Collective Poisson process with periodic rates: applications in physics from micro- to nanodevices

BY ROBERTO DA SILVA1,*, LUIS C. LAMB1 AND GILSON INACIO WIRTH2

1 Institute of Informatics, Federal University of Rio Grande do Sul, Avenida Bento Gonçalves 9500, 91501-970 Porto Alegre, RS, Brazil
2 School of Engineering, Federal University of Rio Grande do Sul, Avenida Osvaldo Aranha 103, 90035-190 Porto Alegre, RS, Brazil

Continuous reductions in the dimensions of semiconductor devices have led to an increasing number of noise sources, including random telegraph signals (RTS) due to the capture and emission of electrons by traps at random positions between oxide and semiconductor. The models traditionally used for microscopic devices become of limited validity in nano- and mesoscale systems since, in such systems, distributed quantities such as electron and trap densities, and concepts like electron mobility, become inadequate to model electrical behaviour. In addition, current experimental works have shown that RTS in semiconductor devices based on carbon nanotubes lead to giant current fluctuations. Therefore, the physics of this phenomenon and techniques to decrease the amplitudes of RTS need to be better understood. This problem can be described as a collective Poisson process under different, but time-independent, rates, \( t_c \) and \( t_e \), that control the capture and emission of electrons by traps distributed over the oxide. Thus, models that consider calculations performed under time-dependent periodic capture and emission rates should be of interest in order to model more efficient devices. We show a complete theoretical description of a model that is capable of showing a noise reduction of current fluctuations in the time domain, and a reduction of the power spectral density in the frequency domain, in semiconductor devices as predicted by previous experimental work. We do so through numerical integrations and a novel Monte Carlo Markov chain (MCMC) algorithm based on microscopic discrete values. The proposed model also handles the ballistic regime, relevant in nano- and mesoscale devices. Finally, we show that the ballistic regime leads to nonlinearity in the electrical behaviour.

Keywords: stochastic processes; Monte Carlo Markov chain simulation; low-frequency noise reduction

1. Introduction

In random Si–SiO₂ interfaces, traps are able to capture electrons travelling in the drain current due to a fluctuation known as random telegraph signals (RTS) [1], as seen in figure 1. Recently, RTS have become increasingly relevant

*Author for correspondence (rlasilva@inf.ufrgs.br).

One contribution of 17 to a Theme Issue ‘Nonlinear dynamics in meso and nano scales: fundamental aspects and applications’.
Figure 1. A simple scheme of a semiconductor where RTS occur: a complementary metal oxide semiconductor device. The electrons (small circles) speed up by voltage $V_{DS}$ while the voltage $V_{GS}$ just works to attract the electrons to the interface between the semiconductor and the oxide. The collective capture and emission of electrons by traps (large circles) generate important discontinuities that are basic sources of failures in flash memories, for example.

owing to the characteristics of the nanodimensions of semiconductor devices. In particular, shocking physical effects have been pointed out by several authors (e.g. [2]). The problems caused by RTS noise include: flash memory failures [3], mobile phone malfunctions and other important failures in micro–meso–nanodevices. Although this problem has been amply explored in complementary metal oxide semiconductor (CMOS) physics research in micro- and mesoscales, understanding RTS noise in nanoscale is crucial. For example, giant RTS have been experimentally verified in single-wall carbon nanotube field-effect transistors [4].

The capture (emission) of an electron by a trap along a drain current is a Poisson process, where the probability that exactly one transition (capture or emission) occurs during the time interval $[t, t + dt]$ is, respectively,

$$p(0 \rightarrow 1) \, dt = \frac{dt}{\tau_c} \quad \text{(capture)}$$

and

$$p(1 \rightarrow 0) \, dt = \frac{dt}{\tau_e} \quad \text{(emission)},$$

where $\lambda_c = 1/\tau_c$ and $\lambda_e = 1/\tau_e$ are the rate parameters of the Poisson process. Such rates have a natural interpretation, i.e. we deduce that the probability that state 1 will not make a transition for an exact time $t$ (respectively, for state 0) is $p_1(t) = \lambda_e \exp(-\lambda_e t)$ and $p_0(t) = \lambda_c \exp(-\lambda_c t)$. Therefore, the average residence times in the states 1 and 0 are, respectively,

$$\langle t \rangle_1 = \int_0^\infty t p_1(t) \, dt = \tau_c \quad \text{and} \quad \langle t \rangle_0 = \int_0^\infty t p_0(t) \, dt = \tau_e.$$

