The role of coherent vortices near the turbulent/non-turbulent interface in a planar jet

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The role of coherent vortices near the turbulent/non-turbulent (T/NT) interface in a turbulent plane jet is analysed by a direct numerical simulation (DNS). The coherent vortices near the jet edge consist of large-scale vortical structures (LSVSs) maintained by the mean shear and intense vorticity structures (IVSs) created by the background fluctuating turbulence field. The radius of the LSVS is equal to the Taylor micro-scale \( R_{\text{svs}} \approx \lambda \), while the radius of the IVS is of the order of the Kolmogorov micro-scale \( R_{\text{ivs}} \sim \eta \). The LSVSs are responsible for the observed vorticity jump at the T/NT interface, being of the order of the Taylor micro-scale. The coherent vortices in the proximity of the T/NT interface are preferentially aligned with the tangent to the T/NT interface and are responsible for the viscous dissipation of kinetic energy near the T/NT interface and to the characteristic shape of the enstrophy viscous diffusion observed at that location.

Keywords: turbulent flows; turbulent jets; turbulent entrainment; coherent vortices

1. Introduction

Turbulent entrainment is one of the most important features observed in free shear flows, such as mixing layers, wakes and jets, and governs some of the most important characteristics of these flows such as their spreading and mixing rates. Turbulent entrainment takes place across a sharply contorted interface—the turbulent/non-turbulent (T/NT) interface—that divides the T from the NT or irrotational flow regions [1]. Understanding of the physical mechanisms taking place at the T/NT interface is important to many natural and engineering flows since important exchanges of mass, momentum and passive or active scalar quantities, e.g. heat, take place across the T/NT interface.

The important role played by the coherent vortices in the context of the turbulent entrainment mechanism was recognized a long time ago [2]. Past studies assumed that turbulent entrainment is mainly caused by large-scale ‘engulfing’ motions induced by the coherent vortices in the vicinity of the T/NT interface [2]. It is clear that these structures drive many of the important process that take place near a T/NT interface [3–6]. Specifically, it is thought that the entrainment rate is somehow imposed by the motion of these large-scale eddies near this interface.

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However, it has been shown recently that the entrainment is mainly caused by small-scale eddy motions (‘nibbling’) acting on the T/NT interface [7,8], as suggested originally by Corrsin & Kistler [1]. This recent discovery motivated a number of investigations on the mechanisms behind the large- and small-scale features responsible for the turbulent entrainment.

Bisset et al. [3] observed that the vorticity components undergo a sharp jump at the T/NT interface. Westerweel et al. [8,9] observed the existence of a jump in the streamwise velocity and passive scalar field across this interface, and confirmed that the engulfment is not the dominating process for the entrainment. They also analysed the dynamics of the flow relative to the T/NT interface and investigated the eddy viscosity near that location. Holzner et al. [10,11] analysed the dynamics of the enstrophy and strain near a T/NT interface generated by an oscillating grid. They observed the existence of an intense kinetic-energy dissipation outside the turbulent region near the T/NT interface, and also found that the net effect of viscosity at that location causes an increase of the total enstrophy. Subsequently, it was observed by Holzner [11] and da Silva & Pereira [12] that it is the viscous diffusion which is responsible for this positive viscous contribution to the enstrophy, while the enstrophy viscous dissipation remains negative across the jet, as expected. Recently, Holzner et al. [13] decomposed the entrainment velocity into two terms: a viscous and an inviscid term. It was observed that the viscous term dominates the entrainment velocity and that its magnitude is comparable to the magnitude of the local Kolmogorov velocity. The universal small-scale features such as the geometry of the dissipation, the geometry of the straining (or deformation) of the fluid elements and the behaviour of the invariants of the velocity gradient tensor—$Q$ and $R$—during the entrainment process were studied by da Silva & Pereira [14,15]. They observed that the characteristic ‘tear drop’ shape in the $(R, Q)$ map is not formed at the T/NT interface, but that it needs about one Taylor micro-scale into the turbulent region in order to form completely. Finally, the challenges faced by the subgrid-scale models near the T/NT interface were analysed by da Silva [16]. The classical subgrid-scale models may need particular modifications near the edge of a jet if fine capturing of the Reynolds stresses at this location is required, e.g. in order to predict accurately the mixing rates near the T/NT interface of a jet.

However, many aspects associated with the characteristics and dynamics of the T/NT interface remain unknown. One of these aspects consists of the existence of a viscous super-layer where viscous processes dominate, as suggested a long time ago by Corrsin & Kistler [1]. A related problem concerns the thickness of the sharp vorticity jump observed at the T/NT interface. Reynolds [17] predicted that a vorticity jump is to be expected to occur at the T/NT interface. However, the values observed for the thickness of this jump vary in the literature, e.g. the thickness of the turbulent front generated from an oscillating grid is of the order of the Kolmogorov micro-scale [10], whereas the experimental results from round jets by Westerweel et al. [8,9] and the numerical simulations of plane jets by da Silva & Pereira [14,15] show clearly that the thickness of the vorticity jump at the T/NT interface is of the order of the Taylor micro-scale.

