Fractional quantum Hall effect in suspended graphene probed with two-terminal measurements

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Recently, fractional quantization of two-terminal conductance was reported in suspended graphene. The quantization, which was clearly visible in fields as low as 2 T and persistent up to 20 K in 12 T, was attributed to the formation of an incompressible fractional quantum Hall state. Here, we argue that the failure of earlier experiments to detect the integer and fractional quantum Hall effect with a Hall-bar lead geometry is a consequence of the invasive character of voltage probes in mesoscopic samples, which are easily shorted out owing to the formation of hot spots near the edges of the sample. This conclusion is supported by a detailed comparison with a solvable transport model. We also consider, and rule out, an alternative interpretation of the quantization in terms of the formation of a p–n–p junction, which could result from contact doping or density inhomogeneity. Finally, we discuss the estimate of the quasi-particle gap of the quantum Hall state. The gap value, obtained from the transport data using a conformal mapping technique, is considerably larger than in GaAs-based two-dimensional electron systems, reflecting the stronger Coulomb interactions in graphene.

Keywords: fractional quantum Hall effect; correlated electron systems; quantum transport

1. Introduction

Since its first isolation (Novoselov et al. 2004), graphene, a one-atom-thick membrane of crystalline carbon, has broken many records with its unusual physical properties: it is the stiffest material known, it has exquisite chemical sensitivity, enabling detection of a single adsorbed atom, and, owing to the relativistic nature of its charge carriers, the highest room temperature...
mobility known (Castro Neto et al. 2009). Yet, despite strong unscreened Coulomb forces between these carriers, the expected manifestations of collective behaviour (Apalkov & Chakraborty 2006; Peres et al. 2006; Toke et al. 2006; Yang et al. 2006; Goerbig & Regnault 2007; Khveshchenko 2007; Wang et al. 2008) have been conspicuously difficult to detect. Here, we discuss requirements for the sample preparation and measurement geometry that are necessary to observe correlated electron behaviour such as the fractional quantum Hall effect (FQHE). One condition is elimination of substrate-induced perturbations that introduce fluctuations on energy scales larger than those characteristic for electron correlations, which can be accomplished using suspended graphene (SG). In addition, owing to the mesoscopic character of SG samples, the measurement geometry and placing of probes are critical for the ability to observe the integer quantum Hall effect (IQHE) and FQHE in SG (Du et al. 2008, 2009; Bolotin et al. 2009; Abanin et al. 2010).

The IQHE is a property of non-interacting two-dimensional electron systems (2DESs) in a perpendicular magnetic field, $B$, marked by plateaus in the transverse (Hall) conductance, $\sigma_{xy} = e^2/\hbar h$. Here, $e$ is the electron charge, $h$ is Planck’s constant, $n = n_s h/eB$ is the filling factor and $n_s$ is the carrier density. For graphene, the sequence of IQHE plateaus is anomalous, with $n = \pm 4(n + 1/2)$, $n = 0, 1, 2, \ldots$, reflecting the fourfold spin and valley degeneracy, the electron–hole symmetry and the number of filled Landau levels, $n$. The $1/2$ shift is a unique property of the relativistic charge carriers in graphene (Castro Neto et al. 2009). Correlations between electrons can lead to additional integer QH plateaus outside this sequence corresponding to lifting of the spin or valley degeneracy. For non-relativistic 2DESs, a strong correlation between the quasi-particles gives rise to the FQHE (see Chakraborty & Pietilainen (1995) and references therein) manifested as new plateaus at fractional filling: $n = m/(2pm \pm 1)$, where $p$ and $m$ are integers. The FQHE reflects the formation of an incompressible condensate that can be viewed as the IQHE for composite particles consisting of an electron with $2p$ flux lines attached to it (Jain 1989). Excitations from this condensate produce fractionally charged quasi-particles, with charge $q^* = e/(2pm \pm 1)$, at an energy cost of the excitation gap, $\Delta$. The gap can be estimated as $\Delta = \beta E_{\text{c}}$, with $E_{\text{c}} = e^2/4\pi\varepsilon_0\varepsilon l_B$ the Coulomb energy, $\varepsilon_0$ the permittivity of free space, $\varepsilon$ the effective dielectric constant, $l_B = (\hbar/eB)^{1/2}$ the magnetic length and $\beta$ a numerical constant that depends on the filling factor. For $v = 1/3$, $\beta \sim 0.1$ according to theory but is much smaller (approx. 0.01) in actual measurements (Boebinger et al. 1987).

