Furthermore, this linear approximation causes an error in the posterior mean and covariance of the transformed GRV. This results in suboptimal
performance and may lead the state to diverge over time. To overcome the limitations with the EKF, the unscented transform (UT) was developed as a method to propagate mean and covariance information through a nonlinear transformation (Julier & Uhlmann 2004). The Kalman filter based on the UT is called the unscented Kalman filter (UKF). The UKF works on the principle that it is easier to approximate a Gaussian distribution when compared with approximating a nonlinear function. The UKF uses a deterministic sampling approach to capture the true mean and covariance with a minimal set of carefully chosen sample points. In the UKF, the mean and covariance of the state estimate for any nonlinearity are accurate up to second order, as opposed to the EKF, which only achieves first-order accuracy. For a Gaussian distribution, the UKF has third-order accuracy and the EKF by contrast achieves first-order accuracy. Remarkably, the computational complexity of the UKF is the same order as that of the EKF. Considering the advantages of the UKF over the EKF, in this paper, the UKF is used as an inverse solver to estimate the non-stationary phase interface. In EIT, a random-walk model is used as a state evolution model in which the rate of evolution is governed by the covariance of the process noise. The modelling uncertainty of the random-walk model causes poor reconstruction performance. In order to improve the modelling accuracy, dynamic evolution of the fluid can also be taken into consideration. These dynamic evolution models, commonly known as kinematic models, can be used to track the settling curves in sedimentation, as the settling curves change with constant speed between the measurements.

The remainder of the paper is organized as follows. The mathematical model for three-dimensional EIT is briefly outlined in §2. In §3, the boundary parametrization used for the sedimentation process is presented and the Jacobian is derived. The state-space estimation approach using the EKF is formulated in §4a and the formulation for the UKF is given in §4b. In §5, the results for numerical experiments are provided and the UKF is compared with the EKF. Finally, conclusions and suggestions for future work is given in §6.

2. Mathematical model

In sedimentation monitoring using EIT, electric currents are injected into electrodes attached on the periphery of the tank. When electric currents $I_l (l=1, 2, \ldots, L)$ are injected into an object $\Omega \in \mathbb{R}^3$ with internal structure through the electrodes $e_l (l=1, 2, \ldots, L)$ attached on its boundary $\partial \Omega$, the conductivity distribution $\sigma$ is known for $\Omega$. Then the corresponding electrical potential $u$ on $\Omega$ can be determined uniquely from a partial differential equation, which can be derived from the Maxwell equations as

$$\nabla \cdot (\sigma \nabla u) = 0 \quad \text{in} \quad \Omega, \quad (2.1)$$

with the following boundary conditions based on the complete electrode model:

$$u + z_l \sigma \frac{\partial u}{\partial n} = \bar{U}_l \quad \text{on} \quad e_l, \quad l = 1, 2, \ldots, L, \quad (2.2)$$

$$\int_{e_l} \sigma \frac{\partial u}{\partial n} \, dS = I_l, \quad l = 1, 2, \ldots, L. \quad (2.3)$$
where \( z_l \) is the effective contact impedance between the \( l \)th electrode and the electrolyte; \( \bar{U}_l = U_l(t_k) \) is the potential on the \( l \)th electrode at time \( t_k \); \( I_l = I_l(t_k) \) is the injected current on the electrode at time \( t_k \); \( e_l \) is the \( l \)th electrode; \( n \) is the outward unit normal; and \( L \) is the total number of electrodes. Furthermore, the following two additional constraints for the injected currents and measured voltages are needed to ensure the existence and uniqueness of the solution:

\[
\sum_{l=1}^{L} I_l = 0, \\
\sum_{l=1}^{L} \bar{U}_l = 0.
\]

The computation of the potential \( u \) on \( \Omega \) and voltages \( \bar{U}_l \) on the electrodes for the given conductivity distribution \( \sigma \) and boundary conditions is called the forward problem. In general, it is difficult to solve the forward problem analytically; thus we have to resort to numerical methods. In this paper, we have employed the finite-element method (FEM) to obtain a numerical solution. In the FEM, the domain is discretized into small tetrahedral elements each having a constant resistivity distribution. The finite-element approximation of the forward problem is given in Tossavainen et al. (2006a,b, 2007) and is again presented here for the formulation of the inverse solver.

The finite-element approximation to the potential distribution inside the domain is given by

\[
u \approx u^h(x, y, z) = \sum_{i=1}^{N} \alpha_i \varphi_i(x, y, z),
\]

and the potential on the electrodes is represented as

\[
U^h = \sum_{j=1}^{L-1} \beta_j n_j,
\]

where \( N \) is the number of nodes in the finite-element mesh; \( \varphi_i \) is the three-dimensional first-order basis function; and the bases for the measurement are \( n_1 = (1, -1, 0, \ldots, 0)^T \), \( n_2 = (1, 0, -1, 0, \ldots, 0)^T \in \mathbb{R}^{L \times 1} \), etc. Here, \( \alpha_i \) and \( \beta_j \) are the nodal and boundary voltages, which are the unknowns to be determined.