Another issue concerns the role of the coherent vortices near the T/NT interface in jets and wakes, as opposed to mixing layers [9]. Recent work has addressed the flow motion induced by the presence of these structures near the jet edge. Bisset et al. [3] studied the shape of the streamlines caused by the large-scale
motions near the T/NT interface. They observed that the streamlines of the *entrainment wind* only cross the T/NT interface where they are normal to the interface surface. The characteristics and dynamics of the shear layers bounding the turbulent and the irrotational flow regions in shear flows are determined by the presence of the nearby vortices. In this context, many useful estimates were derived using linearized calculations by Hunt et al. [4–6]. Recently, Hunt et al. [18] discussed the structure of the shear layers between adjacent eddies. The process of ‘nibbling’ is for the first time described as a two-scale process whereby the vortices that lie within a shear layer with thickness equal to the Taylor micro-scale undergo local straining, causing their thickness to be of the order of the Kolmogorov micro-scale. However, many aspects of the dynamics of these structures are still unclear. If the entrainment rate is imposed by the large scales, but ultimately is felt at the small-scale ‘nibbling’ motions, how exactly does this process occur i.e. how do the large- and small-scale motions interact to cause the turbulent entrainment?

The goal of the present work is to shed light into the role of the coherent vortices in a turbulent jet near the T/NT interface. For this purpose, a direct numerical simulation (DNS) of a turbulent plane jet will be used. The coherent vortices of jets share many common features with the structures commonly found in other free shear layers such as azimuthal tube vortices (generated by the Kelvin–Helmholtz instability in the case of jets and mixing layers), and the existence of streamwise secondary vortices between these primary structures whose ‘foot prints’ are still discernible at the far-field fully developed turbulent state. Therefore, it is expected that the results obtained in the present study display some universal qualitative features of the turbulent entrainment that exist in free shear flows in general.

This article is organized as follows. In §2, we describe the plane-jet DNS used in the present work and the procedure used here to obtain the conditional statistics in relation to the T/NT interface and to detect the intense vorticity structures (IVSs). Section 3 describes the results. These concern (i) the thickness of the vorticity jump near the T/NT interface, (ii) the orientation of the coherent vortices in respect to the surface defining the T/NT interface, (iii) the kinetic-energy viscous dissipation near the jet edge, and (iv) the ‘anomalous’ enstrophy viscous diffusion near the T/NT interface. The work ends with an overview of the main results and conclusions.

2. Direct numerical simulation of a turbulent plane jet

In the present work, a DNS of a turbulent plane jet is used to analyse the effect of the coherent structures near the T/NT interface. This DNS is described in detail by da Silva & Pereira [14] and da Silva [16].

The Navier–Stokes solver uses a pseudo-spectral scheme for spatial discretization, and a third-order, three-step, Runge–Kutta scheme for temporal advancement. The number of collocation points along the streamwise (*x*), normal (*y*) and spanwise (*z*) directions is equal to \((N_x \times N_y \times N_z) = (256 \times 384 \times 256)\), and the extent of the computational domain attains \((L_x, L_y, L_z) = (4H, 6H, 4H)\), where \(H\) is the initial jet slot width.
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Figure 1. (a) Sketch of the T/NT interface indicating the vorticity surface (solid line) and the interface envelope position $Y_I(x)$ (dashed line), with the coordinate system of the plane jet ($x, y$) and the one used in the conditional statistics ($y_I$). The ‘hole’ represents a region of irrotational fluid inside the turbulent region and is removed from the statistical sample. (b) Mean conditional profiles of $\langle|\omega|\rangle_I$, $\langle|\omega_x|\rangle_I$, $\langle|\omega_y|\rangle_I$ and $\langle|\omega_z|\rangle_I$ (all normalized by $U_1/H$).

Results from the validation for this simulation showed that the present DNS is accurate at the large and small scales of motion and representative of a fully developed turbulent plane jet [14]. The self-similar regime is obtained at $T/T_{\text{ref}} \approx 20$, where $T_{\text{ref}} = H/(2U_x)$ and $U_x$ is the initial velocity at the centreline. At the self-similar regime, the Reynolds number based on the Taylor micro-scale $\lambda^2 = \langle u'^2 \rangle / \langle (\partial u/\partial x)^2 \rangle$, and on the root mean square of the streamwise velocity $u' = \langle u'^2 \rangle^{1/2}$ is equal to $Re_\lambda = u'\lambda/\nu \approx 120$.

(a) Detection of the turbulent/non-turbulent interface: conditional statistics in relation to the distance from the turbulent/non-turbulent interface

The T/NT interface is detected using a procedure based on the vorticity norm threshold. A similar procedure was used in several previous works (e.g. [3,8,9,14,16]), and is briefly outlined here since it was already described in detail in da Silva & Pereira [14] and da Silva [16].