The FQHE in graphene is expected to deviate in important ways from that in GaAs-based 2DESs. First, electrons in graphene are considerably more two dimensional than in GaAs quantum wells, where the well widths range from 10 to 30 nm. This makes the interaction at short distances in graphene much stronger than in quantum wells. The interactions in SG are further enhanced owing to the absence of substrate screening ($\varepsilon \sim 1$) in contrast to GaAs where $\varepsilon \sim 13$. Stronger interaction leads to a larger energy gap (Apalkov & Chakraborty 2006), and thus higher temperatures at which the FQHE can be observed. Another difference arises from the fourfold spin and valley degeneracy. Because of this degeneracy, the situation in graphene is more similar to that realized in double quantum wells rather than in single quantum-well systems. However, electron interactions in graphene correspond to nearly identical intra- and...
inter-well interactions, a regime nearly impossible to achieve in GaAs systems. This leads to an SU(4) symmetry of the interaction Hamiltonian, and the prediction of new FQHE states with no analogue in GaAs (Toke et al. 2006; Wang et al. 2008).

In the most common graphene devices, fabricated by mechanical exfoliation of graphite onto an Si substrate capped with SiO$_2$, the IQHE is extremely robust and can persist up to room temperature (Peres et al. 2006). In contrast, the FQHE was never observed in these devices, in spite of intense efforts. Its absence was attributed to substrate intrusiveness that, by introducing fluctuations on energy scales larger than the correlations, can prevent the formation of the quite delicate FQHE states. Based on the historical precedent in the GaAs-based 2DESs where the observation of the FQHE became possible only after achieving better material quality (see Chakraborty & Pietilainen (1995) and references therein), it was expected that once the substrate disturbances were eliminated, correlation effects would also be seen in graphene.

Recently, a significant improvement in sample quality was demonstrated in SG, where all substrate-induced perturbations were eliminated (Bolotin et al. 2008; Du et al. 2008). The combination of ballistic transport and low carrier density that was achieved in these samples has raised hopes to observe the correlated behaviour of the relativistic charge carriers. Surprisingly, measurements of the Hall conductance in SG using the standard Hall-bar lead geometry failed to exhibit the FQHE. In fact, not only was the FQHE not observed, but even the normally much more robust IQHE was absent (Bolotin et al. 2008). By contrast, as we show below, measurements of the SG samples using a two-terminal lead geometry do exhibit both the IQHE and the FQHE.

2. Experimental system: failure of multi-terminal measurements

In the present study, the magneto-transport of SG devices was investigated in both two-terminal and Hall-bar lead geometries. The SG devices were fabricated from conventional devices, mechanically exfoliated onto Si/SiO$_2$ substrates, by removing the SiO$_2$ layer with chemical etching (Du et al. 2008). In the final device, the graphene sample is suspended from the Au/Ti leads (figure 1a and inset figure 2a). An estimate of the contact resistance of these leads was obtained through a multi-terminal measurement of non-suspended samples with leads having contact area and geometry similar to that in the SG samples. Its value, typically approximately 200 $\Omega$, is significantly less than the quantum resistance with no measurable gate-voltage dependence. Following fabrication, the samples were baked at 200$^\circ$C in flowing forming gas (Ar:H$_2$, 9:1) for 2 h and current annealed (Moser et al. 2007) in vacuum at low temperature to further improve sample quality. To ensure mechanical integrity, we focused on short devices, less than 1 $\mu$m long. Longer devices tend to collapse during fabrication or when subject to a gate voltage. The carrier density in the suspended devices, $n_s \sim 2 \times 10^{14}$ $V_g$ (me$^{-2}$), was controlled by the gate voltage $V_g$ (measured in volts; Du et al. 2008).