From (2.7) and (2.8), the FEM solution can be represented as a set of linear equations

\[
Ab = \tilde{I},
\]

where

\[
A = \begin{pmatrix} B & C \\ C^T & D \end{pmatrix}, \quad b = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \quad \text{and} \quad \tilde{I} = \begin{pmatrix} 0 \\ \zeta \end{pmatrix},
\]

where \( \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_N)^T \in \mathbb{R}^{N \times 1} \); \( \beta = (\beta_1, \beta_2, \ldots, \beta_{L-1})^T \in \mathbb{R}^{(L-1) \times 1} \); and \( \mathbf{0} \in \mathbb{R}^{N \times 1} \).
The reduced current vector is defined as
\[ \zeta = (I_1 - I_2, I_1 - I_3, \ldots, I_1 - I_L)^T = N^T \hat{I} \in \mathbb{R}^{(L-1) \times 1}, \] (2.11)
where \( \hat{I} \) is the current vector given by \( \hat{I} = (I_1, I_2, \ldots, I_L)^T \in \mathbb{R}^{L \times 1} \).

The elements of stiffness matrix \( A \) are of the form
\[
B(i, j) = \int_{\Omega} \sigma \nabla \varphi_i \cdot \nabla \varphi_j \, d\Omega + \sum_{l=1}^{L} \frac{1}{z_l} \int_{e_l} \varphi_i \varphi_j \, dS, \quad i, j = 1, 2, \ldots, N, \] (2.12)
\[
C(i, j) = -\frac{1}{z_1} \int_{e_1} \varphi_i \, dS + \frac{1}{z_{j+1}} \int_{e_{j+1}} \varphi_i \, dS, \quad i = 1, 2, \ldots, N, \quad j = 1, 2, \ldots, L-1, \] (2.13)
\[
D(i, j) = \begin{cases} 
\frac{|e_i|}{z_1}, & i \neq j, \\
\frac{|e_i|}{z_1} + \frac{|e_{j+1}|}{z_{j+1}}, & i = j,
\end{cases} \quad i, j = 1, 2, \ldots, L-1, \] (2.14)
where \(|e_j|\) is the area of the electrode \( j \).

In some cases, the voltages are not measured at every electrode. Also, the selected electrodes may be different for each current pattern. Incorporating this constraint, the measured voltages at the measurement electrodes \( \hat{U} \) can be obtained as
\[
\hat{U} = M^T U^h = M^T N \beta \in \mathbb{R}^{E \times P}, \] (2.15)
where \( E \) is the number of the measurement electrodes; \( P \) is the number of current patterns; and \( M \in \mathbb{R}^{L \times E} \) is the measurement matrix. Furthermore, \( U^h \) can be extracted directly from \( b \) by introducing the extended mapping matrix \( \tilde{N} \),
\[
\tilde{N} = (0, N) \in \mathbb{R}^{L \times (N+L-1)} \quad \text{and} \quad U^h = \tilde{N} b, \] (2.16)
where \( 0 \in \mathbb{R}^{L \times N} \). Therefore, we have
\[
\hat{U} = M^T U^h = M^T \tilde{N} b = \tilde{M} b, \] (2.17)
where the extended measurement matrix is defined as
\[
\tilde{M} = M^T \tilde{N} \in \mathbb{R}^{E \times (N+L-1)}. \] (2.18)

### 3. Boundary representation

The sedimentation tank consists of three layers of different phases separated by sharp interfaces (figure 1). The conductivity functions are parametrized by the locations of the phase interfaces \( (\gamma_k, k = 1, 2) \) and the conductivities of the phase layers \( (\sigma_k, k = 1, 2 \text{ and } 3) \). The phase interfaces \( C_q (q = 1, 2) \) at height \( \gamma_q \) can be
represented by a horizontal plane as
\[ C_q(\gamma_q) = \{ w \in \mathbb{R}^3, (w - r_q)^T n = 0 \}, \quad (3.1) \]
where \( n = (0, 0, 1)^T \) is the normal vector of the plane; and the position vector is of the form \( r_q = (0, 0, \gamma_q)^T \). Here, apart from the conductivities, the phase interfaces are also estimated. This parametrization affects the system matrix in the FEM formulation. We present the FEM formulation briefly. The complete description of this FEM formulation can be found elsewhere (Kolehmainen et al. 2001; Tossavainen et al. 2006a, b, 2007).

Let us assume that the region \( \Omega \) is divided into disjoint regions \( S_k \) given by
\[ \Omega = \bigcup_{k=1}^{3} S_k, \quad (3.2) \]
where three regions exist in the domain \( (k=1, 2, 3) \) as shown in figure 1. If \( \chi_k(r) \) denotes the characteristic function of subregion \( S_k \), the conductivities of each layer can be expressed as
\[ \sigma = \sum_{k=1}^{3} \sigma_k\chi_k(r). \quad (3.3) \]

By substituting (3.3) into (2.12) we get
\[ B(i, j) = \sum_{k=1}^{3} \int_{\text{supp}(\varphi_i, \varphi_j) \cap S_k} \sigma_k \nabla \varphi_i \cdot \nabla \varphi_j \, d\Omega + \sum_{l=1}^{L} \frac{1}{z_l} \int_{e_l} \varphi_i \varphi_j \, dS, \quad i, j = 1, 2, \ldots, N, \quad (3.4) \]
where supp($\varphi_i, \varphi_j$) is the part of domain $\Omega$ where both the basis functions are non-zero. The implementation of the integrals of the form (3.4) has been previously described (Kolehmainen et al. 2001; Tossavainen et al. 2006a,b, 2007). As a first step, mesh elements $\Omega_m$ are classified into sets of elements that intersect the interface and those elements that lie above and below the phase interface. The elements that do not intersect the interface boundary are assigned their corresponding conductivity values ($\sigma_k$, $k=1, 2, 3$). However, for the elements that lie on the interface, the volume-weighted conductivity values are assigned as (Kolehmainen et al. 2001)

$$
\sigma_e = \frac{\sigma_u|\hat{S}_u| + \sigma_l|\hat{S}_l|}{|\hat{S}_u| + |\hat{S}_l|},
$$

(3.5)

where $\hat{S}_u = S_u \cap \Omega_m$, $\hat{S}_l = S_l \cap \Omega_m$ and $\hat{S}_u \cap \hat{S}_l = \Omega_m$. Here $\hat{S}_u$ and $\hat{S}_l$ are the volumes of the upper and lower parts of the split element. The subscripts ‘u’ and ‘l’ refer to the upper and lower parts of the element $\Omega_m$ with respect to the phase interface, respectively.