Similarly, the standard deviations are given by

$$\sigma_0 = \sqrt{\int_0^\infty t^2 p_0(t) \, dt - \tau_e^2} = \tau_c \quad \text{and} \quad \sigma_1 = \sqrt{\int_0^\infty t^2 p_1(t) \, dt - \tau_c^2} = \tau_e,$$

which is a particular characteristic of this kind of process.
From this simple approach, we can show that the power spectral density (PSD) for the frequency $f$, caused by a single trap, has a Lorentzian shape [1,5], which is given by

$$S(f) = \frac{4\delta^2}{(\tau_e + \tau_c)[(1/\tau_e + 1/\tau_c)^2 + f^2]}.$$  \hfill (1.3)

Here, $\delta$ denotes the amplitude of the current fluctuation (which depends on the position of the trap in the oxide) caused by an electron capture or emission; we define $\tau_{eq}$ by $1/\tau_{eq} = 1/\tau_e + 1/\tau_c$. Microscopically, the constants $\tau_c$ and $\tau_e$ are written as

$$\tau_c = 10^p(1 + e^{(E_t - E_f)/k_B q}) \quad \text{and} \quad \tau_e = 10^p(1 + e^{(E_f - E_t)/k_B q}),$$

where $p \in [p_{\text{min}}, p_{\text{max}}]$ is uniformly distributed among the traps of the device, which implies that $\tau_{eq} = 10^p$ and is therefore uniformly distributed on a log scale. Here $E_t$ is the energy of the trap and $E_f$ corresponds to the Fermi level, which is proportional to $V_{GS}$ (gate voltage). Here $\theta$ denotes the temperature.

Previous results developed by da Silva et al. [6] showed that the contribution of many traps, where the number of the traps per device is a Poisson random variable, is given by

$$\langle S \rangle \propto \frac{1}{f} \langle A \rangle,$$

where $A = \delta^2\beta/(1 + \beta)^2$. Here, $\beta = \tau_e/\tau_c = e^{(E_t - E_f)/k_B q}$, where $\langle \cdot \rangle$ represents an average over trap energies ($E_t$) and over different amplitudes ($\delta$) due to different captures and emissions.

As long as the basic, relevant assumption remains valid, the $1/f$ behaviour remains unchanged, as shown in our previous work. This assumption states that the capture and emission time constants are uniformly distributed on a log scale. The experimental data available in the literature point out that this assumption remains valid at the nanoscale.

The assumption about the number of traps $N_t$ is that it is a Poisson distribution. From a theoretical point of view and from experimental data available in the literature, it is also expected that this assumption remains valid at the nanoscale.

The computation of the probability distribution of amplitudes $\delta$ is an open problem in the literature. Hence, we avoid making assumptions about this distribution. However, we will address the impact of common assumptions made on the macroscopic scale, and show that they may lead to nonlinearities in electrical behaviour. In §4b, we show why the ballistic regime leads to nonlinear behaviour.

While the distribution of amplitudes is an open problem, the density of states $g(E_t)$ seems to behave as a $U$-shaped distribution [7]. In previous work, we have shown that a normalized quadratic function is a suitable choice to model this phenomenon:

$$g(E_t) = aE_t^2 - a(E_c + E_v)E_t + \frac{1}{(E_c - E_v)} \left( \frac{1}{6} aE_c^3 - \frac{1}{6} aE_v^3 - \frac{1}{2} aE_c E_v^2 + \frac{1}{2} aE_c^2 E_v + 1 \right),$$  \hfill (1.4)

where $a$ is a fitting parameter and naturally $\int_{E_c}^{E_v} g(E_t) \, dE_t = 1$, and the band-gap (range of trap energies) is represented by $[E_v, E_c]$.
In general, $V_{GS}$ is a constant voltage (in the stationary regime). However, periodic gate voltages should be considered, making the Fermi level a square cyclostationary excitation:

$$E_f(t) = \begin{cases} E_{ON} & \text{if } 0 \leq t < \alpha T, \\ E_{OFF} & \text{if } \alpha T \leq t < T, \end{cases}$$

(1.5)

where $T$ is the period of excitation and for the sake of simplicity $\alpha = 1/2$. The idea of noise reduction in our approach is completely restricted to reducing the term $\langle A \rangle$, making the Fermi level a time-dependent function (1.5).

