Dynamical evolution plays a key role in shaping the current properties of star clusters and star cluster systems. A detailed understanding of the effects of evolutionary processes is essential to be able to disentangle the properties that result from dynamical evolution from those imprinted at the time of cluster formation. In this review, I focus my attention on globular clusters, and review the main physical ingredients driving their early and long-term evolution, describe the possible evolutionary routes and show how cluster structure and stellar content are affected by dynamical evolution.

Keywords: globular clusters: general; N-body simulations; stellar dynamics

1. Introduction

Globular star clusters have long been considered the ideal astrophysical objects to explore many aspects of stellar dynamics and, in particular, to study the evolution of stellar systems as driven by the effects of two-body relaxation.

Only recently, however, the actual complexity of globular cluster dynamics has emerged from the wealth of new observational and theoretical studies that have clearly shown the close interplay between stellar dynamics, stellar evolution, the clusters’ stellar content and the dynamics and properties of the host galaxy. The notion of an ‘ecology of star clusters’, introduced by Heggie (1992), nicely illustrates the close interplay among the different elements of the astrophysics of star clusters.

To use star clusters as tools to advance and guide our understanding of star formation in galaxies, it is essential to understand to what extent the current properties of star clusters and star cluster systems are the result of evolutionary processes. We also need to be able to discern the signatures of properties imprinted by formation processes from those determined by dynamical evolution.

In §2, I discuss the early evolution of clusters, i.e. the processes that can affect their stellar content early in their life and lead to their rapid dissolution. In §3,
I review the long-term evolution of clusters as driven, primarily, by two-body relaxation, focusing my attention on the evolution of cluster structure, mass and stellar content. Conclusions are summarized in §4.

2. Early dynamical evolution

Although much progress has been made in recent years (e.g. Clarke 2010; Lada 2010), a full understanding of the formation and the initial structural properties of star clusters is still lacking.

Analytical and numerical studies of globular cluster evolution often adopt initial conditions that resemble the current structural properties of clusters (e.g. purely stellar, spherical and isotropic King models), rather than the possibly more complex structures suggested by theoretical and observational studies of very young star clusters (e.g. Elmegreen 2000; Bonnell et al. 2003; Allen et al. 2007). Figure 1 shows a few snapshots of a hydrodynamical simulation modelling the formation of a small star cluster (Bonnell et al. 2003) and nicely illustrates the clumpy and irregular structure of a young star cluster still embedded in the primordial gas from which its stars are forming.

In addition, the standard paradigm according to which clusters are ‘simple stellar populations’ composed of stars of the same age and chemical composition has recently been challenged by the increasing observational evidence of the presence of multiple stellar populations in globular clusters (e.g. Bedin et al. 2004; Gratton et al. 2004; Piotto et al. 2007; Carretta et al. 2008, 2009a,b; D’Antona & Caloi 2008; Milone et al. 2008; see also the reviews of Bruzual 2010; Kalirai & Richer 2010; van Loon 2010). This is a major paradigm shift and significantly increases the complexity of models following the formation and dynamical evolution of globular clusters (e.g. D’Ercole et al. 2008; Renzini 2008; and references therein).

Although the exact structural and kinematic properties of newly formed clusters are still uncertain, a number of studies have explored the early evolution of clusters and shown their possible evolutionary routes. In the following subsections, I present an overview of the early dynamical evolution of clusters, based on analytical calculations and N-body simulations.

(a) Evolution of clumpy systems

A number of observational and numerical studies suggest that star clusters might form with irregular clumpy structures and thus out of virial equilibrium. These studies mostly concern low-mass clusters and it is not clear whether such initial conditions are shared by more massive globular-cluster-like objects. Many clusters as young as approximately 50–100 Myr already exhibit smooth surface-brightness profiles (e.g. Elson et al. 1987; Larsen 2004), which are well fitted by the ‘Elson–Fall–Freeman’ profile (hereafter EFF; Elson et al. 1987),

\[ \Sigma \propto \frac{1}{(1 + (r/r_c)^2)^{\gamma/2}}, \]  

(2.1)

where \( \Sigma \) is the cluster’s surface brightness and \( r_c \) its core radius. A number of studies have investigated the dynamics of clusters starting from clumpy initial
Figure 1. Formation and early evolution of a star cluster from a hydrodynamical simulation following the hierarchical fragmentation of a turbulent molecular cloud. (Reproduced with permission from Bonnell et al. 2003.)

conditions and explored if, and on what time scale, early dynamical evolution can drive them towards properties similar to those of older clusters with a smoother density profile.