Assuming that the basis functions $\varphi_i$ and $\varphi_j$ are constant within the whole element $\Omega_m$, the volume integration in (3.4) can be treated as

$$
\frac{\sigma_u|\hat{S}_u| + \sigma_l|\hat{S}_l|}{|\hat{S}_u| + |\hat{S}_l|} \int_{\Omega_m} \nabla \varphi_i \cdot \nabla \varphi_j \, d\Omega.
$$

(3.6)

The detailed description with regard to implementation of the volume integral (3.6) is presented in Tossavainen et al. (2007).

(a) Derivation of Jacobian

In general, EIT has a nonlinear relationship between the boundary voltages and the conductivity distribution, so the measurement equation is linearized before use in the conventional inverse solver. This leads to derivation of the Jacobian matrix. The Jacobian determines how sensitive the measurements are with respect to change in conductivity inside the domain. In the current implementation, the Jacobian represents the change in boundary voltages with respect to phase interfaces, their corresponding velocities and conductivities.

The Jacobian can be formulated as follows. If measurement matrix $\mathbf{M} \in \mathbb{R}^{L \times E}$ is used, the voltages on the electrodes can be given by (2.18). The Jacobian can then be defined as

$$
J = \frac{\partial \tilde{U}}{\partial \theta} = \frac{\partial (\tilde{M} \mathbf{b})}{\partial \theta} = \tilde{M} \frac{\partial A^{-1}}{\partial \theta} \tilde{I}.
$$

(3.7)

The pseudo-resistance matrix is defined as

$$
\tilde{R} = A^{-1} \tilde{M}^T \in \mathbb{R}^{(N+L-1) \times E} \text{ or } A \tilde{R} = \tilde{M}^T.
$$

(3.8)

The pseudo-resistance matrix can be easily obtained by the solution of the system equation

$$
A(\tilde{R} \quad \mathbf{b}) = (\tilde{M}^T \quad \tilde{I}^T).
$$

(3.9)
Using (2.18) and (2.10) the above equation can be written as

\[
A \begin{pmatrix} \tilde{R}_1 \\ \tilde{R}_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ N^T M & N^T I \end{pmatrix},
\]

(3.10)

where

\[
\tilde{R}_1 = \tilde{R}(i,j) \in \mathbb{R}^{N \times E} \quad (i = 1, 2, \ldots, N; \quad j = 1, 2, \ldots, E),
\]

\[
\tilde{R}_2 = \tilde{R}(i,j) \in \mathbb{R}^{(L-1) \times E} \quad (i = N + 1, \ldots, N + L - 1; \quad j = 1, 2, \ldots, E).
\]

The Jacobian \( \frac{\partial \tilde{U}}{\partial \theta_i} \) is given by

\[
\frac{\partial \tilde{U}}{\partial \theta_i} = \tilde{R}^T \frac{\partial \tilde{A}}{\partial \theta_i} A^{-1} \mathbf{B} = \tilde{R}^T \frac{\partial \tilde{A}}{\partial \theta_i} A^{-1} \mathbf{B} = \tilde{R}^T \frac{\partial \tilde{A}}{\partial \theta_i} A^{-1} \mathbf{B} = \tilde{R}^T \frac{\partial \tilde{A}}{\partial \theta_i} A^{-1} \mathbf{B}.
\]

(3.12)

Here, \( \theta_i \) is the state vector, which includes the settling curves, velocities and phase conductivities. The evaluation of the term \( \frac{\partial \mathbf{B}}{\partial \gamma_{i=1,2}} \) with respect to the phase interfaces and \( \frac{\partial \mathbf{B}}{\partial \sigma_{i=1,2,3}} \) with respect to the phase conductivities is given in Tossavainen et al. (2007).

## 4. Inverse problem

In the inverse problem, the phase interface location, the rate of change of the interface and the conductivity distribution for each phase are estimated based on the injected currents and the measured voltages. The underlying inverse problem is treated as a state estimation problem (Kim et al. 2001), which consists of a state equation that models the temporal evolution of the state, and an observation equation that gives the relationship between the state and the boundary voltages.

First of all, consider the state evolution model. The state equation is assumed to be of linear form defined as

\[
\theta_t = F_{t-1} \theta_{t-1} + D_{t-1} w_{t-1}.
\]

(4.1)

The measurement equation corresponding to boundary voltages is given by

\[
\tilde{U}_t = h_t(\theta_t) + v_t,
\]

(4.2)

where subscript \( t \) represents the state index; \( F_t \in \mathbb{R}^{N \times N} \) is the state transition matrix; \( N \) is the number of unknown state parameters; \( h_t \) is the observation matrix; and \( w_t \in \mathbb{R}^{N \times 1} \) and \( v_t \in \mathbb{R}^{L \times 1} \) denote the process noise and measurement noise, respectively. They are assumed to be zero-mean Gaussian noise with
covariance $Q = E[\omega \omega^T]$ and $R = E[\nu \nu^T]$, respectively. Also, $D_t$ represents the transition matrix of the process noise; $h_t(\theta_t)$ represents the forward solution through FEM.