In a previous work [8], we have shown that an alternative approach is to consider the time dependence of $\langle A \rangle$ after integration over the density of states:

$$\langle A \rangle = \alpha \delta^2 \int_{E_c}^{E_{c+}} \frac{\exp[(E_t - E_{OFF})/k_B\theta]}{1 + \exp[(E_t - E_{OFF})/k_B\theta] + 1} dE_t$$

$$+ (1 - \alpha) \delta^2 \int_{E_c}^{E_{c+}} \frac{\exp[(E_t - E_{ON})/k_B\theta]}{1 + \exp[(E_t - E_{ON})/k_B\theta] + 1} dE_t.$$  

(1.6)

In this previous contribution, we showed that noise is reduced in the cyclostationary regime, which corroborates the main experimental results from the literature (e.g. [9]). However, in recent work we considered capture and emission probabilities as time-dependent periodic functions. We showed, considering in particular the case of square-wave excitation, that, for one trap, a generalization of the Machlup formula can be obtained under a regime where the emission or capture probabilities are much smaller than the excitation period in such a way that few transitions occur during a period. In this case, we have that time averages come in the Lorentzian form for the PSD, for one trap [10]:

$$S = 4\delta^2 \frac{\langle \beta \rangle}{(1 + \langle \beta \rangle)^2} \frac{1}{f_0} \frac{1}{1 + (f/f_0)^2},$$

(1.7)

where $f_0 = \langle 1/\tau_c \rangle + \langle 1/\tau_e \rangle = 10^{-p}$ and here $\langle \cdot \rangle = (1/T) \int_0^T \cdot dt$ is the time average. It would then be interesting to formulate the superposition of many traps under many devices. In this paper, using this new alternative approach, we developed a formulation to describe the PSD for the noise corresponding to many traps, by averaging over many different samples. Our previous approach in the frequency domain [10] has shown that noise can be reduced. However, this reduction can be improved since previous results correspond to a particular case denoted by modulation (25%). It is known that such reduction can be larger than estimated, and other models must be considered to cover more realistic situations.

As an alternative to this spectral study of RTS, direct current fluctuations caused by occupation level fluctuations of the traps can be studied. Further, alternatives to obtain a decrease of the noise by looking directly to stochastic dynamics established for successive capture and emissions over time can also be analysed.

In this paper, we aim to show a theoretical description of a model that describes a reduction of the noise in semiconductor devices in two alternative and important ways: time domain (fluctuations in noise current) and frequency domain (fluctuations in PSD) based on microscopic values. Moreover, we show...
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that our model is also able to handle the ballistic regime, relevant in nano- and mesoscale devices, leading to nonlinearity in the electronic behaviour. Our contributions and the method used in each part of this paper are the following:

— We numerically integrate the equations for the PSD $S(E_{\text{ON}}, E_{\text{OFF}})$ (in the cyclostationary regime), where the Fermi level changes periodically between two values, $E_{\text{ON}}$ and $E_{\text{OFF}}$. We analyse the ratio $\eta = S(E_{\text{ON}}, E_{\text{OFF}})/S(E_{\text{ON}}, E_{\text{ON}})$ that corresponds to reduction of noise in relation to the stationary case ($E_{\text{ON}} = E_{\text{OFF}}$).

— In the time domain, we present a detailed Monte Carlo Markov chain (MCMC) algorithm to describe the time evolving noise current. We show that the amplitude logarithmic behaviour of the current as a function of time decreases when the cyclostationary regime is assumed, and the gain is larger as $E_{\text{OFF}}/E_{\text{ON}}$ becomes smaller.

— To explore the effects of nanoscale in semiconductor devices, we raise a conjecture for amplitudes $d$ in the ballistic regime, and we estimate the factor $\gamma = \langle d \rangle_{\text{nano}}/\langle d \rangle_{\text{micro}}$ that measures the amplification of the noise in the ballistic regime (nanodevices) in relation to the non-ballistic regime (micro-devices). We show that, at room temperature ($\theta = 293$ K) and over a drain-to-source voltage $V_{\text{DS}}$ equal to 0.5 V, an amplification of a factor of 3 is verified.

In §2, we describe the MCMC algorithm that produces as output the current as a function of time obtained via computer simulations of an ensemble of devices in the cyclostationary regime. Based on a logarithmic law experimentally observed [11] and analytically demonstrated in da Silva & Wirth [12], we explore some preliminary results reproduced by the algorithm for the stationary regime. In §4a, we show the cyclostationary effects on the logarithmic law keeping $E_{\text{ON}} = 0.9 \, \text{eV}$ fixed and changing $E_{\text{OFF}}$ from 0.3 up to 0.9 eV.

In §3, we obtain a closed formula for the computation of the PSD of a superposition of traps by considering the approximate equation for the PSD for one single trap, under time-dependent capture and emission rates (see equation (1.7)). We also present a formula for the relative error and the formula for the gain ($\eta$) in the cyclostationary condition in relation to the stationary one. In §4a the PSD under cyclostationary excitation and its relative error amounts are drawn using three-dimensional plots, as a function of different pairs $(E_{\text{ON}}, E_{\text{OFF}})$, showing that noise minimization corresponds to maximization of the relative error. Further, in §4b, we introduce an explicit formula to show the nanoeffects on the current fluctuations of semiconductor devices. Section 5 concludes the paper.