In figure 1, we show the results obtained for an SG device with a Hall-bar lead geometry. The filling factor dependence of the Hall and longitudinal conductance is shown in figure 1b. Surprisingly, $G_{xy}$ does not show clear plateaus, and its value is significantly lower than that expected for the QHE plateaus, $n e^2/h$. At

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Figure 1. (a) Scanning electron microscope (SEM) image of SG sample with Hall-bar lead geometry. (b) Longitudinal and Hall conductance for the sample in (a) obtained from the measured resistances \( G_{xx} = R_{xx}/(R_{xx}^2 + R_{xy}^2) \), \( G_{xy} = R_{xy}/(R_{xx}^2 + R_{xy}^2) \). (c) Equipotential line distribution in a sample with \( W/L = 1.78 \), for a large Hall angle \( \sigma_{xy}/\sigma_{xx} = 20 \) illustrating the regions of sharp potential drop (hot spots) at opposite corners of the sample. \( V_s \) and \( V_d \) denote source and drain voltage. (d) Shorting of Hall voltage by invasive leads. In the QH regime, the carriers move in ballistic edge channels originating from one current lead and traversing a region of potential gradient, ‘hot spot’ (grey circle), before entering the opposite lead. A voltage lead covering the hot-spot region (hashed rectangles) shorts out the Hall voltage as observed in (b).

At the same time, \( G_{xx} \), instead of vanishing for \( \nu \) corresponding to the QHE, develops plateau-like features. This indicates that, although the QH effect does occur in our system, the Hall voltage probes are short-circuited through the source and drain leads.

Since in our devices, the size of the voltage probes is comparable to the sample size, it is important to take into account the effects of geometry. The behaviour of magneto-transport in such devices is to be contrasted with that in Hall-bar devices, conventionally used for studying Hall transport. In long Hall bars, the electric field is mostly transverse, and, as a result, the Hall voltage is practically insensitive to the positions of the Hall probes. This can be understood in an intuitive manner by considering the edge-state model of QH transport, in which transport is due to one-dimensional chiral edge channels. This model predicts that the voltage probes on the opposite sides of the sample measure the chemical potential of the top (bottom) chiral edge state, which is equal to the source (drain) potential. As a result, the measured Hall voltage is quantized under the QH effect conditions.
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Figure 2. (a) Comparison of measured density dependence of resistivity in zero field (grey) with the calculated curve (black) for an ideal ballistic graphene junction with the same dimensions (Tworzydlo et al. 2006). Upper inset: false-colour SEM image of SG device with $L = 0.6 \mu m$ and $W = 1.4 \mu m$. Lower inset: Drude mobility versus carrier density. (b) Gate-voltage dependence of two-terminal resistance at $T = 1.2 K$ for the indicated field values. Dashed lines mark the resistance values on the QHE plateaus. In addition to the usual non-interacting IQHE, we note a plateau at $v = -1$ and a newly observed one at $v = -1/3$, clearly visible on the hole side ($V_g < 0$). (c) $G(v)$ obtained from scaled electron density, $n_s h/eB$, in (b). All the curves collapse together, giving QH plateaus at $G(v) = v e^2/h$ centred at the correct filling factors in the hole sector. The N-shape distortions characteristic of the two-terminal measurements with $L < W$ are clearly seen (Abanin & Levitov 2008). (d) Evolution of QHE resistance plateaus with magnetic field for indicated values of gate voltage for an SG sample with $W = 2.6 \mu m$, $L = 0.5 \mu m$. Horizontal lines indicate theoretical resistance values for QH plateaus.

In contrast, for the case of a sample with an aspect ratio of order one, the existence of so-called hot spots situated on the sides of the sample (Wakabayashi & Kawaji 1978; Klass et al. 1991; Komiyama et al. 2006; Ikushima et al. 2007) near the contacts, where the component of the field parallel to the edge is large, makes the measured Hall voltage strongly dependent on the arrangement of the Hall probes. The voltage probe placed in the vicinity of the hot spot senses a potential which is intermediate between that of source and drain. This can cause a significant reduction in the measured Hall voltage between opposite voltage probes. Thus, in contrast to the case of a long Hall bar, the measured Hall voltage is not quantized.
As a concrete model illustrating this behaviour, we consider the rectangular geometry shown in figure 1c,d. In this case, the potential distribution can be found exactly by the conformal-mapping technique (Rendell & Girvin 1981). In the limit of large Hall angle, and for the sample aspect ratio of order one, the distribution of potential and current becomes highly non-uniform, in contrast to the more familiar case of a long Hall bar. This distribution is characterized by two accumulation regions at the opposite corners of the samples (hot spots), where field lines are concentrated. In the hot-spot corners, which are located at the opposite corners \((x = 0, y = 0), (x = L, y = W)\), where the \(x\)-axis is horizontal and \(y\)-axis is vertical, the electric field exhibits a power-law singularity (Rendell & Girvin 1981),