The sketch in figure 1a shows the T/NT interface separating the turbulent and the irrotational flow regions at the upper shear layer of the plane jet. In the present work, the T/NT interface location $Y_I(x)$ is defined using the vorticity norm $\omega = (\omega_x, \omega_y)^{1/2}$, where $\omega_i = \nabla \times u_i$ is the vorticity field and the detection threshold is $\omega = 0.7U_1/H$, which is the same threshold used in Bisset et al. [3] and Mathew & Basu [7]. The envelope location $Y_I(x)$ is determined using a linear interpolation along the $y$ direction, for each one of the $N_x$ grid points along the $x$ direction in the original coordinate system. The conditional statistics are made in a new (local) coordinate system ($y_I$), which is defined at the interface location. In this coordinate system, the T/NT interface is at $y_I = 0$, while the irrotational and turbulent regions are defined by $y_I < 0$ and $y_I > 0$, respectively. ‘Holes’ of ‘ambient fluid’ that appear inside the jet, as well as ‘islands’ of rotational fluid inside the turbulent region are removed from the statistical sample. The same procedure is used also for the lower shear layer and the statistics use each one of the $N_z$
spanwise planes and \( N_T = 11 \) instantaneous fields taken from the fully developed turbulent regime between \( T/T_{\text{ref}} = 20.2 \) and 27.0. The conditional statistics are denoted by \( \langle \rangle \).

Figure 1b shows conditional mean profiles of \( \langle |\omega_x| \rangle \), \( \langle |\omega_y| \rangle \) and \( \langle |\omega_z| \rangle \) in relation to the distance from the T/NT interface, showing a sharp jump across the T/NT interface with thickness roughly equal to \( \delta_\omega \sim 20\eta \). For the present simulation inside the turbulent region, we have \( \lambda \approx 20\eta \), where \( \lambda \) is the Taylor micro-scale; therefore, this jump is equal to the \( \delta_\omega \sim \lambda \), in agreement with other experimental and numerical work [8,9,19].

(b) The coherent vortices near the turbulent/non-turbulent interface

In turbulent flows, the regions of concentrated vorticity consist of either (i) vortical tube-like structures (vortex tubes) or in (ii) structures exhibiting a sheet-like shape (vortex sheets) [20]. Whereas in the tube-like structures, vorticity and strain are comparable, the sheet-like structures are dominated by vorticity with relatively low levels of strain. The sheet-like structures exhibit smaller lifetimes than the tube structures, and therefore it is probable that the tubes will have more possibilities of influencing the dynamics of the flow near the T/NT interface than the relatively short-lived sheet structures [21].

For this reason in the present paper, we focus on the tube structures and we divide these structures into two classes: IVSs and large-scale vortical structures (LSVSs).

The IVSs are defined as structures with particularly strong vorticity, i.e. structures comprised of flow points with vorticity greater than a particularly high vorticity threshold. Jiménez et al. [22] and Jiménez & Wray [23] defined this vorticity threshold as equal to the vorticity defining the 1 per cent of flow points with the highest vorticity. In isotropic turbulence, the IVSs are the well-known ‘worms’ described by Siggia [24]. The IVSs are similar in many different flows such as mixing layers, wakes, jets and also boundary layers, ducts and channel flows [25–28]. For instance, the IVSs in these flows do not exhibit any particular spatial orientation and their vortex core radius is \( R_{\text{ivs}}/h \approx 4–5 \), where \( h \) is the Kolmogorov micro-scale.

We define by LSVSs all the remaining vortex structures, i.e. structures of concentrated vorticity with tubular shape with vorticity smaller than the IVSs. The LSVSs are the largest vortical structures that are present in a particular flow. Often originating in the particular instabilities from that flow, their characteristics such as the vortex core radius, azimuthal velocity and lifetime are deeply related to these processes, and therefore are quite different from flow to flow. However, the dynamics of these structures share some common features with LSVSs from other turbulent flows, e.g. they consist of structures with roughly tubular shape and are approximately governed by the same simple inviscid laws.

Although there is some arbitrariness in these definitions, the separation between the two classes of vortices is important since the two types of structures are fundamentally different.

Figure 2 displays both the LSVSs and the IVSs near the T/NT interface for the present turbulent plane jet simulation. The flow coherent structures from the simulated jet are similar to many previous DNSs of turbulent plane jets (e.g. [29,30]). The LSVSs are identified here using low-pressure iso-surfaces.
Figure 2. Large-scale vortical structures (LSVSs) and intense vorticity structures (IVSs) near the T/NT interface from the upper shear layer of the planar turbulent jet (the flow is from left to right): the T/NT interface (translucent orange) displays a contorted shape dictated by the LSVSs underneath its surface. The LSVSs are defined by low-pressure iso-surfaces (white), while the IVSs (solid yellow iso-surfaces) are defined by points where the vorticity $\omega = |\omega|$ is above the vorticity threshold $\omega_{tr}$, which is equal to the vorticity of the points with highest enstrophy that are contained in 1% of the total volume. The radii of the IVSs shown here correspond to the exact radii of these structures. (Online version in colour.)