2. Time domain analysis

Let us now consider the temporal evolution of devices that initially have a number of non-occupied traps. So, we can write the drain current at time $t$ as

$$\langle I(t) \rangle = \sum_{k=0}^{N_{\text{tr}}} \delta_{i} \sigma_{i}(t) = \langle \delta \rangle \sum_{k=0}^{N_{\text{tr}}} k \Pr(k|n(0), t), \quad (2.1)$$

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where we consider that amplitudes are equally distributed for the $N_{tr}$ traps, i.e. $\langle \delta_i \rangle = \langle \delta \rangle$. Considerations about this average value are outlined in §4, when we perform some predictions specific to nanodevices. For now, we consider

$$\sum_{k=0}^{N_{tr}} k \Pr(k|n(0), t).$$

Here, it is important to note that this leads to the probability that a particular trap initially at state 0 occupies state 1 after a time $t$ given by

$$P_{01}(\tau_c, \tau_e, t) = \tau_e(\tau_e + \tau_c)^{-1}[1 - e^{-t/\tau_{eq}}]$$

and similarly

$$P_{11}(\tau_c, \tau_e, t) = \tau_e(\tau_e + \tau_c)^{-1}[\tau_e + \tau_c e^{-t/\tau_{eq}}].$$

In a recent paper [12], we have shown (for the stationary case) that the probability of $k$ traps to be occupied, denoted by $\Pr(k|n(0) = 0, t)$, starting from an initial state where no traps are occupied, is simply given by the binomial

$$\Pr(k|n(0) = 0, t) = \binom{n}{k} P_{01}(\tau_c, \tau_e; t)^k \cdot P_{00}(\tau_c, \tau_e; t)^{N_{tr} - k}$$

after averaging over microscopic details of the trap constants. Here, $P_{00} = 1 - P_{01}$ and

$$\tau := \int \int f(\tau_c) g(\tau_e) \, d\tau_c \, d\tau_e \rightarrow \int_{\tau_{min}}^{\tau_{max}} \int_{E_v}^{E_c} g(\cdot) \, dE_t \, dE_t.$$

We can show that for the stationary case a log behaviour is observed for the temporal evolution:

$$\langle I(t) \rangle \sim \langle \delta \rangle (p_{max} - p_{min})^{-1} \left( \int_{E_v}^{E_c} \frac{dE_t \, g(E_t)}{1 + e^{-(E_t - E_f)/k_B T}} \right) \int_{0}^{t} \frac{(e^{-u} - 1)}{u} \, du \sim \langle \delta \rangle \log t.$$

Our interest is to analyse the deviations of this law (considering the stationary case) looking at the cyclostationary cases. So we introduce an MCMC algorithm that is initially tested to check this expected law [12]. In the sequel, we will use this algorithm to show the noise reduction in §4.

Firstly, we have to set up the number $n_{sample}$ of different runs that will be performed, which mimics the number of devices that will be simulated. For each run, we select a number of traps of the device (denoted by $k$), which follows a Poisson distribution with average $\lambda$ (input). We consider $\lambda = 150$ for our experiments, i.e. the expected number of traps for each device.

The number $k$, is selected according to the following procedure.

---

**Routine 1: select number of traps per device**

1. $k := 0; \text{ sum } = 0$
2. $x := \text{rand}[0, 1]$
3. while (sum $< x$) then
4. \hspace{1cm} $\text{sum } := \text{sum } + \frac{\exp(-\lambda) \lambda^k}{k!}$
5. \hspace{1cm} $k = k + 1$
6. endwhile

Here, rand[0, 1] denotes the uniform random deviation generated in interval [0, 1].
Since $k$ was selected by this routine, for each trap indexed by $i = 1, \ldots, k$, we determined if it is occupied or empty according to initial density $\rho_0$ (input) and we also determined its energy according to the U-shape $g(E_t)$ from equation (1.4). This procedure is described by the following procedure (this is basically a trapezoidal integration).

**Routine 2: select energy of the trap (trapezoidal rule)**

1. $w := \text{rand}[0,1]$
2. $h := (E_c - E_v) / n_{\text{steps}}$
3. sum2 := 0;
4. $E_t = E_v;$
5. $c := (1/6) * a * E_v^3 - (1/6) * a * E_c^3 - (1/2) * a * E_c * E_v^2$
   $\quad + (1/2) * a * E_v * E_c^2 + 1)/(E_c - E_v);$
6. while (sum2 < $w$) then
7. aux1 := $a * (E_t - h)^2 - a(E_v + E_c) * (E_t - h);$
8. aux2 := $a * E_t^2 - a(E_v + E_c) * E_t;$
9. sum2 := sum2 + (aux1 + aux2 + 2 * c) * $h/2;$
10. endwhile

Here $n_{\text{steps}}$ is the number of resolution steps in the integration; we consider $n_{\text{steps}} = 2000$. In our simulations, we have used the values $E_v = 0.2$, $E_c = 1.0$, resulting in a band-gap of size 0.8 eV, which is compatible with more recent technologies. For our simulations we consider $a \approx 10$, which is more compatible with experimental U-shapes (see da Silva et al. [8] for a discussion on this issue).