\[
E \propto e^{i\theta} \left[ \tan \left( \frac{\pi \Delta Z}{2L} \right) \right]^{-2\theta/\pi},
\]

where \(\theta\) is the Hall angle, \(E = E_x - iE_y\) is the complex electric field and \(\Delta Z = \Delta x + i\Delta y\) is the complex distance to the corner of the sample. In the other two corners, the field does not exhibit a divergence,

\[
E \propto e^{i\theta} \left[ \tan \left( \frac{\pi \Delta Z}{2L} \right) \right]^{2\theta/\pi}.
\]

The difference in the longitudinal electric field on the opposite sides of the sample, evident from equations (2.1) and (2.2), will result in a large distortion of the measured Hall voltage.

Another manifestation of current and field accumulation in the hot-spot regions, which can affect the measured Hall voltage, is the strong heating of these regions. Indeed, the Joule heating is strongly enhanced by the singular behaviour of the electric field in the hot-spot equation (2.1) in the limit of the large Hall angle. The heating can affect the microscopic conductivity tensor in the vicinity of the hot spot, giving rise to the increased \(\sigma_{xx}\) in that region. The increased conductivity may lead to the short-circuiting between the source (drain) and the voltage probe places in the vicinity of the hot spot.

Another interesting mechanism, which may further increase conductivity near the corners of the sample, can be due to a strain-induced pseudo-magnetic field (Prada et al. 2010). The strain of the SG sheet, which may result, e.g. from the electrostatic interaction with the gate (Prada et al. 2010), is strongest near the corners; the pseudo-magnetic field is expected to exhibit a power-law singularity near the corners. The estimated magnitude of the pseudo-magnetic field near the corners, of the order of a few tesla, is comparable to the external magnetic field in our experiments. The large pseudo-magnetic field makes the effective filling factor non-uniform across the sample. Because of that, even when the filling factor in the bulk of the sample corresponds to an incompressible QH state with vanishing \(\sigma_{xx}\), the regions near the corners may be conducting, which would lead to the short-circuiting between the source (drain) and voltage probes. This would decrease the measured Hall voltage, which would no longer be quantized.

We believe that the absence of quantization in our six-terminal measurements is owing to the effects associated with hot spots, discussed above. In our mesoscopic samples, the lead dimensions are comparable to sample size. Because of that,
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it is likely that the shorting of the Hall voltage observed in SG devices is because of the fact that the voltage leads were placed within the hot-spot regions. In future experiments, this difficulty may be circumvented by fabricating samples with a larger aspect ratio $L/W > 1$ and placing voltage probes further away from the hot-spot regions.

3. Two-terminal measurements and fractional quantum Hall effect

Although, at present, reliable Hall-bar measurements are not feasible in SG, it is still possible, as we show in the next section, to obtain the Hall conductance on QH plateaus as well as the longitudinal conductivity, $\sigma_{xx}$, from a two-terminal conductance ($G$) measurement, where the leads completely avoid the hot spots.