(a) Formulation of the EKF

In the EKF, we estimate the state $\theta_t$ based on all the measurements taken up to the time step $t$ (Kim et al. 2001, 2004, 2007a,b). With the Gaussian assumption, the required estimate is obtained by minimizing the cost functional, which is formulated based on the above-mentioned state and measurement equations (4.1) and (4.2), respectively. The cost functional for the EKF is of the form

$$G(\theta_t) = \frac{1}{2} \left\{ ||\theta_t - \theta_{t\mid t-1}||^2_{P_{t\mid t-1}} + ||\tilde{U}_t - J_t(\theta_{t\mid t-1}) \theta_t||^2_{R_{t-1}} \right\}, \quad (4.3)$$

where $||x||_A$ denotes $x^T A x$, and $P_{t\mid t-1} \in \mathcal{R}^{N \times N}$ is the time-updated error covariance matrix defined as

$$P_{t\mid t-1} = E[(\theta_t - \theta_{t\mid t-1})(\theta_t - \theta_{t\mid t-1})^T]. \quad (4.4)$$

Minimizing the cost functional (4.3) and solving for the updates of the associated covariance matrices, a recursive EKF algorithm is obtained, which consists of the following steps.

**Step 1. Predicted (a priori) state**

$$\theta_{t\mid t-1} = F_{t-1} \theta_{t-1\mid t-1}. \quad (4.5)$$

**Step 2. Predicted (a priori) error covariance**

$$P_{t\mid t-1} = \text{cov}(\epsilon_{t\mid t-1}) = F_{t-1} P_{t-1\mid t-1} F_{t-1}^T + D_{t-1} Q_{t-1} D_{t-1}^T. \quad (4.6)$$

**Step 3. Predicted measurement error**

$$\epsilon_t = \tilde{U} - h_t \theta_{t\mid t-1}. \quad (4.7)$$

**Step 4. Predicted measurement error covariance**

$$S_t = \text{cov}(\epsilon_t) = J_t P_{t\mid t-1} J_t^T + R_t. \quad (4.8)$$

**Step 5. Kalman gain**

$$K_t = P_{t\mid t-1} J_t^T S_t^{-1}. \quad (4.9)$$

**Step 6. Updated (a posteriori) state**

$$\theta_{t\mid t} = \theta_{t\mid t-1} + K_t \epsilon_t = \theta_{t\mid t-1} + K_t (\tilde{U}_t - h_t(\theta_{t\mid t-1})). \quad (4.10)$$

**Step 7. Updated (a posteriori) error covariance**

$$P_{t\mid t} = (I - K_t J_t) P_{t\mid t-1}. \quad (4.11)$$

Here $J_t$ represents the Jacobian with respect to the previous state ($\theta_{t\mid t-1}$). One of the main tasks in the inverse solver is the computation of the Jacobian.
(b) Formulation of the UKF

As already mentioned, the EKF uses a GRV to estimate the state distribution and uses the Jacobian. The UKF, on the other hand, uses a deterministic sample-and-propagate approach (UT) to capture the mean and variance estimates with a minimum set of carefully chosen sample points (Julier & Uhlmann 2004; Ijaz et al. 2008a,b; Kandepu et al. 2008). Considering the linear state equation and the nonlinear measurement equation, the underlying algorithm for the UKF is explained as follows.

Given the state vector at step \( t-1 \), generate a set of sigma points, and store them in columns of the \( \bar{N} \times (2\bar{N} + 1) \) sigma point matrix \( \chi_{t-1} \), where \( \bar{N} \) is the dimension of the state vector. The sigma points are selected to lie on the principal component axes of the covariance \( P_{t-1} \), and include one extra point for the mean \( \theta \) of the distribution:

\[
\bar{\theta} = E[\theta_0],
\]

\[
P_0 = E[(\theta_0 - \bar{\theta})(\theta_0 - \bar{\theta})^T].
\]

For \( t \in \{1, \ldots, \infty \} \), calculate the sigma points,

\[
\chi_{t-1} = \begin{bmatrix}
\bar{\theta}_{t-1} \\
\bar{\theta}_{t-1} + \sqrt{(\bar{N} + \lambda)P_{t-1}} \\
\bar{\theta}_{t-1} - \sqrt{(\bar{N} + \lambda)P_{t-1}}
\end{bmatrix},
\]

where \( \sqrt{(\bar{N} + \lambda)P_{t-1}} \) is the \( t \)th column of the matrix square root; \( \lambda \) is a scaling parameter that determines how far the sigma points are spread from the mean and is defined by

\[
\lambda = \alpha^2(\bar{N} + \kappa) - \bar{N}.
\]

Here \( \alpha \) determines the spread of the sigma points around \( \bar{\theta} \) and is usually set to a small positive value (e.g. \( 1 \leq \alpha \leq 10^{-4} \)); and \( \kappa \) is a secondary scaling parameter that approximates the higher-order terms and is usually set to either 0 or \( 3 - \bar{N} \) (van der Merwe 2004). The prediction step or time update is performed by propagating the generated sigma points through the nonlinear state equation. The propagated sigma points are then combined with associated weights to produce the predicted state and covariances.