Thus, the current can be computed through the main algorithm (table 1). A simple test can be performed starting from $\rho_0 = 0$ (stationary case) with conditions $E_{\text{ON}} = E_{\text{OFF}} = 0.9 \text{ eV}$. We also consider $p_{\text{min}} = 0$ and $p_{\text{max}} = 5$, and our test was performed for $n_{\text{sample}} = 1$ corresponding to only one device.

In figure 2, we show the temporal evolution of the noise current as a function of time. We can see the pattern of convergence to the logarithmic law, which is expected from analytical results. In §4, we will describe simulations that corroborate the noise reduction under cyclostationary excitation.

### 3. Frequency domain analysis

Let us consider the superposition of power spectral densities of $N_{\text{tr}}$ traps, according to equation (1.7):

$$S_{N_{\text{tr}}}(f) = \sum_{i=1}^{N_{\text{tr}}} S_i = \sum_{i=1}^{N_{\text{tr}}} 4\delta_i^2 \frac{\langle \beta_i \rangle}{(1 + \langle \beta_i \rangle)^2} \frac{1}{10^{-p_i} + (f/10^{-p_i})^2}.$$  

Here $\langle \beta_i \rangle = (\beta) = a\beta_{\text{ON}} + (1 - a)\beta_{\text{OFF}}$, where $\beta_{\text{ON}} = e^{(E_i - E_{\text{ON}})/k_B\theta}$ and $\beta_{\text{OFF}} = e^{(E_i - E_{\text{OFF}})/k_B\theta}$. Taking the average, and considering the U-shaped distribution of $E_t$, $p$ a uniform random variable in $[p_{\text{min}}, p_{\text{max}}]$, and $N_{\text{tr}}$ Poisson-distributed over...
Table 1. MCMC algorithm for the temporal evolution of the current under cyclostationary excitation.

<table>
<thead>
<tr>
<th>Main algorithm</th>
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<tr>
<td><strong>input:</strong> $n_{\text{sample}}$, $\rho_0$, $n_{\text{mcsteps}}$, $E_{\text{ON}}$, $E_{\text{OFF}}$, $E_v$, $E_c$, $p_{\text{min}}$, $p_{\text{max}}$</td>
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1. for $i_{\text{sample}} = 1, \ldots, n_{\text{sample}}$
2. select the number of traps $k$, of the device $i_{\text{sample}}$, by routine 1
3. for $i_k = 1, \ldots, k$ (for each trap $i_k$ of the device $i_{\text{sample}}$)
   1. $ikey := 0$;
   2. $icount := 1$;
   3. $p := p_{\text{min}} + \text{rand}[0, 1] * (p_{\text{max}} - p_{\text{min}})$;
   4. $x := \text{rand}[0, 1]$ (we decide the initial state of the trap)
   5. if ($x < \rho_0$) then
      1. $\sigma = 1$;
   6. else
      1. $\sigma = 0$;
      7. endif
   8. compute the energy of the trap $i_k$ by routine 2
   9. $q_{\text{ON}} = (E_t - E_{\text{ON}}) / k_B \theta$; $q_{\text{OFF}} = (E_t - E_{\text{OFF}}) / k_B \theta$;
   10. for $i_{\text{MCstep}} = 1, \ldots, n_{\text{MCsteps}}$ (start the Monte Carlo steps)
     1. $y := \text{rand}[0, 1]$;
     2. $icount := icount + 1$;
     3. if ($ikey = 0$) go to 19;
     4. if ($ikey = 1$) go to 26;
     5. if ($icount < \alpha T$) then
        1. $\tau_c = 10^p[1 + \exp(q_{\text{ON}})]$; $\tau_e = 10^p[1 + \exp(-q_{\text{ON}})]$;
     6. else
        1. $\tau_c = 10^p[1 + \exp(q_{\text{OFF}})]$; $\tau_e = 10^p[1 + \exp(-q_{\text{OFF}})]$;
        7. endif
     8. $ikey = 1$; $icount = 0$;
     9. goto 32
   11. if ($icount < (1 - \alpha) T$) then
     12. $\tau_c = 10^p[1 + \exp(q_{\text{OFF}})]$; $\tau_e = 10^p[1 + \exp(-q_{\text{OFF}})]$;
     13. else
     14. $\tau_c = 10^p[1 + \exp(q_{\text{OFF}})]$; $\tau_e = 10^p[1 + \exp(-q_{\text{OFF}})]$;
     15. endif
     16. goto 32
   17. endif
   18. $\tau_{eq} = \tau_c \tau_e / (\tau_c + \tau_e)$
   19. if ($\sigma_{-1} = 0$) then
      1. $p_{01} = [\tau_c / (\tau_c + \tau_e)] (1 - e^{-t / \tau_{eq}})$
      2. if ($y < p_{01}$) $\sigma := 1$;
   20. else
      1. $p_{10} = [\tau_e / (\tau_c + \tau_e)] (1 - e^{-t / \tau_{eq}})$
      2. if ($y < p_{10}$) $\sigma := 0$;
   21. endif
   22. $\text{current}(i_k) = \text{current}(i_k) + \delta$
   23. endfor
   24. endfor