The LSVSs (white) consist of the classical big rollers and of the streamwise vortices that are known to exist in a turbulent plane jet. The rollers in particular can be seen as remnants of the Kelvin–Helmholtz vortices generated during the transition to turbulence in the jet, while the streamwise vortices correspond to the secondary instabilities. A very large Kelvin–Helmholtz roller can be seen at the right-hand side of the figure, extending through the whole height of the figure, with several legs from streamwise vortices that are seen to be emerging from this roller. The observed radius of the bigger LSVSs, i.e. the rollers, is close to the Taylor micro-scale $R_{lsvs} \approx \lambda$. Notice that the LSVSs not only exhibit some level of preferential spatial orientation close to

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the T/NT interface, but also are clearly responsible for the characteristic size of
the convolutions observed on the T/NT interface, i.e. the observed convolutions
of the T/NT interface are clearly associated with the presence of a nearby
LSVS underneath. Consequently, the average length of these convolutions is
deeply influenced by the geometry of the LSVSs. Specifically, the radius of these
structures is clearly proportional to the average length of the convolutions of the
T/NT interface, in agreement with previous observations (e.g. [2]).

The IVSs (yellow) are identified here using a similar tracking procedure as
the one developed in Jiménez et al. [22] and Jiménez & Wray [23], where the
IVSs are defined as consisting of points where the vorticity \( \omega = |\boldsymbol{\omega}| \) is above a
particular vorticity threshold \( \omega_{tr} \) defining the points with highest enstrophy that
are contained in 1 per cent of the total volume. Once the axis points for the IVSs
were identified, the IVS radius \( R \) for any point on the axis is calculated by fitting
the axial distribution to \( \omega(r) = \omega_0 e^{-r^2/R^2} \), where \( \omega_0 \) is the value of vorticity at
the axis point. As can be seen in figure 2, the IVSs exhibit a much smaller radius
than the LSVSs and also a more ‘random’ spatial orientation. The calculated core
radius of the observed IVS is \( R_{ivs} \approx 4.6 \eta \), where \( \eta \) is the Kolmogorov micro-scale,
which agrees with the values observed in isotropic turbulence [22, 23]. Notice,
however, that there is of course some spatial overlap between the LSVSs and the
IVSs since the IVSs are extreme events of LSVSs. It is interesting to observe that
this figure resembles the sketch of the LSVSs and IVSs described in Westerweel
et al. [9], where the IVSs are described as structures surrounding the LSVSs near
the jet edge.

Following a suggestion from Javier Jiménez (2009, private communication), we
argue here that the T/NT interface in fact consists of (or is made up from) the
borders of the coherent vortices sitting at that location. Notice that the vortex-
sheet structures mentioned in §1 are also part of the T/NT interface, but their
contribution to the T/NT interface statistics is small due to their smaller lifetime
and coherence compared with the vortices. For the same reason, we expect the
geometrical characteristics of the T/NT interface to be more dominated by the
LSVSs than the IVSs simply because the bigger the radius, the bigger the lifetime
of a given vortex, and thus the bigger the chances of defining the geometry and
statistics of the T/NT interface. In the same vein, we expect the larger of the
LSVSs, e.g. the rollers, to have more influence than the smaller LSVSs, i.e. the
streamwise vortices. However, instantaneously, all the ranges of LSVSs and IVSs
of course contribute to the definition and characteristics of the T/NT interface.
Figure 2 supports this view in that it shows that the geometry of the T/NT
interface is more influenced by the larger LSVSs than by the smaller LSVSs or
the IVSs lying underneath its surface.

In order to show this, figure 3a,b is used to show that the IVSs have only a
limited influence on the characteristics of the T/NT interface, i.e. ‘something
else’ has to exist near the jet edge in order to cause the observed T/NT interface
characteristics such as its thickness, which, as we saw, is equal to the Taylor
micro-scale \( \delta_\omega \approx \lambda \). Figure 3a shows the Gaussian vorticity profile used to fit the
vorticity distribution for three different IVSs with \( R/\eta = 4.5 \) and 6 using the
method described in Jiménez & Wray [23]. \( l_\omega \) is defined as the distance from the
axis of the IVS to the point where the vorticity equals the threshold used here
to defined the T/NT interface, e.g. for an IVS with a radius equal to \( R/\eta = 6 \),
the radial distance \( r \) to which the vorticity \( \omega(r) \) is equal to the threshold used to
defined the T/NT interface is $l_\omega/\eta \approx 12$. Figure 3b shows the conditional profile for $l_\omega$, where $l_\omega$ is computed for each one of the detected IVSs. Inside the region $0 < y_I/\eta < 20$, the conditional profile of $l_\omega$ is approximately linear; however, the slope of this line is not equal to 1, e.g., $\delta l_\omega/\delta y_I \approx 0.2 \neq 1$. Therefore, we conclude that the geometry of the T/NT interface is not imposed by the shape of the IVS. On the contrary, this result suggests that these vortices have a small impact on the determination of the vorticity jump across the T/NT interface. In line with this, conditional statistics of the number of detected IVSs as a function of the distance from the T/NT interface were determined (not shown), and indicate that no IVS axis exists between the T/NT interface and a distance of roughly 5$\eta$ into the interior of the turbulent region. This is not surprising since the radius of the IVS is of the order of 5$\eta$, naturally only IVSs with radii much smaller than 5$\eta$ could fit in the region defined by $0 < y_I/\eta < 5$.