The time update is

\[
\theta^*_{t|t-1} = F_{t-1} \cdot \chi_{t-1} + \chi^*_t,
\]

\[
\theta^m_t = \sum_{i=0}^{2\bar{N}} W_i^{(m)} \theta^*_{i,t|t-1},
\]

\[
P_t = \sum_{i=0}^{2\bar{N}} W_i^{(c)} [\theta^*_{i,t|t-1} - \theta^m_t][\theta^*_{i,t|t-1} - \theta^m_t]^T + Q_t,
\]

where \( F_{t-1} \) denotes the state transition matrix; and \( W_i^{(m)} \) are the weights defined by

\[
W_0^{(m)} = \frac{\lambda}{(\bar{N} + \lambda)},
\]

\[
W_0^{(c)} = \frac{\lambda}{(\bar{N} + \lambda)} + (1 - \alpha^2 + \beta),
\]
\[ W_i^{(c)} = W_i^{(m)} = \frac{1}{2(N + \lambda)}, \quad i = 1, \ldots, 2N. \]  

(4.21)

Here, \( Q_t \) is the process noise covariance; \( \beta \) is used to incorporate prior knowledge of the distribution of \( \theta \) and for a Gaussian distribution, \( \beta = 2 \) is optimal (Julier & Uhlmann 2002, 2004).

To compute the measurement update, the sigma points are transformed through the nonlinear measurement equation to obtain the predicted voltage estimates,

\[ \psi_{t|t-1} = h_t(\theta^*_{i,t|t-1}) + x^w_{i-1}, \]  

(4.22)

\[ \hat{U}_t^{-} = \sum_{i=0}^{2N} W_i^{(m)} \psi_{i,t|t-1}. \]  

(4.23)

With the transformed state vector \( \hat{U}_t^{-} \), we compute the a priori state estimate using

\[ \hat{\theta}_t = \theta^p_t + K_t(\hat{U}_t - \hat{U}_t^-), \]  

(4.24)

where \( K_t \) is the Kalman gain and is defined as

\[ K_t = P_{\theta_i,\hat{U}_t} \hat{U}_t^{-} P_{\theta_i,\hat{U}_t}^{-1}; \]  

(4.25)

\[ P_{\theta_i,\hat{U}_t} = \sum_{i=0}^{2N} W_i^{(c)} [\psi_{i,t|t-1} - \hat{U}_t^-] [\psi_{i,t|t-1} - \hat{U}_t^-]^T + R, \]  

(4.26)

\[ P_{\theta_i,\hat{U}_t} = \sum_{i=0}^{2N} W_i^{(c)} [\theta^*_{i,t|t-1} - \theta^p_t] [\psi_{i,t|t-1} - \hat{U}_t^-]^T. \]  

(4.27)

Two significant covariance matrices, \( P_{\hat{U}_t,\hat{U}_t} \) and \( P_{\theta,\hat{U}_t} \), are used. During the iterative process \( P_{\hat{U}_t,\hat{U}_t} \) will be reduced so that the transformed sigma points move towards the cluster mean. With the introduction of the measurement data \( \hat{U}_t \), the cluster mean will then move further towards the true mean and, as a consequence, \( P_{\theta,\hat{U}_t} \) will be reduced. In (4.26), \( R \) is the measurement noise covariance matrix. Finally, the a posteriori estimate of the error covariance is given by

\[ P_t = P_t^- - K_t P_{\hat{U}_t,\hat{U}_t} K_t^T. \]  

(4.28)

Here, it should be noted that, if the interface remains static during the application of some current patterns, then the transformed sigma points will move towards the true mean during the time the interface is static. However, the sigma points spread again whenever the interface changes. Therefore, the trade-off from the performance perspective is the number of current patterns and settling time.

(c) State evolution model

In EIT, the evolution of the state is governed by (4.1). The random-walk model is usually considered, in which the modelling uncertainty is compensated by the process noise. A better approach is to use kinematic models, which are popular in motion tracking (Bar-Shalom & Li 1993; Li 1997; Kim et al. 2007a,b). By representing the sedimentation process using Newtonian kinematic laws,
the phase interfaces are assumed to evolve with constant velocity between consecutive measurements. The evolution model for location parameters $\gamma_{t,i}$ ($i=1,2$) can be described as

$$
\gamma_{t+1,i} = \gamma_{t,i} + \dot{\gamma}_{t,i} \Delta T + \frac{1}{2} \Delta T^2 w_{t,i},
$$

(4.29)

$$
\dot{\gamma}_{t+1,i} = \dot{\gamma}_{t,i} + \Delta T w_{t,i},
$$

(4.30)

where $\dot{\gamma}_{t,i}$ is the rate of change of the location of the phase interfaces (settling velocities), and $\Delta T$ denotes the time difference between the time indices $t+1$ and $t$. For conductivities ($\sigma_k$, $k=1,2,3$), the random-walk model is still used as an evolution model.