Phil. Trans. R. Soc. A (2011)
Figure 2. Different temporal evolutions of the current of the simulated device for different \( \lambda \) values (average number of traps per device). We used (a) \( \lambda = 2 \), (b) \( \lambda = 6 \), (c) \( \lambda = 10 \) and (d) \( \lambda = 100 \). Here, to observe in detail the evolution to log behaviour, we fixed only one sample. For the last plot (\( \lambda = 100 \)) a semi-log scale is used.

different devices, from similar methods developed in da Silva et al. [6], we expect that the value of \( S_{N_{tr}}(f) \) is:

\[
E[S(f)] = \lambda \langle \delta^2 \rangle \int_{E_v}^{E_c} dE_t \, g(E_t) \frac{\alpha e^{(E_t-E_{ON})/k_B\theta} + (1 - \alpha) e^{(E_t-E_{OFF})/k_B\theta}}{[1 + \alpha e^{(E_t-E_{ON})/k_B\theta} + (1 - \alpha) e^{(E_t-E_{OFF})/k_B\theta}]^2} \frac{1}{(p_{\text{max}} - p_{\text{min}})} \int_{p_{\text{min}}}^{p_{\text{max}}} dp \frac{10^p}{1 + (10^p f)^2},
\]

where \( \lambda = \sum_{N_{tr}=0}^{\infty} N_{tr} e^{-\lambda} / N_{tr}! = \rho WL \) is the average number of traps per device, and \( \rho, W \) and \( L \) are, respectively, the density of traps per device and their characteristic dimensions (see figure 1).

For small and large values of \( p_{\text{min}} \) and \( p_{\text{max}} \), we have the expected \( 1/f \) noise

\[
E[S(f)] \sim \frac{B}{f},
\]

from which our interest in the noise reduction is reduced to analysing the amplitude:

\[
B = \delta^2 \int_{E_v}^{E_c} dE_t \, g(E_t) \frac{\alpha e^{(E_t-E_{ON})/k_B\theta} + (1 - \alpha) e^{(E_t-E_{OFF})/k_B\theta}}{[1 + \alpha e^{(E_t-E_{ON})/k_B\theta} + (1 - \alpha) e^{(E_t-E_{OFF})/k_B\theta}]^2},
\]

\( \text{(3.1)} \)
In our previous model [8], the noise reduction was reduced to analyse (A) described by equation (1.6). In our new approach, it is interesting to define the noise reduction ratio

$$\eta(E_{ON}, E_{OFF}) = \frac{B(E_{ON}, E_{OFF})}{B(E_{ON}, E_{ON})}$$

(3.2)

that expresses the yield by using a cyclostationary input instead of the stationary state ($E_{ON} = E_{OFF}$, a constant Fermi level). In our analysis, it is also interesting to analyse the relative error of the PSD. For that purpose, we have to calculate the second moment of the PSD:

$$E[S(f)^2] = \sum_{i=1}^{N_{tr}} \frac{e^{-\lambda} N_{tr}}{N_{tr}!} \left( \sum_{i=1}^{N_{tr}} S_i^2 + \sum_{i=1}^{N_{tr}} \sum_{j=1}^{N_{tr}} S_i^2 S_j^2 \right)$$

$$= S^2 \sum_{i=1}^{N_{tr}} \frac{N_{tr}}{N_{tr}!} e^{-\lambda} N_{tr} + S^2 \sum_{i=1}^{N_{tr}} (N_{tr} - 1) \frac{e^{-\lambda} N_{tr}}{N_{tr}!}$$

$$= \bar{S}^2 + \lambda^2 \bar{S}^2,$$

where $\bar{}$ denotes the average over $E_t$ and $p$. So if we have that $E[S(f)] = \bar{S}$, the variance $\text{var}[S] = E[S(f)^2] - E[S(f)]^2$ is equal to $\lambda \bar{S}^2$, and the relative error is then calculated via $e(S) = (\text{var}[S]/E[S(f)])^{1/2} = (\bar{S}^2/\bar{S})^{1/2}$. So it is easy to show that

$$e(E_{ON}, E_{OFF}) \sim \frac{1}{(WL)^{1/2}} \left( \int_{E_c}^{E_v} f(E_t, E_{ON}, E_{OFF})^2 \, dE_t \right)^{1/2},$$