The results of the two-terminal measurements on SG samples are shown in figure 2. From the zero-field resistivity (figure 2a), we extract the Drude mobility, $\mu = (\rho ne)^{-1}$, and find that its density dependence, $\mu \propto n_s^{-1/2}$, closely follows that of a ballistic device with the same dimensions (figure 2a, lower inset) for hole carriers (negative $V_g$), with a somewhat less good agreement on the electron side. In a finite magnetic field, the two-terminal conductance exhibits well-defined plateaus, $G = |n|e^2/h$ for $n = \pm 2, \pm 6, \pm 10, \pm 14, \ldots$, seen already below 1T (figure 2b). In between the plateaus, we observe conductance maxima consistent with theory for our device geometry ($W/L > 1$) (Abanin & Levitov 2008; Williams et al. 2009). When $G$ is plotted versus the filling factor, the curves for all fields lie on top of each other (figure 2c) and exhibit a series of well-defined IQHE plateaus, consistent with the high carrier mobility (Abanin & Levitov 2008) and negligible contact resistance. Above 2T, additional plateaus develop at $n = \pm 1, 3$, reflecting interaction-induced lifting of the degeneracy in the $n = 0$ and $n = 1$ Landau levels. This is in sharp contrast to earlier measurements on non-suspended samples, where the $n = \pm 1$ plateaus were only seen above 26T (Zhang et al. 2006) and the one at $n = 3$ was absent. At low temperatures, we observe an FQHE plateau at $n = -1/3$ with $G = (1/3)e^2/h$, which becomes better defined with increasing field (figure 2b). Zooming into the low $n$ regime in figure 3a, we note that the plateaus at $n = -1/3, -1$ show accurate values of the quantum Hall conductance. Crucially, the densities at which these plateaus occur scale linearly with the $B$ field, $n_s = neB/h$, corresponding to constant filling factor values. The FQHE at $n = -1/3$ is quite robust; it appears already at approximately 2T at low temperatures and persists up to approximately 20K at 12T. At even lower filling factors, the conductance becomes vanishingly small, indicating that the FQHE in graphene competes with an insulating phase at $n = 0$. The properties of this phase including temperature dependence and activation gaps are discussed in Du et al. (2009). It is not obvious a priori that the correlated state leading to the FQHE for the relativistic charge carriers in graphene is the same as that for the 2DESs in semiconductors. In order to gain insight into the nature of the FQHE state in graphene, it is desirable to obtain an accurate measure of the excitation gap. Experimentally, the gap is usually extracted from the temperature dependence of $\sigma_{xx}$ that can be obtained, together with $\sigma_{xy}$, from a multi-probe measurement, such as a Hall-bar lead geometry. However, as discussed above, this method fails in mesoscopic samples.

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\( T = 1.2 \text{ K} \)

\( G = \frac{e^2}{h} \)  
\( G = \frac{1}{3}(\frac{e^2}{h}) \)

Figure 3. Field and temperature dependence of the \( n = -1/3, -1 \) plateaus. (a) Zoom into the low \( \nu \) segment of the data in figure 2c. Well-defined conductance plateaus at \( G = \frac{e^2}{h} \) and \( G = \frac{1}{3}(\frac{e^2}{h}) \) occurring at the precise filling factors \( \nu = -1, -1/3 \) are already visible at 2T. (b) Temperature dependence of the QH plateaus. On the plateaus, the conductance increases with temperature faster than off the plateaus. Both plateau features remain visible up to \( T \sim 20 \text{ K} \).

4. Fractional quantum Hall effect versus integer quantum Hall effect

Here, we discuss to what extent the interpretation of the fractional quantization in terms of a Laughlin FQH state is unambiguous. Recently, it was found that integer QHE in locally gated graphene devices, such as p–n and p–n–p junctions, can manifest itself in fractional quantization of the two-terminal conductance (Ozyilmaz et al. 2007; Williams et al. 2007). Such fractional quantization was observed for the spatially varying density profile in which incompressible QH states with different integer filling factors \( \nu, \nu^* \) are created in different parts of the device. The fractional quantization was understood (Abanin & Levitov 2007) in terms of edge-state transport, in which the QH edge states propagate along sample edges as well as along p–n, p–p’ and n–n’ boundaries.

Thus, a natural question arises whether our observations can be explained in terms of the formation in SG of a p–n–p junction system. In this section, we shall investigate whether a model involving a p–n–p system with integer QH states that result from the lifted degeneracies of Landau levels (LLs) could account for the fractional conductance observed in our experiment. On very general grounds, we conclude that such a model is inconsistent with our observations, which rules out the p–n–p junction scenario, and suggests that, in fact, a genuine fractional QH effect has been observed.