The evolution of clusters with different levels of clumpiness, clump spatial distributions and initial velocity distributions has been studied in general surveys (e.g. Aarseth et al. 1988; Goodwin 1998; Goodwin & Whitworth 2004; McMillan et al. 2007) as well as in investigations aimed at modelling the evolution of specific clusters (e.g. Scally & Clarke 2002). Recently, Hurley & Bekki (2008) made further progress towards more realistic models and explored the stellar dynamical evolution of an inhomogeneous, clumpy stellar system with initial conditions produced by a hydrodynamical simulation of a turbulent giant molecular cloud.

The results of all these numerical studies show that, in most cases, an initially clumpy system undergoes a rapid phase of violent relaxation, which erases any initial substructure and leaves the cluster with a density profile that is very similar.
to the EFF profile. Similar simulations have also been carried out in the context of early galaxy evolution: see the first simulations by van Albada (1982) and McGlynn (1984) and see Trenti et al. (2005, and references therein) for a more recent study. In fact, even before the EFF profile was used to fit the observed surface-brightness profiles of young clusters in the Large Magellanic Cloud, it was adopted by McGlynn (1984) in a study of early galaxy evolution to fit the density profiles emerging at the end of the violent-relaxation phase of initially clumpy systems.

The time scale to reach final equilibrium without any significant remaining trace of the initial substructures depends on the details of the initial conditions, but it is, in general, roughly \(1-10(t_{\text{dyn}})\), where \(t_{\text{dyn}} = \sqrt{3\pi/16G\rho}\) is the cluster dynamical time (and \(\rho\) the cluster’s average density). Fellhauer et al. (2009) recently showed that the clump-merging time scale may be shorter than the time scale of primordial-gas expulsion, thus enhancing the cluster’s chances of surviving the disruptive effects of gas expulsion (see §2c).

Statistical tools to measure cluster substructure more rigorously have been used by Cartwright & Whitworth (2004), Schmeja & Klessen (2006) and Kumar & Schmeja (2007), and provided a quantitative measure of the evolving level of clumpiness in observed young clusters and star-forming regions, as well as in the numerical results of hydrodynamical simulations of star cluster formation.

(b) Clumpy systems and initial mass segregation

Although the evolution of clumpy stellar systems is rapid and occurs on time scales of the order of a few dynamical time scales, it can have important implications for the generation of mass segregation early in a cluster’s evolution. Mass segregation, the tendency of more massive stars to sink towards the cluster’s central regions, is a consequence of energy equipartition driven by two-body relaxation and one of the processes normally associated with the long-term evolution of clusters. However, a number of observational studies (e.g. Hillenbrand 1997; Hillenbrand & Hartmann 1998; Fischer et al. 1998; de Grijs et al. 2002; Sirianni et al. 2002; Gouliermis et al. 2004; Stolte et al. 2006; Sabbi et al. 2008; Allison et al. 2009a; Gouliermis et al. 2009; see also Ascenso et al. 2009) have found evidence of mass segregation in clusters with ages shorter than the time needed to produce the observed segregation by two-body relaxation (see also de Grijs 2010).

It has been suggested in a number of theoretical studies that the observed segregation in young clusters would be primordial and imprinted by the star-formation process (Bonnell et al. 1997, 2001; Bonnell & Davies 1998; Klessen 2001; Bonnell & Bate 2006). Specifically, the mechanism responsible for primordial mass segregation would be the higher accretion rate for stars in the central regions of young clusters. The actual efficiency of this mechanism is still a matter of debate (Krumholz et al. 2005; Krumholz & Bonnell 2009), but if the individual clumps are indeed mass segregated, it has been shown by McMillan et al. (2007) that such primordial mass segregation would not be erased in the violent-relaxation phase during which clumps merge. The final system would preserve the mass segregation of the original clumps (see also Fellhauer et al. 2009; Moeckel & Bonnell 2009).
In the same study, McMillan et al. (2007) also presented an alternative scenario for a dynamical origin of early mass segregation in young clusters. Even if the clumps are not initially segregated, if their internal segregation time scale is shorter than the time needed for the clumps to merge, they will segregate through standard two-body relaxation and preserve this segregation after they have merged. The multi-scale dynamical evolution of clumpy systems is, in this case, responsible for rapidly leading to mass segregation in young clusters without invoking any mechanism associated with the star-formation process.