(3.3)

where

$$f(E_t, E_{ON}, E_{OFF}) = \frac{\alpha e^{(E_t - E_{ON})/k_B \theta} + (1 - \alpha) e^{(E_t - E_{OFF})/k_B \theta}}{[1 + \alpha e^{(E_t - E_{ON})/k_B \theta} + (1 - \alpha) e^{(E_t - E_{OFF})/k_B \theta}]^2}.$$ 

The relative error assumes a maximum value when the noise is minimum. This fact has already been claimed by some authors [9].

### 4. Results

#### (a) General results

We begin by simulating the noise current for the cyclostationary cases via MCMC simulations based on the main algorithm. In our simulations, we have used $T = 60$ MC steps (the period of cyclostationary excitation) and we considered $E_{ON} = 0.9 \, \text{eV}$. In figure 3, we show the noise current, in a semi-log plot, produced for $n_{MC\text{steps}} = 5000$ and $n_{\text{sample}} = 100$ devices, with $\lambda = 150$ traps.

For $E_{OFF} = 0.6 \, \text{eV}$ and $0.3 \, \text{eV}$, we observe a decrease of the noise when compared to the stationary case (here $E_{ON} = 0.9 \, \text{eV}$) represented by the continuous line (robust logarithmic behaviour). On the other hand, the frequency
Figure 3. Temporal evolution of noise current in the cyclostationary regime. We fixed $E_{ON} = 0.9\,\text{eV}$. On changing $E_{OFF}$, we can observe a decrease of noise current for $E_{OFF} = 0.6\,\text{eV}$ (circles) and $0.3\,\text{eV}$ (squares) when compared with the robust logarithmic behaviour of the stationary case $E_{OFF} = 0.9\,\text{eV}$ (solid line).

Figure 4. Noise reduction ratios for the approach presented in this paper (squares) compared with da Silva et al. [8] (circles; classical reduction predicted by modulation theory). We can see a noise reduction for the cyclostationary cases $E_{OFF} < 0.9\,\text{eV}$ in both cases. However, the reduction is greater using the approach of the current paper, showing a better matching with the expected experimental results from the literature.

domain approach can also be used to show this decrease of the noise. We analysed the noise reduction ratio given by equation (3.2) and the same ratio also was calculated using the previous approach developed in da Silva et al. [8].

In figure 4, we show a comparison of the noise reduction ratio using the current approach with that of da Silva et al. [8]. From the approach developed in da Silva et al. [8], we can see a maximum reduction around 25 per cent that represents the expected reduction predicted by modulation theory (circles in figure 4). Experimental results suggest that the gain must be larger. Our current
Figure 5. (a) Average PSD (more precisely described by \(B(E_{ON}, E_{OFF})\) in equation (3.1)) as a function of \(E_{ON}\) and \(E_{OFF}\). (b) A similar plot for the relative error. We observe that minimum noise corresponds to maximum relative error in PSD.

The approach developed in this paper (squares in figure 4) shows a reduction of almost 60 per cent. To complete our analysis we prepare plots of \(E[S(f)]\) (more precisely expressed by constant \(B\)) and \(e(S)\) as a function of \(E_{ON}\) and \(E_{OFF}\).

Figure 5 depicts the variation of PSD numerically obtained by direct integration of equation (3.1). We can see that a minimum PSD (figure 5a) corresponds to maximum relative error, which is directly obtained by integration of equation (3.3).

(b) Nanoscale effects in semiconductor devices: ballistic effects

Many authors have showed that ballistic or quasi-ballistic transport is highly expected in nanodevices, different from microdevices [2]. However, in the traditional approach, the amplitude distribution (\(\delta\)) is evaluated assuming the macroscopic behaviour modelled through the mobility. Therefore, for the mobility equation to be valid, the number of scattering events along the electron path must be large, and the energy gained from the electron field is small, if compared with the thermal energy. For nanoscale devices, the path from source to drain is on the nanometre scale, and the electrical field is extremely high.