The unknown state parameters to be estimated then become

$$
\theta_t = (\gamma_{t,1}, \dot{\gamma}_{t,1}, \gamma_{t,2}, \dot{\gamma}_{t,2}, \sigma_{t,1}, \sigma_{t,2}, \sigma_{t,3})^T \in \mathbb{R}^{7 \times 1}.
$$

(4.31)

The corresponding state and noise transition matrices $F_t$ and $D_t$ are modified as

$$
F_t = \begin{pmatrix}
1 & \Delta T & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & \Delta T & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix},
$$

$$
D_t = \begin{pmatrix}
\frac{1}{2} \Delta T^2 & 0 & 0 & 0 & 0 \\
\Delta T & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2} \Delta T^2 & 0 & 0 & 0 \\
0 & \Delta T & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}.
$$

(4.32)

From (4.31) it can be seen that the settling velocities are embedded as state variables. Therefore, they can be estimated directly in the state vector as opposed to by post-differentiation of the location of the phase interfaces. Since the settling velocities are constant, therefore, the corresponding column in the Jacobian matrix is also set to 0.

5. Results and discussion

In this section, the numerical results for three-dimensional sedimentation monitoring using the UKF are presented. A stratified flow of three immiscible liquids in a cylindrical tank of radius 5 cm and height 30 cm is considered. A total of 32 electrodes arranged in four vertical arrays of eight electrodes each are mounted on the tank as shown in figure 1. The size of each electrode is 1 cm$^2$ and the vertical separation between the electrodes’ centres is 2.5 cm. A simple current injection protocol comprising four current patterns is used. In each current injection, current is injected through the top and bottom electrodes of the vertical array and the voltage is measured across the adjacent vertical electrodes. Only non-current-carrying electrodes are involved in the measurement, in order to mitigate the effect of skin resistance (Cheng et al. 1990).
Therefore, a total of 104 $(4 \times 26)$ measurements are available at each time step. A contact impedance of $0.35 \, \Omega \, \text{cm}^2$ is used. The time between consecutive measurements is set to 1 min. For the generation of data, the domain volume is discretized into 19 672 finite tetrahedral elements and 4401 nodes, whereas in the inverse solver a coarse mesh with 2459 tetrahedral elements and 771 nodes is used (figure 2). The sedimentation tank is filled with liquid and solid sediment and is allowed to settle. As a test case, the settling velocities and phase conductivities are kept constant. The true conductivities of the top (liquid), middle (diluted slurry) and bottom (solid sediment) layers are set to 0.15, 0.125 and 0.1 mS cm$^{-1}$, respectively. We consider a scenario in which the phase interface locations of the top and bottom interfaces are at 15 and 2 cm, respectively, and the phase interface evolves with time. The rate at which the bottom interface evolves is $0.1 \, \text{cm min}^{-1}$ and that of the top interface is $-0.3 \, \text{cm min}^{-1}$. The initial estimate of the phase conductivities is obtained by solving the least squares problem. The global conductivity was obtained as 0.128 mS cm$^{-1}$. The initial estimates for the phase interfaces are $\gamma_{t,1} = 1.5 \, \text{cm}$ and $\gamma_{t,2} = 16 \, \text{cm}$, respectively, and the initial estimates of the settling velocities of the top and lower interfaces are taken as 0, i.e.

$$\theta_{0|0} = (1.5, 0, 16, 0, 0.128, 0.128, 0.128).$$

The number of unknown state parameters to be determined is $\tilde{N} = 7$; therefore a total of 15 $(2\tilde{N} + 1)$ sigma points are generated in the UKF. An alternative is to use the augmented form in which the total number of sigma points depends on the size of the state vector, process noise vector and measurement noise vector, and will increase to 93, i.e. $2(2\tilde{N} + L) + 1$.

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For numerical experiments, four test cases are considered, and the parameters are shown in tables 1–4. In the first case (case 1), the same mesh is used in the forward and inverse solvers and the simulated measurement data are free from noise. Figures 3–5 show the reconstructed results for case 1. Reconstruction

Table 1. Simulation parameters for test case 1.

<table>
<thead>
<tr>
<th>parameters</th>
<th>EKF</th>
<th>UKF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>$\text{diag}([2, 10^{-1}, 10^{-1}, 10^{-5}] \times 10^{-5})$</td>
<td>$\text{diag}([2, 10^{-1}, 10^{-1}, 10^{-5}] \times 10^{-5})$</td>
</tr>
<tr>
<td>$R$</td>
<td>$5 \times 10^{-6}$</td>
<td>$1 \times 10^{-6}$</td>
</tr>
<tr>
<td>$P_0$</td>
<td>$\text{diag}([5, 5, 5, 5, 5, 5] \times 10^{-4})$</td>
<td>$\text{diag}([1, 10, 10, 10, 10, 1] \times 10^{-5})$</td>
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<td>—</td>
<td>2</td>
</tr>
<tr>
<td>$\kappa$</td>
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</tr>
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Table 2. Simulation parameters for test case 2.

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>$\text{diag}([1, 10^{-3}, 10^{-3}, 10^{-3}] \times 10^{-5})$</td>
<td>$\text{diag}([1, 10^{-4}, 10^{-4}, 10^{-4}] \times 10^{-5})$</td>
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<tr>
<td>$R$</td>
<td>$4 \times 10^{-6}$</td>
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</tr>
<tr>
<td>$P_0$</td>
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<td>$\text{diag}([1, 1, 1, 30^{-1}, 30^{-1}, 30^{-1}] \times 10^{-5})$</td>
</tr>
<tr>
<td>$\alpha$</td>
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<td>—</td>
<td>2</td>
</tr>
<tr>
<td>$\kappa$</td>
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Table 3. Simulation parameters for test case 3.