In a recent study, Allison et al. (2009b) showed that mass segregation can be rapidly produced dynamically also in the high-density core formed during the collapse of cold clumpy clusters. Bastian et al. (2008) found observational evidence of a strong expansion in the first 20 Myr of evolution of six young M51 clusters and pointed out that this expansion could also lead to a rapid variation in the cluster relaxation time. As pointed out by Bastian et al., using the current relaxation time might lead to an underestimate of the possible role played by two-body relaxation in generating mass segregation in the early phases of a cluster’s dynamical evolution.

Regardless of the mechanism producing mass segregation, the presence of segregation very early in a cluster’s life can have a significant impact on its dynamical evolution. I will discuss some of the implications of initial mass segregation in the next subsection.

\[ \text{(c) Early mass loss: primordial gas expulsion and stellar evolution} \]

Expulsion of the primordial residual gas in which a young cluster is still embedded in the very early stages of its life, along with mass loss due to stellar evolution, can have a significant impact on a cluster’s structure and survival chances.

Early analytical calculations by Hills (1980) based on the virial theorem showed that clusters in virial equilibrium losing impulsively more than half of their mass would rapidly expand and dissolve in response to this mass loss. Subsequent studies by Boily & Kroupa (2003a, b) refined Hills’ results by calculating the fraction of stars remaining bound in a cluster with a given velocity distribution function after the impulsive removal of a given amount of gas. Boily & Kroupa’s study showed that, for clusters with a high-binding-energy massive core, up to 70 per cent of the cluster mass can be removed impulsively without leading to complete cluster dissolution (see also the simulations in Kroupa et al. 2001, and references therein).

Goodwin & Bastian (2006) explored, by means of N-body simulations, the evolution of the structural properties of clusters in the stages following gas expulsion and showed that the observed luminosity profiles of a few young massive clusters differ from EFF and King models in their outer regions, consistent with their simulations. These could, therefore, be examples of clusters currently expanding in response to gas expulsion.

Baumgardt & Kroupa (2007) carried out a survey of N-body simulations to explore the dependence of the response to residual gas expulsion on the star-formation efficiency, the expulsion time scale and the strength of the host galaxy’s tidal field. Slow gas removal and a weak tidal field can increase the
amount of mass a cluster can lose without undergoing complete disruption. However, in most cases a cluster undergoes significant expansion and needs to be initially much more compact than observed today to survive this phase. As shown in a number of numerical studies (e.g. Portegies Zwart & McMillan 2002; Gürkan et al. 2004; Portegies Zwart et al. 2004; for a review see also McMillan 2008, and references therein), very high initial central densities might lead to a rapid core collapse and segregation of massive stars, and trigger a runaway merger of massive stars, leading to the formation of an intermediate-mass black hole (IMBH); but see Glebbeek et al. (2009) for a recent study of the stellar evolution of runaway merger products, showing how mass loss due to stellar winds might significantly affect this process and prevent the formation of an IMBH.

Early mass loss due to stellar evolution (for example, expulsion of Type II supernova ejecta) can also have a significant impact on early cluster evolution. Semi-analytical calculations (e.g. Applegate 1986; Chernoff & Shapiro 1987) followed by Fokker–Planck (e.g. Chernoff & Weinberg 1990) and $N$-body simulations (e.g. Fukushige & Heggie 1995; Portegies Zwart et al. 1998) have shown that mass loss due to stellar evolution triggers an expansion, which can lead to rapid cluster dissolution (see also Kalirai & Richer 2010).

The extended surveys of Chernoff & Shapiro (1987) and Chernoff & Weinberg (1990) showed that mass loss due to stellar evolution can cause the rapid dissolution of clusters with a low initial concentration and/or a flatter stellar initial mass function (IMF).