These results agree with the visualizations shown in figure 2, which further support the idea that the T/NT interface is, in fact, the physical line defined by the borders of the range of vortices near the edge of the jet, from LSVSs down to IVSs; however, the geometry of the T/NT interface is likely to be more affected by the bigger LSVSs than to the smaller LSVSs or the IVSs, since the bigger vortices evolve slowly and have longer lifetimes and thus are able to influence the T/NT interface in a more permanent way.

3. The role of coherent vortices near the turbulent/non-turbulent interface

This paper focuses on the effect of the LSVSs and IVSs on the turbulence dynamics and characteristics of the flow near the T/NT interface. The next subsections address the thickness of the vorticity jump, orientation of the coherent vortices, energy viscous dissipation and enstrophy viscous diffusion near the T/NT interface.

(a) The thickness of the vorticity jump at the turbulent/non-turbulent interface

An interesting and long-standing issue regarding the T/NT interface has been the magnitude of its thickness $\delta_\omega$. In the original work by Corrsin & Kistler [1], it was postulated that a viscous super-layer must exist outside the T/NT interface.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{(a) Sketch explaining the determination of the theoretical distance $l_\omega$ and (b) conditional profile of $l_\omega$. Solid line, $R/\eta = 6$; dashed line, $R/\eta = 5$; dotted line, $R/\eta = 4$.}
\end{figure}
where the viscous diffusion of vorticity responsible for the growth of the shear layer takes place. Assuming that this thickness depends only on the viscosity $\nu$ and on the enstrophy inside the shear layer $\omega'$, simple dimensional reasoning yields $\delta_\omega = \delta_\omega(\nu, \omega') = \sqrt{\nu/\omega'} = (\nu^3/\epsilon)^{1/4} = \eta$. Holzner et al. [10,11] showed that in the T/NT interface originated from an oscillating grid, the thickness of the vorticity jump is of the order of the Kolmogorov micro-scale; however, both the experimental results in round jets from Westerweel et al. [8,9] and the DNS of planar jets by da Silva & Pereira [14,15] show that the thickness of this jump is equal to the Taylor micro-scale.

A justification for these two apparently conflicting results can be given if one realizes that the T/NT interface is largely made up from the coherent vortices ‘sitting’ near the T/NT interface. Arguably, the T/NT interface is ‘made up’ both from vorticity from the whole range of eddies near the T/NT interface and from some ‘incoherent’ vorticity that is shed by these vortices as they travel into the T/NT interface. However, this ‘incoherent vorticity’, as is well known, is short lived, i.e. its lifetime is small compared with the lifetime of the coherent vortices, and moreover it is difficult to assign a proper length scale to these ‘islands’ of vorticity ‘lost’ by the eddies. As explained before from all the sea of eddies defining the T/NT interface, the bigger eddies from the class defined here as LSVSs should be the ones responsible for the T/NT interface characteristics, since they are more permanent, evolve slower than the other vortices and are associated with larger lifetimes.

Assuming that the length scale of this vorticity layer caused by the vortices $\delta_\omega$ depends only on the viscosity and on the strain rate acting on the LSVSs, we can write $\delta_\omega = \delta_\omega(\nu, S)$, where $S$ is the strain rate acting on the LSVSs. It can be argued that the magnitude of the strain rate acting on an LSVS near the jet edge is $S \sim u'/L_{11}$, where $u'$ is the fluctuating velocity field and $L_{11}$ is the integral scale of turbulence [4,18]. If the Reynolds number associated with the integral scale is $Re_0 = u'L_{11}/\nu$, the length scale associated with this vorticity jump caused by the vortices is then equal to

$$\delta_\omega = \sqrt{\frac{\nu}{u'/L_{11}}} = L_{11} Re_0^{-1/2} \sim \lambda. \quad (3.1)$$

In a situation such as the flow generated in an oscillating grid [10], no mean shear exists, and therefore the coherent vortices should be similar to the ‘worms’ observed in simulations of isotropic turbulence. For these structures, the strain rate acting on the vortices comes mainly from the background turbulence field and is equal to $S \sim u'/\lambda$. Therefore, the associated length scale of the vorticity jump, i.e. the scale of the vortices near the T/NT interface is equal to

$$\delta_\omega = \sqrt{\frac{\nu}{u'/\lambda}} = \lambda Re_0^{-1/2} \sim \eta, \quad (3.2)$$

where $Re_0 = u'\lambda/\nu$ is the Reynolds number based on the Taylor micro-scale. Thus, the problem of determining the scale of the T/NT interface becomes the problem of estimating the size of the larger coherent vortices existing near the edge of the T/NT interface.