Before we proceed, let us briefly discuss the possible origin of a p–n–p junction in our graphene devices. The most likely mechanism of p–n–p junction formation is contact doping (Giovannetti et al. 2008; Huard et al. 2008). The proximity to metal is expected to pin the electron density below the contact to either a positive or negative value, depending on the type of the metal (Giovannetti et al. 2008). Therefore, generally the density across the device is non-uniform, with p–n, n–n’ or p–p’ junctions formed in the vicinity of the contacts. The resulting electron–hole asymmetry of the transport has been verified experimentally (Huard et al. 2008). Alternatively, p–n–p junctions in our experiments may stem from density inhomogeneity induced by disorder potential.
The conductance of a p–n–p junction, analysed in Ozyilmaz et al. (2007) for incompressible QH states, was shown to be a simple function of filling factors in the inner and outer regions,

\[ G = \frac{|\nu\nu^*|}{|\nu^*| + |\nu^* - \nu|} \frac{e^2}{\hbar}, \]  

(4.1)

where \( \nu \) is the filling factor in the inner region, and \( \nu^* \) is the filling factor in the outer region. A similar result can be obtained in the ideal Hall effect limit (large Hall angle; using a method similar to that used in Abanin & Levitov (2008)). In the regime where LL degeneracies are lifted, such that \( n, n^* = 0, \pm 1, \pm 2, \pm 3, \ldots \), this formula predicts the following quantized values:

\[ G = \frac{1}{3}, 2/3, 2/5, 1/2, \ldots, \frac{e^2}{h}. \]

The quantized value of \( G = (1/3)(e^2/h) \), which is most interesting to us, occurs when \( \nu = 1, \nu^* = -1 \) or \( \nu = -1, \nu^* = 1 \). We consider the situation where the p–n–p junction is formed by contact doping, and model the behaviour of \( \nu, \nu^* \) as a function of back-gate voltage \( V_{bg} \) as follows: \( \nu = \alpha V_{bg} \) and \( \nu^* = \alpha V_{bg} + \beta V_c \), where \( V_c \) is a parameter that describes the contact doping. In that, assuming that \( V_c \) is constant and \( V_{bg} \) is varying, \( \nu \) and \( \nu^* \) change sign at different values of \( V_{bg} \).

As a result, in the region where \( \nu, \nu^* \) have opposite signs, the behaviour of \( G(V_{bg}) \), predicted by equation (2.1) is non-monotonic. Hence, the one-third plateau, which is realized when \( \nu \) and \( \nu^* \) have different signs, is positioned between two plateaus with zero conductance, \( G = 0 \), which occur when \( \nu = 0 \) or \( \nu^* = 0 \). Similar behaviour will arise when \( V_c \) depends smoothly on \( V_{bg} \). Such non-monotonic behaviour is inconsistent with our experiment, where only one \( G = 0 \) plateau is observed, and function \( G(V_{bg}) \) is monotonic.

To further illustrate this point, we model the behaviour of the conductance of a p–n–p junction in the LL-split regime as a function of parameters \( V_{bg}, V_c \). We have employed a method similar to that used in Abanin & Levitov (2008). The result is illustrated in figure 4, where the experimentally measured conductance trace is given by the vertical cross section (dashed line) of a two-dimensional plot, with a non-monotonic behaviour around one-third. It is clear from the figure that allowing \( V_c \) to vary with \( V_{bg} \) would not change these conclusions.

Another discrepancy with the p–n–p model, which is evident from figure 4, is that the fractional conductance plateaus are positioned at \( V_{bg} \) values of \( V_{bg} \) that correspond to integer filling factors in the central region. By contrast, in our experiment, the conductance behaviour is monotonic, with fractional plateaus found at values of \( V_{bg} \) corresponding to fractional filling factors. Thus, we conclude that the p–n–p junction formation is inconsistent with observations. The only viable explanation of the observed fractional conductance, in our view, is that a spatially uniform FQH state is realized in our SG samples.

5. Extracting the longitudinal and transverse conductivity from two-terminal conductance

Since, at present, there are no reliable Hall-bar measurements in SG, here, we attempt to obtain \( \sigma_{xx} \) and \( \sigma_{xy} \) from the two-terminal conductance \( G \). Since this quantity depends on \( \sigma_{xx}, \sigma_{xy} \) and sample geometry, ‘deconvolving’ \( G \) requires an additional input. Such input is provided by the conformal invariance of the