Fukushige & Heggie (1995) used $N$-body simulations to explore the early evolution of clusters dissolving due to mass loss associated with stellar evolution. Their results are in general, qualitative agreement with those of the Fokker–Planck models of Chernoff & Weinberg (1990); see also Takahashi & Portegies Zwart (2000) for an extensive comparison between $N$-body simulations and Fokker–Planck models. Fukushige & Heggie also explored the mechanism behind this rapid cluster dissolution and showed it to be the result of a loss of equilibrium as the cluster expands and reaches structural properties for which no virial equilibrium is possible.

The expansion triggered by early mass loss due to stellar evolution is stronger if the cluster is initially mass segregated. For a given amount of mass loss, preferentially removing mass from the central regions (where massive stars tend to be located in mass-segregated clusters) increases the heating and strengthens the subsequent expansion. $N$-body simulations by Vesperini et al. (2009) show that, as the degree of initial mass segregation increases, so does the strength of the initial cluster expansion. As a result, clusters differing only in the degree of initial mass segregation can have very different lifetimes. Mackey et al. (2007, 2008) showed that the stronger early expansion of mass-segregated clusters, along with the subsequent heating from a population of stellar black holes, can explain the radius–age trend observed for massive clusters in the Magellanic Clouds.

A strong early loss of stars plays a key role in the evolution of multiple stellar populations in clusters. Although different origins have been suggested for the gas from which second-generation stars might have formed (e.g. Ventura et al. 2001; Decressin et al. 2007; see also Renzini 2008, and references therein), most models require the first-generation population to have an initial mass that is at least
10 times higher than its current mass. As shown by D'Ercole et al. (2008), early cluster expansion and mass loss can be responsible for the escape of such a large fraction of first-generation stars.

Finally, although this review focuses on the internal dynamics of clusters rather than on cluster systems (see Harris 2010; Larsen 2010), it is important to point out that evolution and disruption of clusters due to gas expulsion and stellar evolution may play an important role in the evolution of the mass function of globular cluster systems (hereafter GCMF).

Young cluster systems have a power-law GCMF (e.g. Zhang & Fall 1999) while older clusters follow a bell-shaped distribution (e.g. Ashman & Zepf 1998; Harris 2001; Brodie & Strader 2006). While evaporation due to two-body relaxation can transform a power law into a bell-shaped GCMF, most GCMF evolution models show that, when GCMF evolution is driven only by two-body relaxation, a galactocentric dependence of the GCMF turnover not found in observational data is also produced (e.g. Vesperini 1997, 1998, 2000, 2001; Baumgardt 1998; Fall & Zhang 2001; Vesperini et al. 2003; Baumgardt et al. 2008a; McLaughlin & Fall 2008; see Zepf 2008 for a recent review).

It has been shown that early cluster dissolution due to gas expulsion and mass loss from stellar evolution can preferentially destroy low-mass clusters and significantly flatten the low-mass end of the initial power-law GCMF, without introducing any strong dependence on galactocentric distance. Two-body relaxation and tidal shocks, while still playing a key role and leading to disruption of a significant number of clusters, will act on the GCMF already flattened by early evolutionary processes without giving rise to any radial trend of GCMF properties, inconsistent with observations (Vesperini & Zepf 2003; Baumgardt et al. 2008a; Parmentier et al. 2008).

### 3. Long-term evolution of clusters

For most old globular clusters, the core and half-mass two-body relaxation time scales are shorter than the cluster ages. Two-body relaxation plays a major role in driving the long-term evolution of clusters and shaping their current structural properties, as well as their stellar content.

In this section, I focus my attention on the effects of two-body relaxation. I point out, however, that the evolution of clusters on eccentric orbits and clusters in spiral galaxies can be further affected by the tidal shocks during passages through the disc or near the host galaxy’s central bulge. Although in most cases two-body relaxation is the dominant evolutionary process, depending on the properties of the host galaxy and the cluster’s orbital parameters, tidal shocks can significantly speed up cluster dissolution and either accelerate or slow down the evolution towards core collapse (e.g. Chernoff et al. 1986; Spitzer 1987; Weinberg 1994; Gnedin et al. 1999). Encounters with giant molecular clouds and passages through spiral arms can also affect cluster evolution and mass loss (Gieles et al. 2006, 2007).