An alternative solution can be formulated if one writes from the outset that the vorticity jump observed at the T/NT interface is caused exclusively by the LSVSs from the flow: $\delta_\omega \approx Re_{lev}$. Since, for long-lived vortices, the time scale associated
with the radial viscous diffusion of vorticity is roughly balanced by the axial stretching caused by the local strain-rate field, $S$, acting on the vortices—as in a Burgers vortex—the radius of the LSVS has to be of the order of the Burgers radius, $R_{\text{LSVS}} \approx R_B = (v/S)^{1/2}$. In a jet, the magnitude of the strain rate acting on an LSVS near the jet edge is $S \sim u'/L_{11}$, leading to $R_{\text{LSVS}} \sim \sqrt{v/(u'/L_{11})} = L_{11}Re_0^{-1/2} \sim \lambda$, whereas in a very high Reynolds number jet or in a shear free flow, where the only visible structures are IVSs (since the LSVSs are non-existent or are too much fragmented), the strain is imposed only by the background turbulence and is $S \sim u'/\lambda$, whereby $R_{\text{IVS}} \sim \sqrt{v/(u'/\lambda)} = \lambda Re_\lambda^{-1/2} \sim \eta$.

(b) The orientation of the coherent vortices with respect to the turbulent/non-turbulent interface

In this section, we study the orientation of the IVSs in relation to the surface defining the T/NT interface. The IVSs are a particular set of vortices taken from the sea of eddies of different sizes, but since the LSVSs were often seen to have IVSs at their core, this orientation reflects to some extent the orientation of the LSVSs. A similar study, but focusing on the ‘whole’ vorticity field, as opposed to the vorticity contained on the axis of the LSVSs was carried out by Holzner et al. [13].

The orientation of the IVSs with respect to the T/NT interface is studied through the conditional statistics and the probability density function (PDF) of $\cos(\theta_0) = \omega_0 \cdot n_{\text{int}} / (|\omega_0| \cdot |n_{\text{int}}|)$, where $\omega_0$ is the axial vorticity vector for an identified IVS and $n_{\text{int}}$ is the local tangent to T/NT interface in the $(x, y)$-plane containing this IVS axis. The conditional mean profile of $\cos(\theta_0)$ is shown in figure 4a. Figure 4b shows the PDF of $\cos(\theta_0)$ using data from three sets of data, each containing information from three intervals (distances) from the T/NT interface: $[5,10]\eta$, $[30,40]\eta$ and $[90,100]\eta$.
Very close to the T/NT interface at \( y_I/\eta = 5\textendash 10 \), we have \( \langle \cos(\theta_0) \rangle_I \approx 0.8 \) (figure 4a), which shows that the IVSs are strongly aligned with the local tangent to the T/NT interface. The PDFs of \( \cos(\theta_0) \) confirm the existence of a strong tendency for an alignment between the IVSs and the tangent to the T/NT interface, and show that this trend weakens as the centre of the jet shear layer is approached.

These results are not surprising: since the IVSs cannot exist outside the turbulent region and cannot end inside the fluid (Helmholtz’s second theorem), the only vorticity components that can exist very close to the T/NT interface are either \( \omega_x \) or \( \omega_z \), but not \( \omega_y \). The fact that \( \cos(\theta_0) > 0.5 \) inside the turbulent region can be explained by the existence of some remaining legs from streamwise vortices. These results are also consistent with the conditional mean vorticity profiles described in figure 1b. Furthermore, the strong tendency for an alignment of the IVSs with the tangent to the T/NT interface is consistent with the anisotropy of the viscous-dissipation rate observed by da Silva & Pereira [14]: the presence of dominating axial and azimuthal vortices near the T/NT interface implies that the most intense local gradient near the interface is \( \partial u/\partial y \), since the axial vortices emerge in opposite pairs, cancelling their effect on the velocity gradient near the T/NT interface. From this, it follows that \( S_{12}^2 > S_{13}^2 \approx S_{23}^2 \), as observed by da Silva & Pereira [14], who support the idea that the ‘irrotational’ viscous dissipation observed near the T/NT interface is induced by large-scale eddy motions near the jet edge. This is further supported by the analysis of the LSVSs near the T/NT interface, as described below, and agrees with the recent results reported by Holzner et al. [13] (see also [31]).