Thus, in order to predict these effects, we can look at the amplitude distribution (\(\delta\)). A simple hypothesis used by other authors is to consider that \(\delta\) in a capture depends linearly on \(x\) (depth of the trap in the oxide—see figure 1). By this hypothesis (supported by [13]), we can show that

\[
\delta(x, y) = \frac{e}{WLC_{ox}} \left(1 - \frac{x}{T_{ox}}\right) f(y),
\]
where here we conjecture that \( f(y) \) is the amplification of discontinuity caused by different positions in the \( y \)-direction. Here \( W, L \) and \( T_{\text{ox}} \) are the dimensions of the device (figure 1), \( C_{\text{ox}} \) is the capacitance of the oxide and \( e \) is the fundamental electron charge.

In nanodevices, electrons are terminally injected and now are ballistically or quasi-ballistically accelerated, and in the worst case (ballistic) we can show that the dependence on \( y \) is given by

\[
f(y) = \sqrt{1 + \frac{e V_{\text{DS}}}{k_B \theta} y}.
\]

Here, \( V_{\text{DS}} \) corresponds to the voltage from drain to source (as in figure 1). Supposing that traps and electrons are uniformly distributed in space (over the oxide and along the drain current, respectively), we have a simple integration:

\[
\langle \delta \rangle_{\text{ballistic}} = \frac{e}{W L C_{\text{ox}}} \int_0^{T_{\text{ox}}} \left( 1 - \frac{x}{T_{\text{ox}}} \right) \frac{1}{L} \int_0^L \left( 1 + \frac{e V_{\text{DS}}}{k_B \theta} y \right)^{1/2} \, dy
\]

\[
= \frac{k_B \theta}{3 V_{\text{DS}} W L C_{\text{ox}}} \left[ \left( 1 + \frac{e V_{\text{DS}}}{k_B \theta} \right)^{3/2} - 1 \right].
\]

So we can estimate the amplification factor of the ballistic (nanoscale) regime in relation to the non-ballistic (supposing a micro–mesoscale transistor) in the current of RTS:

\[
\gamma = \frac{\langle \delta \rangle_{\text{nano}}}{\langle \delta \rangle_{\text{micro}}} = \frac{2 k_B \theta}{3 e V_{\text{DS}}} \left[ \left( 1 + \frac{e V_{\text{DS}}}{k_B \theta} \right)^{3/2} - 1 \right].
\]

We obtain \( \gamma \approx 3 \) for room temperature \( \theta = 293 \text{ K} \) and a drain–source voltage \( V_{\text{DS}} = 0.5 \text{ V} \), i.e. on average we have three times more noise when the ballistic regime is assumed.

Although this amplification factor of the current is highly pictorial, since many other points are not considered in this approximation, several works have claimed catastrophic effects in nanoscale CMOS (e.g. [2]) and in other nanodevices that correspond, even qualitatively, to our prediction. In another type of semiconductor device, the so-called single-wall carbon nanotube field-effect transistors, Liu et al. [4] present results claiming ‘giant telegraph signals’ and the amplitude of the RTS is up to 60 per cent of the total current. So the noise reduction techniques presented here, which consider cyclostationary inputs for the Fermi level in semiconductor devices, will certainly be extremely important to correct the anomalous distortions caused by nanoscales.

Therefore, in both domains (frequency and time), our approach shows a noise reduction in the cyclostationary regime. These results have motivated many applied scientists and the microelectronics industry (actually micro–meso–nanoelectronics) to invest in circuits that consider periodic excitations in order to obtain more reliable devices that make use of nanodimensions.

*Phil. Trans. R. Soc. A* (2011)
5. Conclusions

We have presented a new model for the time and frequency domain that considers a cyclostationary square excitation for the gate voltage ($V_{GS}$). This makes the Fermi level a discontinuous periodic function described by a square wave. Mathematically, we consider that traps in a random medium (the interface between oxide and semiconductor) generate an independent, but collective, Poisson process where the rates are time-dependent periodic functions. The superposition of these collective Poisson processes generates a noise in the drain current that causes inefficiency in the functioning of some important circuits based on CMOS devices (CMOS transistors).

Our approach models a decrease of noise in several aspects. In the time domain, based on a logarithmic law for the stationary case, we observe deviations from this law when the Fermi level is a time-dependent periodic function using an MCMC algorithm. In the frequency domain, we analyse the variations of the Fermi level in the PSD (the Fourier transform of the autocorrelation) extended for all traps.

We show that minimum noise corresponds to a maximization of the relative error in the PSD. Finally, we obtain estimates for prediction of the amplification of noise in the current when the ballistic regime (nanotransistors) is considered. This model also handles the ballistic regime, relevant in nano- and mesoscale devices. We also show that the ballistic regime leads to nonlinearity in the electronic behaviour. We believe our study presents a theoretical framework for an important topic relevant in applied physics: the physics of RTS micro- and nanoscale semiconductor devices.

This research is partly supported by the Brazilian Research Council CNPq.

References