In the following subsections, I focus my attention on mass loss due to two-body relaxation, the implications of mass loss for the evolution of the cluster stellar mass function and the cluster’s structural evolution.
Mass loss and cluster dissolution

As a star exchanges energy with other single and binary stars in a cluster due to close and distant encounters, it can reach an energy in excess of the escape energy and escape from the cluster.

Mass loss due to two-body relaxation plays a key role in a broad range of issues related to the evolution of globular clusters and globular cluster systems. Specifically, for clusters surviving the early evolutionary processes described in §2, mass loss due to two-body relaxation is the main process determining their lifetimes. It affects the clusters’ stellar content and stellar mass function and determines the fraction of stars in the host galaxy’s field population contributed by clusters.

Although mass loss due to two-body relaxation has been the subject of a large number of studies since the very early investigations of star cluster dynamics (see for a review Heggie 2001, and references therein), only recently a number of investigations have shed light on some fundamental aspects of this process and its dependence on cluster structural parameters and the strength of the host galaxy’s tidal field.

The external tidal field of the host galaxy plays a key role in the process of mass loss. While isolated clusters undergo mass loss due to the combined effects of close and distant encounters (e.g. Giersz & Heggie 1994; Heggie 2001; Baumgardt et al. 2002; Heggie & Hut 2003), their dissolution time scale is extremely long. The \( N \)-body simulations of isolated star clusters of Baumgardt et al. (2002) show that it takes about \( 10^3 t_{\text{rh}}(0) \) for a cluster to lose about half of its initial mass, where \( t_{\text{rh}}(0) \) is the cluster’s initial half-mass relaxation time.

For clusters evolving in the external tidal field of the host galaxy, on the other hand, the mass-loss rate is much faster and the dissolution time significantly shorter: by lowering the escape speed and truncating the cluster sizes, the external tidal field significantly enhances the mass-loss rate. The process of mass loss due to two-body relaxation and the dissolution time associated with this process depend on the strength of the external tidal field. Fokker–Planck (e.g. Chernoff & Weinberg 1990; Takahashi & Portegies Zwart 2000) and \( N \)-body simulations (e.g. Vesperini & Heggie 1997; Aarseth & Heggie 1998) have shown that the mass-loss rate and dissolution time \( T_d \) of a cluster containing an initial number of stars \( N_i \), evolving on a circular orbit at a galactocentric distance \( R_g \) in a host galaxy with circular velocity \( v_c \) is proportional to \( N_i R_g / \log(N_i) v_c \). For given values of \( N_i \), \( v_c \) and \( R_g \), the mass-loss rate depends weakly on the cluster’s internal structural properties (e.g. concentration and half-mass radius). An extensive survey of \( N \)-body simulations was recently carried out by Gieles & Baumgardt (2008). Their results clearly show that, for a cluster with a given \( N_i \), the strength of the tidal field is the dominant factor determining a cluster’s lifetime.

If the presence of the tidal field is properly modelled (as opposed to being treated in a simplified way by introducing, for example, a spatial or an energy cut-off), stars can escape the cluster only through one of the Lagrangian points of the galaxy–cluster system. Fukushige & Heggie (2000) and Baumgardt (2001) showed that the time needed for a star with an energy greater than the escape energy to flow through one of the Lagrangian points and actually escape from the cluster is not negligible and can, depending on the star’s orbital parameters, be even longer than a Hubble time. Fukushige & Heggie (2000), Baumgardt (2001)
and Baumgardt & Makino (2003) showed that the escape time scale modifies the scaling of the cluster dissolution time with cluster mass, and that $T_d$ is actually proportional to $(N_i/\log(N_i))^{0.75}$; see also Lamers et al. (2005) for analytical fits to the mass-loss rates resulting from the $N$-body simulations of Baumgardt & Makino (2003).