(c) The kinetic-energy viscous dissipation near the turbulent/non-turbulent interface

An interesting issue regarding the dynamics of the turbulent quantities near the T/NT interface is the existence of non-negligible kinetic-energy viscous dissipation, \( \varepsilon = 2\nu S_{ij}S_{ij} \), outside the turbulent region and very close to the T/NT interface, as indicated by the experimental results from Holzner et al. [10,11] and the numerical simulations from da Silva & Pereira [14]. The ‘irrotational dissipation’ can be seen as ‘anomalous’ since no viscous effects can exist in irrotational flow, as described below. Notice however, that in practice, both in the experiments and in the simulations cited above, the vorticity is different from zero in the irrotational flow because the T/NT interface is defined by points where the vorticity is greater than a given (although small) vorticity threshold.

It can be argued that no viscous dissipation can exist inside a potential flow region since [32, p. 77] the Navier–Stokes equations can be written as

\[
\frac{\partial u_i}{\partial t} = -\frac{\partial}{\partial x_i} \left( \frac{p}{\rho} + \frac{1}{2} u_j u_j \right) + \varepsilon_{ijk} u_j \omega_k - \nu \varepsilon_{ijk} \frac{\partial \omega_k}{\partial x_j},
\]

where the last term on the right-hand side of the equation is responsible for viscous effects. This equation shows that viscous effects are absent from irrotational flow since \( \omega_k = 0 \), implying that the last term of the above equation is zero inside the irrotational flow region. Notice however, that the governing
equation for the kinetic energy can be obtained by multiplying $u_i$ by the above equation, which yields

$$\frac{\partial}{\partial t} \left( \frac{1}{2} u_i u_i \right) = - \frac{\partial}{\partial x_i} \left( \frac{u_ip}{\rho} + \frac{1}{2} u_i u_j u_j \right) + \varepsilon_{ijk} u_i u_j \omega_k - \nu \varepsilon_{ijk} u_i \frac{\partial \omega_k}{\partial x_j}, \quad (3.4)$$

where again the effects of molecular viscosity are contained in the last term on the right-hand side of the equation. Recall that in a turbulent flow, the molecular viscosity is always associated with two effects: viscous diffusion and viscous dissipation. Indeed, what the last equation implies is that the sum of these two effects must be zero in an irrotational flow. Therefore, the effect of viscous dissipation alone is not excluded from the equations. However, for viscous dissipation to be active, the net result of the viscous diffusion has to be positive. At least for the enstrophy, a positive net viscous-diffusion contribution has been observed to exist near the T/NT interface inside the irrotational region by Holzner et al. [11] and da Silva & Pereira [12]. A possible explanation for this ‘anomalous dissipation’ has been suggested by da Silva & Pereira [14]: the analysis of the individual terms contained in $S_{ij} S_{ij}$ in the plane turbulent jet has showed that the observed viscous dissipation is anisotropic outside the turbulent region, which is consistent with it being caused by the large-scale motions imposed by the LSVSs near the T/NT interface (see [14]). This picture is confirmed here in figure 5a,b. Figure 5a shows contours of intense values of viscous dissipation $\varepsilon$ near the T/NT interface and the LSVSs of the flow. The visualizations do confirm that intense values of $\varepsilon$ near the T/NT interface tend to occur whenever nearby LSVSs are present. Figure 5b shows instantaneous profiles of $\varepsilon$ and $|\omega|$ across the shear layer. Many similar
Figure 6. (a) Conditional profiles of enstrophy viscous diffusion—$T_4 = \nu(\partial^2/\partial x_l \partial x_l)(1/2\omega_i \omega_i)$—in relation to the distance from the T/NT interface. (b) Profile of a model LSVS (equation (3.7)) and the corresponding values of enstrophy viscous diffusion $\nu(\partial^2/\partial x_l \partial x_l)(1/2\omega_i \omega_i)$ (equation (3.8)). (c) Instantaneous profile of vorticity norm and enstrophy viscous diffusion near the T/NT interface at the upper shear layer. (a) Solid line; $\langle|\omega|\rangle_I$, dashed line, $\langle T_4 \rangle_I$. (b) Solid line, $\omega_{\text{diff}}(r)/\omega_0$; dashed line, $\omega(r)/\omega_0$. (c) Solid line, $|\omega| \times 10$; dashed line $T_4$.

instantaneous profiles for these quantities were observed. The figure shows a clear link between the vorticity field associated with a strong LSVS near the T/NT interface and intense values of dissipation originated in the pure shear motion caused by the LSVSs. Thus, the observed ‘irrotational’ dissipation originates in the vicinity of the LSVSs that are close to the T/NT interface.