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<td>$\text{diag}([1, 10^{-3}, 10^{-3}, 10^{-3}] \times 10^{-5})$</td>
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<td>$R$</td>
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<td>$\alpha$</td>
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<tr>
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Table 4. Simulation parameters for test case 4.

<table>
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<tr>
<td>$R$</td>
<td>$7 \times 10^{-8}$</td>
<td>$3 \times 10^{-7}$</td>
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<tr>
<td>$P_0$</td>
<td>$\text{diag}([10, 10, 1, 1, 1, 1, 1] \times 10^{-2}] \times 10^{-5})$</td>
<td>$\text{diag}([1, 1, 1, 1, 10^{-1}, 10^{-1}, 10^{-3}] \times 10^{-4})$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>—</td>
<td>0.2</td>
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results for the phase interface location are given in figure 3. As can be seen, the UKF has a better estimation of the phase interface than the EKF. The upper interface is well estimated by both the UKF and EKF, whereas for the lower interface the UKF has a better estimation compared with the EKF. It can also be observed that the EKF has a higher transition period and therefore the convergence of the EKF is slow. Figure 4 shows the reconstruction
results for settling velocities. Both the UKF and EKF were able to estimate the rate of change of the phase interface. Figure 5 shows the reconstruction results for the phase conductivities. It can be seen that the UKF has a better estimation of phase conductivities as compared with the EKF. Again, a longer transition period can be observed for the EKF. To compare the UKF and EKF quantitatively, we calculated the root mean square error (r.m.s.e.) for estimated state parameters. The r.m.s.e. for the phase interfaces and conductivities are calculated as

\[
\text{r.m.s.e.}(\gamma) = \frac{\|\gamma_{\text{estimated}} - \gamma_{\text{true}}\|}{\|\gamma_{\text{true}}\|}, \quad (5.1)
\]

\[
\text{r.m.s.e.}(\sigma) = \frac{\|\sigma_{\text{estimated}} - \sigma_{\text{true}}\|}{\|\sigma_{\text{true}}\|}. \quad (5.2)
\]

The r.m.s.e. plots for the phase interfaces and conductivities for case 1 are shown in figures 6 and 7. From the r.m.s.e. plots, it can be seen that the UKF has a lower r.m.s.e. for both phase interfaces and conductivities.

In the second case (case 2), different meshes are used in the forward and inverse solvers and the simulated measurement data are free from measurement noise. The results are shown in figures 8–10. Owing to the different meshes, the performance has deteriorated and can be observed. Figure 8 shows the reconstructed results for the phase interfaces. It can be seen that, with the use of a different mesh for forward and inverse solvers, the transition period is longer for both the UKF and EKF. The UKF is able to track both the upper and lower interfaces efficiently. On the other hand, the EKF follows the trend of the interface but still deviates from the true interface. The settling velocities for case 2 are shown in figure 9. Again, the UKF
Figure 6. The r.m.s.e. comparison of the UKF (solid line) and EKF (dotted line) for phase interface locations for case 1: (a) bottom phase interface; and (b) top phase interface.

Figure 7. The r.m.s.e. comparison of the UKF (solid line) and EKF (dotted line) for phase conductivities for case 1: (a) upper phase layer; (b) middle phase layer; and (c) bottom phase layer.

Figure 8. Phase interface location for test case 2. Diamonds and triangles represent the locations of upper and lower interfaces, respectively. Grey line represents the true interface, black line represents the estimated phase interface; solid line represents UKF and dotted line represents EKF.
has a better estimation of settling velocities, whereas the estimation of the lower interface in the EKF has a noticeable lag. Figure 9 shows the estimated velocities for the upper and lower interfaces, respectively. Grey line represents the true velocity, black line represents the estimated velocities; solid line represents UKF and dotted line represents EKF.

Figure 10. Phase conductivities for test case 2. Diamonds, circles and triangles represent the conductivities of upper, middle and lower phase layers, respectively. Grey line represents the true conductivities, black line represents the estimated conductivities; solid line represents UKF and dotted line represents EKF.

The difference between the true and estimated conductivities for the
UKF is approximately 0.01, whereas for the EKF the difference is approximately 0.015. The r.m.s.e. plots for the phase interfaces and conductivities are shown in figures 11 and 12. Higher r.m.s.e. values are reported in case 2 when compared with case 1, as expected. From these figures, it can be seen that in all cases the UKF has lower r.m.s.e. than the EKF.

In the third case (case 3), the same mesh is used in both the forward and inverse solvers and the measurement data are corrupted with 1 per cent relative white Gaussian noise. Figures 13–15 show the results for case 3. The estimation performance of the UKF for all the state parameters (phase interface locations, settling velocities and conductivities) is better than for the EKF. The EKF was only able to estimate the upper interface, the velocity of the upper interface, and the conductivity of the middle and top layers. However, the estimation quality for the lower interface, the velocity of the lower interface, and the conductivity of the lower interface, is bad. The r.m.s.e. plots for phase interfaces and conductivities are given in figures 16 and 17. On average, a performance gain of 2 is observed in the UKF over EKF in terms of r.m.s.e. (γ) and a performance gain of 3 is observed in the case of r.m.s.e. (σ₃).

In the fourth case (case 4), different meshes are used in the forward and inverse solvers and the measurement data are corrupted with 1 per cent relative white Gaussian noise. Figures 18–20 show the reconstruction results for case 4. It can be seen that the UKF has a better estimation of settling curves, settling
velocities and phase conductivities. Both the upper and middle layer conductivities are estimated well by the UKF and the lower phase conductivity only differs by 0.01. The r.m.s.e. plots for case 4 are given in figures 21 and 22. From the r.m.s.e. plots it can be seen that the UKF has lower r.m.s.e. than the EKF in all cases except for the conductivity of the lower phase.