A number of studies based on $N$-body simulations have followed the motion of escaping stars beyond the cluster’s tidal radius and explored the formation, structure and stellar content of the elongated tidal tails that these stars populate and the relationship between their orientation and the cluster’s orbit (e.g. Combes et al. 1999; Johnston et al. 1999; Dehnen et al. 2004; Fellhauer et al. 2007; Montuori et al. 2007; Ernst et al. 2009; Odenkirchen et al. 2009). Observational studies have detected the presence of tidal tails in several clusters (e.g. Leon et al. 2000; Odenkirchen et al. 2003; Grillmair & Dionatos 2006) and also revealed the presence of density clumps in the tidal tails (e.g. Leon et al. 2000). Similar clumps were found by Capuzzo Dolcetta et al. (2005) in $N$-body simulations of clusters on eccentric orbits. More recently, Küpper et al. (2008) and Just et al. (2009) carried out detailed studies of the orbits of escaping stars and showed that overdensities along the tails are expected even for clusters on circular orbits. The observed clumps can be explained in terms of the epicyclic motion of escaping stars and a slowdown in some portions of their orbits as they move away from the cluster.

As I will discuss in the next subsection, many theoretical studies have shown that mass loss due to two-body relaxation leads to the preferential escape of low-mass stars. In agreement with these results, Koch et al. (2004)—in an observational study of the Pal 5 main-sequence stellar luminosity function—found a significant overabundance of low-mass stars in this cluster’s extended tidal tails.

(b) Mass loss and evolution of the stellar mass function

As a result of the tendency towards energy equipartition, massive stars tend to sink to the cluster’s central regions, while low-mass stars populate the outer parts and dominate the population of escaping stars.

A consequence of the preferential loss of low-mass stars is that the stellar mass function flattens as a cluster proceeds in its dynamical evolution and loses mass. A number of $N$-body simulations have revealed a close correlation between the fraction of the initial cluster mass lost and the slope of the low-mass end of the stellar mass function (Vesperini & Heggie 1997; Baumgardt & Makino 2003; Trenti et al. in press). Assuming that clusters are characterized by a universal stellar IMF, the current slope of the mass function is a good indicator of the extent to which a cluster has been affected by dynamical evolution.

The relationship between a cluster’s dynamical evolution, mass loss and stellar mass function is one of the manifestations of the close interplay between different aspects of the astrophysics of star clusters. Observational investigations of the stellar mass function of a number of Galactic globular clusters have found a range of different slopes of the low-mass side of the present-day mass function, which might indeed be the result of differences in the clusters’ dynamical histories (cf. Piotto & Zoccali 1999). A recent observational study by De Marchi et al. (2007) resulted in a puzzling trend between cluster structure and slope of the stellar mass function: clusters with a flatter present-day mass function tend to have low-concentration density profiles. The origin of this trend is not clear—see
Baumgardt et al. (2008b) for a study suggesting that primordial mass segregation might be required to explain the extreme mass function flattening of some low-concentration clusters—and will require further investigation of the evolution of cluster structural properties as they evolve towards complete dissolution.

A consequence of the preferential loss of low-mass stars is that the evolution of a cluster’s mass-to-light ratio, $M/L$, differs from that driven solely by stellar evolution. Specifically, it has been shown by Baumgardt & Makino (2003) that, during most of a cluster’s life, $M/L$ is lower than what the cluster would have without any loss of stars; see also Lamers et al. (2006) and Anders et al. (2009) for recent studies of the photometric evolution of dissolving clusters. Only at the very end of its life, when the cluster approaches complete dissolution, does its stellar population become dominated by dark remnants and $M/L$ rises to values greater than those expected if there were no mass loss. Kruijssen (2008) and Kruijssen & Mieske (2009) further investigated the effect of the preferential loss of low-mass stars on $M/L$ and showed that the effects of mass loss can account for the discrepancy between the observed $M/L$ values for a sample of Galactic globular clusters and those expected on the basis of including only the effects of stellar evolution.

Tidal shocks due to passages near the host galaxy’s bulge for clusters on eccentric orbits, and through the galactic disc, will inject energy into the cluster and further speed up the process of mass loss due to two-body relaxation (e.g. Gnedin & Ostriker 1997; Gnedin et al. 1999). However, mass loss due to tidal shocks is independent of stellar mass, and only combined with mass segregation can mass loss due to tidal shocks contribute to the flattening of the stellar mass function (e.g. Vesperini & Heggie 1997).