(d) The ‘anomalous’ enstrophy viscous diffusion near the turbulent/non-turbulent interface

Another interesting issue concerning viscous effects in the proximity of the T/NT interface concerns the behaviour of the enstrophy viscous diffusion. The transport equation of this quantity is particularly interesting to analyse in the context of the turbulent entrainment since the existence or absence of enstrophy is the most distinguished feature separating the turbulent from the non-turbulent flow regions [1]. The enstrophy transport equation is given by

$$
\frac{\partial ((1/2)\omega_i \omega_i)}{\partial t} + u_j \frac{\partial ((1/2)\omega_i \omega_i)}{\partial x_j} = \omega_i \omega_j S_{ij} + \nu \frac{\partial^2 ((1/2)\omega_i \omega_i)}{\partial x_j \partial x_j} - \nu \frac{\partial \omega_i}{\partial x_j} \frac{\partial \omega_j}{\partial x_j},
$$

(3.5)

where the two terms on the left-hand side of the equation represent the temporal and local variation of enstrophy and the three terms on the right-hand side represent the enstrophy production, viscous diffusion and viscous dissipation, respectively. In most turbulent flows, the viscous-diffusion term is the least important term in the equation in terms of magnitude. In isotropic turbulence, this term is identically zero. However, near the T/NT interface, this term exhibits a curious shape. Experimental data from Holzner et al. [10,11] and numerical simulations by da Silva & Pereira [12] and da Silva et al. [31] show that the enstrophy viscous-diffusion term is negligibly small in most of the turbulent and irrotational regions, except very close to the T/NT interface, with a negative minimum inside the turbulent region at $y_I/\eta \sim 10$ and a positive maximum right at the T/NT interface at $y_I/\eta \sim 0$ (figure 6a).
The mechanisms responsible for this curious shape have been analysed in detail in Holzner et al. [11] in terms of the relationships between strain and enstrophy. However, a more simple explanation can be given if one recalls that the LSVSs are defining the T/NT interface location. The vorticity radial profile of an LSVS can be well approximated by a Gaussian function,

\[
\frac{\omega(r)}{\omega_0} = e^{-r^2/R^2},
\]  

so that the enstrophy radial profile is

\[
\frac{\omega(r)^2}{\omega_0^2} = e^{-2r^2/R^2}. \tag{3.7}
\]

Consequently, the radial profile of enstrophy diffusion is

\[
\frac{\partial^2}{\partial r^2} \left[ \frac{1}{2} \omega(r)^2 \right] = -\frac{4\omega_0^2}{R^2} e^{-2r^2/R^2} \left( 1 - \frac{4r^2}{R^2} \right). \tag{3.8}
\]

Figure 6b shows the radial profiles corresponding to equations (3.7) and (3.8) for the case \( \omega_0 = R = 1 \) for simplicity. As can be seen, the viscous-enstrophy diffusion term has the same shape observed near the T/NT interface, with negative values inside the vortex and positive values just outside. This is, of course, explained by the mechanics of a diffusion term, removing high values of the diffused quantity where they are above the average (inside the vortex, hence the negative value) and injecting it into regions where it is lower than the average, hence the positive value outside the vortex, representing a net ‘production’ of enstrophy, as described in Holzner et al. [11]. Finally, figure 6c shows an instantaneous profile of a vorticity norm and enstrophy viscous diffusion near the T/NT interface at the upper shear layer. The chosen line passed through the centre of two big vortices that can be easily identified by the local peaks of vorticity. As can be seen, the enstrophy viscous diffusion near these structures exhibit a similar structure as predicted by the model enstrophy and enstrophy viscous diffusion in equations (3.7) and (3.8), with (negative) minima at the centre of the vortices and positive maxima just outside the vortex cores.

In summary, these results show that the so-called ‘anomalous’ enstrophy diffusion observed near the T/NT interface corresponds to the viscous-diffusion process taking place around the LSVSs ‘sitting’ near the T/NT interface. It could be argued that the so-called ‘nibbling’ eddy motions responsible for the turbulent entrainment process consist simply of the viscous diffusion of vorticity from the LSVSs near the T/NT interface, but this has to be assessed carefully in future work.

4. Conclusion

A DNS of a turbulent plane jet at \( Re_\lambda = 120 \) was used to analyse the role of the coherent vortices near the T/NT interface. These structures can be divided into LSVSs maintained by the mean shear and IVSs created by the background turbulence.
The shape of the T/NT interface is defined by the characteristics of these structures since, ultimately, it is these structures that define this interface. The characteristic vorticity jump of the T/NT interface, as well as its thickness, is dictated by the radial vorticity distribution of the bigger LSVs near the T/NT interface. Specifically, the radii of these LSVs are equal to the Taylor microscale and explains why this is also the average thickness of the T/NT interface. The coherent vortices in the proximity of the T/NT interface are preferentially aligned with the tangent to the T/NT interface and are responsible for the viscous dissipation of kinetic energy observed outside the turbulent region.

Finally, it is argued that the ‘nibbling’ eddy motions responsible for the turbulent entrainment are the manifestation of the enstrophy viscous diffusion that exists near the edges of the LSVs ‘sitting’ near the T/NT interface. The lifetime and dynamics of these structures will be the subject of a future study.

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