Figure 13. Phase interface location for test case 3. Diamonds and triangles represent the locations of upper and lower interfaces, respectively. Grey line represents the true interface, black line represents the estimated phase interface; solid line represents UKF and dotted line represents EKF.

Figure 14. Settling velocities for test case 3. Diamonds and triangles represent the velocities of upper and lower interfaces, respectively. Grey line represents the true velocity, black line represents the estimated velocities; solid line represents UKF and dotted line represents EKF.

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Figure 15. Phase conductivities for test case 3. Diamonds, circles and triangles represent the conductivities of upper, middle and lower phase layers, respectively. Grey line represents the true conductivities, black line represents the estimated conductivities; solid line represents UKF and dotted line represents EKF.

Figure 16. The r.m.s.e. comparison of the UKF (solid line) and EKF (dotted line) for phase interface locations for case 3: (a) bottom phase interface; and (b) top phase interface.

Figure 17. The r.m.s.e. comparison of the UKF (solid line) and EKF (dotted line) for phase conductivities for case 3: (a) upper phase layer; (b) middle phase layer; and (c) bottom phase layer.
The computational complexities of the UKF and EKF are similar. The most expensive operations in the UKF include calculating the matrix square root, and performing the outer product to compute the covariance of projected sigma points. Both operations are of the same complexity $O(N^3)$. They are computationally

Figure 18. Phase interface location for test case 4. Diamonds and triangles represent the locations of upper and lower interfaces, respectively. Grey line represents the true interface, black line represents the estimated phase interface; solid line represents UKF and dotted line represents EKF.

Figure 19. Settling velocities for test case 4. Diamonds and triangles represent the velocities of upper and lower interfaces, respectively. Grey line represents the true velocity, black line represents the estimated velocities; solid line represents UKF and dotted line represents EKF.
Figure 20. Phase conductivities for test case 4. Diamonds, circles and triangles represent the conductivities of upper, middle and lower phase layers, respectively. Grey line represents the true conductivities, black line represents the estimated conductivities; solid line represents UKF and dotted line represents EKF.

Figure 21. The r.m.s.e. comparison of the UKF (solid line) and EKF (dotted line) for phase interface locations for case 4: (a) bottom phase interface; and (b) upper phase interface.

Figure 22. The r.m.s.e. comparison of the UKF (solid line) and EKF (dotted line) for phase conductivities for case 4: (a) upper phase layer; (b) middle phase layer; and (c) bottom phase layer.

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equal to matrix multiplications in predicted covariance for the EKF (Julier & Uhlmann 2004). In the UKF, apart from tuning the measurement noise, process noise and error covariance matrices, three parameters are tuned additionally. But tuning of some of these parameters \((k \text{ and } \beta)\) can be avoided if the Gaussian assumption is made. In such a case, the only parameter that needs to be adjusted is the spread parameter \(\alpha\). The UKF generates multiple sigma points, which are processed through the state and measurement equations and thus can be computationally intensive. However, in modern hardware GPU implementation can take care of this problem by running part of the UKF code in parallel. The computational burden can be reduced further by using limited measurement data. In this work, we are using the simple UKF rather than the augmented version and hence few sigma points are generated. Therefore, the computational time for the UKF is comparable with the EKF. For purely linear systems, the estimation performance of the UKF is identical to that of the EKF. For nonlinear systems, the UKF produces performance equal to or better than the EKF. The improvement with respect to the EKF depends on the system uncertainties and nonlinearities. Therefore, in view of the discussion above, it can be established that the UKF is well suited for sedimentation monitoring.

6. Conclusions

In this paper, an application of EIT for monitoring sedimentation processes is presented. Settling curves and settling velocities provide necessary information regarding the sedimentation process. The UKF is used as an inverse algorithm to estimate the settling curves and settling velocities. A three-layer sedimentation model is assumed. To locate the interfacial phase boundaries, a first-order kinematic model is used as an evolution model. Furthermore, the rate of change of phase interfaces, i.e. settling velocities, can be incorporated as a state parameter, and therefore settling velocities are obtained directly. Comparison of the UKF with the EKF is done. The UKF uses a deterministic sampling approach to approximate the true posterior mean and covariance, and is free from linearization errors caused in the EKF. Unlike the EKF, the analytical Jacobian is not required in the UKF, as it employs a nonlinear UT, which uses the nonlinear measurement equation as such. The performance of the UKF is better than the EKF in terms of robustness and speed of convergence. This is mainly due to enhanced time-update and measurement-update accuracy. The complexity and computational burden of the UKF is comparable to that of the EKF. Extensive numerical experiments have demonstrated that the UKF approach has better estimation of the phase interface locations, the interface velocities and the conductivity values as compared to the EKF. If the phase interfaces evolve with varying velocity, the estimation performance will be affected, as a first-order kinematic model is used as an evolution model for the phase interfaces. Also, the present sedimentation model will fail if the homogeneous assumption of the phase layers is not made. One solution is to consider several layers instead of only three layers. Future work includes the testing of the proposed model with real EIT measurements taken from industrial sedimentation tanks. Sedimentation modelling based on solids flux theory, where the settling velocity is a function of the concentration of the phase layers, would be of great benefit for further research.
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