**(c) Structural evolution, core collapse and post-core collapse**

Figure 2 shows the results of an $N$-body simulation (Aarseth & Heggie 1998) following the time evolution of the 0.1, 1, 10 and 50 per cent Lagrangian radii and tidal radius of a star cluster orbiting at 4 kpc from the Galactic Centre and with an initial mass of $M_i \simeq 1.5 \times 10^5 M_\odot$. The initial expansion of the cluster is a consequence of mass loss due to stellar evolution. As discussed in §2, clusters with low initial concentrations or a large fraction of massive stars, or with strong initial mass segregation, can undergo significant early expansion and rapidly dissolve as a consequence of mass loss due to stellar evolution.

For clusters surviving early dissolution, figure 2 illustrates the typical structural evolution towards core collapse driven by two-body relaxation. First studied by Hénon (1961) using a Monte Carlo integration of the Fokker–Planck equation, this process, also known as gravothermal catastrophe, has subsequently been explored in a large number of investigations; Antonov (1962) and Lynden-Bell & Wood (1968) are among the first studies that explored the physics of this process (for a review, see Heggie & Hut 2003, and references therein). Without intervention by an energy source balancing the loss of energy from the core, this process would lead to a smaller and smaller core and a diverging central density in a finite time.

Binary stars, either primordial or dynamically formed during close encounters between single stars, can provide the energy needed to halt core collapse and support the core in the post-core-collapse (PCC) phase; see Heggie & Hut (2003,
and references therein) for a review of the gravothermal oscillations that can characterize the PCC phase of clusters. In particular, as shown by Heggie (1975; for reviews see also Heggie & Hut 1993, 2003; Goodwin 2010), binary stars with binding energy $\epsilon_b$ moving in a stellar system of single stars with mass $m$ and one-dimensional velocity dispersion $\sigma$ such that $|\epsilon_b|/m\sigma^2 > 1$ will, on average, increase their binding energy and release the energy lost to the star cluster.

The depth of core collapse and the concentration of a cluster in the PCC phase, as measured, for example, by the ratio of the core to half-mass radius $r_c/r_h$, depends on the energy source. Clusters supported by dynamically formed binaries will, in general, be extremely concentrated, $r_c/r_h \approx 10^{-3}$ (e.g. Heggie & Hut 2003).

The concentration of a cluster supported by primordial binaries depends on the abundance of binaries and, to a smaller extent, on the distribution of binary binding energy. Primordial binaries can support a cluster in the PCC phase with values of $r_c/r_h$ as large as approximately 0.05–0.08 (see the analytical studies of Goodman & Hut 1989; Vesperini & Chernoff 1994; and the Fokker–Planck and $N$-body simulations of McMillan et al. 1990, 1991; Gao et al. 1991; Heggie & Aarseth 1992; Giersz & Spurzem 2000, 2003; Heggie et al. 2006; Fregeau & Rasio 2007; Portegies Zwart et al. 2007; Trenti et al. 2007a).

Observational studies have shown that about 20 per cent of Galactic clusters have cuspy surface-brightness profiles and identified these objects as clusters in the PCC phase (Djorgovski & King 1986; Chernoff & Djorgovski 1989; Djorgovski & Meylan 1994). As predicted by Chernoff & Shapiro (1987), PCC clusters are preferentially located near the Galactic Centre (Chernoff & Djorgovski 1989). The smaller cluster sizes set by the stronger tidal field at small galactocentric distances lead to shorter relaxation times and more rapid evolution towards core collapse.

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While many clusters are still in the pre-core-collapse phase, a few clusters have central and half-mass relaxation times that are short enough that they should have already undergone core collapse (Chernoff & Djorgovski 1989; Trenti in press). These clusters, however, have normal surface-brightness profiles, with a resolved core and values of $r_c/r_h$ greater than can be supported even by a large fraction of primordial binaries—and it is important to note that recent observational studies suggest that the fraction of primordial binaries in several clusters might only be roughly 1–2% (Davis *et al.* 2008a).