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Figure 4. Conductance of a p–n–p junction system versus the back-gate voltage $V_{bg}$, and parameter $V_c$ that describes doping by contacts, modelled using a method similar to that of Abanin & Levitov (2008). Splitting of the fourfold LL degeneracy by the interactions is accounted for by integer incompressible filling factor values $\nu, \nu^* = 0, \pm 1, \pm 2, \pm 3, \ldots$. This gives fractional quantized conductance values, including $G = 1/3, 2/3, 2/5, 1/2, \ldots, e^2/h$ (labelled by numbers in boxes). The measured conductance trace is taken along a cross section of this plot $V_c = V_c(V_{bg})$. Vertical cross section $V_c = \text{constant}$ is indicated by a grey dashed line. The behaviour of the conductance is non-monotonic around the one-third plateau, which is in clear departure from observations.

magneto-transport problem (Abanin & Levitov 2008). In this approach, $\sigma_{xx}$ and $\sigma_{xy}$ are interpreted as a real and imaginary part of a complex number $\sigma = \sigma_{xx} + i\sigma_{xy}$, and thereupon the transport equations become conformally invariant. Applied to a rectangular two-lead geometry, such as that in our experiment, theory (Abanin & Levitov 2008) yields a specific dependence of $G$ on $\sigma_{xx}, \sigma_{xy}$ and the aspect ratio $W/L$. Interestingly, the same dependence describes two-terminal conductance for an arbitrary sample shape, with the ‘effective aspect ratio’ $W/L$ encoding the geometry dependence.

There are several ways to use this approach for determining $\sigma_{xx}$. One is to focus on the plateaus where $\sigma_{xx}$ is small and $\sigma_{xy}$ is quantized, $\sigma_{xy} = ne^2/h$. Expanding $G$ in the small ratio $\sigma_{xx}/|\sigma_{xy}| \ll 1$, the deviation from a quantized value can be expressed as $G = |\nu|e^2/h + k\sigma_{xx}$. The coefficient $k$, obtained from theory (Abanin & Levitov 2008), as a function of $W/L$, is positive (negative) for $L < W$ ($L > W$) and zero for $L = W$. Despite its conceptual simplicity, we found this approach not quite adequate, since the effective $W/L$ may significantly deviate from the geometric aspect ratio of the sample (Williams et al. 2009). Further, since $\sigma_{xx}$ and $\sigma_{xy}$ change with $\nu$, the conductance plateaus exhibit N-shaped distortions, rendering the deviation in $G$ from a quantized value unsuitable for an accurate estimate of $\sigma_{xx}$.

Considerably more reliable results can be obtained by focusing on the N-shaped distortions of QH plateaus (Abanin & Levitov 2008; Williams et al. 2009). The N-shaped features can be described by the density-dependent $\sigma_{xx}(\nu)$ and $\sigma_{xy}(\nu)$ obtained from the so-called semicircle relation (see Ruzin & Feng (1995) and references therein). This relation, which is also rooted in the complex-variable
interpretation of magneto-transport, is known to work well in GaAs-based 2DESs, both in the integer and in the fractional QHE regimes (Shahar et al. 1997; Murzin et al. 2002, 2005). For a plateau-to-plateau transition between incompressible filling factors \( n_1 < n_2 \), the semicircle relation gives \( \sigma_{xx}^2 = (\sigma_{xy} - n_1)(n_2 - \sigma_{xy}) \) (in units of \( e^2/h \)). For fitting the conductance data shown in figure 3 with the incompressible states at \( \nu = 0, -1/3, -1, -2 \), we model the contribution of each QH transition by a Gaussian, \( \sigma_{xx}(n) = (1/2)(n_2 - n_1)e^{-A(n-n_c)^2} \), \( n_1 < n_c < n_2 \), and find the corresponding \( \sigma_{xy} \) from the semicircle relation. This gives a contribution to the longitudinal and Hall conductivity of each of the relevant Landau (sub) levels. The net conductivity, found as a sum of such independent contributions, is then used to calculate \( G(\nu) \) using the approach (Abanin & Levitov 2008) discussed above.

We treat the \( \sigma_{xx} \) peak positions and widths, as well as the effective ratio \( W/L \), as variational parameters. As illustrated in figure 5a, the Gaussian model with individually varying peak widths and positions is found to provide a rather good description of the data, at the same time giving the quantity \( \sigma_{xx}(\nu) \). The approach based on treating \( W/L \) as a variational parameter, in general different from the actual sample aspect ratio, works rather well in the integer QH regime (Williams et al. 2009). It was conjectured (Williams et al. 2009) that variations in the best-fit value of \( W/L \) account for the sample-dependent specifics of the current flow pattern, such as those owing to imperfect contacts and/or contact doping.