Alternative energy sources have been suggested. Trenti (in press) suggested that an IMBH at the centre of these clusters might support the large values of $r_c/r_h$ (for $N$-body simulations of clusters containing an IMBH see Baumgardt *et al.* 2005; Heggie *et al.* 2007; Trenti *et al.* 2007b; Gill *et al.* 2008; Pasquato *et al.* 2009). Heyl (2008), Heyl & Penrice (2009) and Fregeau *et al.* (2009), following recent observational studies suggesting that white dwarfs might receive a kick velocity at the time of their formation (Davis *et al.* 2008b; see also Kalirai & Richer 2010), have explored the effects of these kicks on cluster dynamics and showed that they can indeed represent a significant energy source capable of delaying core collapse and supporting high values of $r_c/r_h$.

Hurley (2007) and Trenti *et al.* (in press) analysed the results of a number of $N$-body simulations, following observational procedures, and showed that adopting the observational definitions of the structural parameters of clusters reduces the differences between observational and theoretical values of parameters such as the ratio of the core to half-mass radius. As numerical models become more realistic, adopting data-analysis procedures consistent with those used in observational studies is important and will help to clarify for which clusters an additional energy source is indeed required to explain the current cluster structure.

The results of detailed $N$-body models for the dynamical evolution of M4 and NGC 6397 (Heggie & Giersz 2008, 2009a, b) show that there are additional factors that can complicate the observational identification of a cluster’s dynamical phase. M4 and NGC 6397 are two Galactic clusters with expected similar dynamical histories, which, however, have different surface-brightness profiles: M4 has a normal King profile (although of high concentration) while NGC 6397 exhibits a cuspy profile and is usually classified as a PCC cluster. According to the $N$-body models of Heggie & Giersz (2008, 2009a, b), both M4 and NGC 6397 are actually in the PCC phase and the differences in the surface-brightness profiles are due to oscillations in the core size, which, in turn, cause the surface-brightness profile to oscillate between cuspy and high-concentration King profiles. Depending on the phase of this oscillatory behaviour, a cluster will be classified either as a normal, high-concentration King model or as a PCC cluster.

The results of all these studies suggest that it is likely that not all clusters in the PCC phase have been classified as PCC clusters. Further exploration of the range of possible observational properties of PCC clusters and of additional observational indicators of the PCC phase is necessary to ensure that the dynamical state of normal and PCC clusters is correctly identified and that a proper study of correlations between a cluster’s dynamical state and other internal (e.g. abundance of exotic objects such as pulsars, blue stragglers, etc.) and external properties (e.g. position in the Galaxy) can be carried out.
4. Conclusions

The study of star cluster dynamics has a long history and significant progress has been made in understanding the physics of the main ingredients driving the dynamical evolution of star clusters. Many fundamental problems, however, are still open and new observations continually present us with new challenges and questions.

The following are just a few of the issues that are still awaiting firm solutions: understanding the role played by early evolutionary processes in the dissolution of star clusters and in shaping the properties of those which survive; how to identify a cluster’s dynamical state; what energy sources support clusters in the PCC phase; and what the role is of evolutionary processes in determining the current properties of globular cluster systems.

As the close interplay between dynamical evolution, the evolution of a cluster’s stellar content and the role played by the host galaxy’s environment has become clear, new and increasingly complex questions crossing the boundaries of different fields have arisen. These are some examples of unsolved problems that require increasingly sophisticated models: understanding the links between the formation and abundance of exotic objects (such as blue stragglers, X-ray sources and IMBHs) and a cluster’s dynamical history; the relationship between a cluster’s stellar mass function, its structure and its past dynamical evolution; the role played by the host galaxy’s tidal field, and the effect of its time variation during galaxy assembly in shaping the current properties of individual clusters and the global properties of cluster populations. The observational evidence of the presence of multiple stellar populations in globular clusters is the latest major challenge and further illustrates the complexity of the formation and dynamical evolution of star clusters.

As clusters evolve, their structural properties and stellar content are modified by evolutionary processes. Clusters in different galaxies and at different galactocentric distances have different dynamical histories. A better understanding of the effects of dynamical evolution is an essential step in our attempts to shed light on the relationship between current star cluster properties and those which were imprinted by star-formation processes.

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