In order to extract \( \sigma_{xx} \) at the plateaus \( \nu = -1/3 \) and \( \nu = -1 \), we analysed a set of fits that best follow the data near these filling factors (figure 5a). In both cases, we found an optimal effective aspect ratio \( W/L \approx 1.7 \) (such deviation from the geometric aspect ratio, which for our sample is close to 2.5, is consistent with the results of Williams et al. 2009)). Best fits were found from the standard deviation in the conductance averaged over the range of densities on the plateau and around it; in the case of the one-third plateau, we excluded a small interval near \( \nu \approx 0.4 \), which we believe is related to another incipient fractional QHE feature.
The value of $\sigma_{xx}$ at the minima of $\sigma_{xx}(\nu)$ was taken as an estimate of the longitudinal conductivity of the incompressible QH states. Statistical error was estimated from the spread in the $\sigma_{xx}$ values found from the fits with standard deviation within 30 per cent of the best fit. The resulting error bar, displayed in figure 5b, is below 10 per cent at 10 K, and increases to 20–25% at 1.2 K as a result of pronounced mesoscopic fluctuations developing at low temperatures.

We used the 12 T data at $T = 1.2, 4.2, 6$ and 10 K to infer the temperature dependence of $\sigma_{xx}$ in the $\nu = -1/3$ and $\nu = -1$ states. The results, displayed in figure 5b, were analysed with a thermal activation model, $\sigma_{xx} \sim \exp(-\Delta/2k_BT)$. For $\nu = -1/3$, we find $\Delta_{\nu=1/3} \sim 4.4$ K. We also considered fits to the variable-range-hopping dependence, $\sigma_{xx} \sim \exp(-(T^*/T)^{0.5})$, but have not found a discernible statistical advantage over the activation dependence. This gap is close to 2.5 times larger than the gap measured in the best samples in the GaAs/GaAs-based 2DESs. Theoretical estimates (Toke et al. 2006) for the gap in the $\nu = 1/3$ state in SG gives $\Delta_{\nu=1/3} \sim 0.1E_c$. Taking the random-phase approximation result for the intrinsic screening function of graphene in zero field (Gonzalez et al. 1999), $\varepsilon \sim 5.24$, the theoretical value of the gap at $B = 12$ T is $\Delta_{\nu=1/3} \sim 42$ K. This implies that it is possible to observe an even more robust FQHE at $\nu = -1/3$ and other fractions by improving the quality of the graphene samples. By contrast, in the GaAs/GaAs-based 2DESs, the gaps are one order of magnitude smaller than in graphene owing to the higher dielectric screening and the finite width of the quantum wells. In the highest quality semiconductor samples, the theoretical gap values are already achieved at high fields so that not much improvement in the robustness of the FQHE can be expected there.

For the $\nu = -1$ state, we find that activated behaviour, $\sigma_{xx} \sim \exp(-\Delta/2k_BT)$, with the energy gap value $\Delta/k_B = 9.6$ K, best fits the data. This is only 5 per cent of the theoretically predicted value (Toke et al. 2006), and we expect it would increase in higher quality graphene samples.

6. Summary

In this work, we analyse the magneto-transport measurements in SG, in which electrons are isolated from environmental disturbances caused by the substrate. We demonstrate that by using two-terminal measurement geometry, the effect of invasive voltage probes can be eliminated, making the collective behaviour of the relativistic two-dimensional carriers readily observable. We present a detailed analysis of recent magneto-transport measurements and show that SG samples exhibit a robust FQHE at $\nu = 1/3$.

From the temperature dependence of the longitudinal conductivity, we found the energy gap of the quasi-particle excitations. Its magnitude, while only 10 per cent of the theoretically predicted value indicating imperfect sample quality, is still more than three times larger than that for the best semiconductor-based 2DESs. The robust character of the FQHE observed in graphene reflects enhancement of the Coulomb interactions owing to lower dielectric screening and to the more two-dimensional nature of electrons in suspended graphene.

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