Interfaces and inhomogeneous turbulence

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A Euromech colloquium, on interfacial processes and inhomogeneous turbulence, was held in London on 28–30 June 2010. Papers were presented describing and analysing the influence of interfaces that separate turbulent/non-turbulent regions, between regions of contrasting fluid properties, or at the edge of boundaries. This paper describes a summary of the work presented, giving a snapshot of the current progress in this area, along with discussions about future research directions.

Keywords: turbulence; interfaces; direct numerical simulations

1. Introduction

Thin, approximately continuous layers and even thinner interfaces that bound these layers have always been known to be characteristic features of turbulent flows separating regions of high and low fluctuations of kinetic energy and at the same time and place regions of high and low concentration of other scalar and vector fields. The importance of these vortical layers in high Reynolds number turbulence was indeed recognized in the discussions on the intermittency of turbulence at the Marseilles conference held in 1961 for many of the founding ‘fathers’ of turbulence [1]. These layers are observed on the edges of clouds or exhaust jets from vehicles or flames (e.g. [2]). Scalar interfaces between regions of high/low concentration are generally also associated with velocity interfaces as is the case with heated jets [3]. Recent research has shown that these layers are significant dynamically because of their internal small-scale motions and their effect on large-scale flows and are not merely passive markers as previously considered. With new experimental methods to measure conditionally sampled velocity fields within and around these interfaces (e.g. [3]), computational power to resolve them with simulations of $4096^3$ grid points [4] and new theoretical models of the turbulent velocity fields across interfaces [5], the dynamics and statistics of different types of layered flows have and can be studied in different types of homogeneous and inhomogeneous turbulent flows. These were the

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primary aims of the Euromech 517 colloquium at University College London, 28–30 June 2010. Since these layers appear to have a dominant role, general descriptions and models of turbulence need to take them into account in order to explain properties of turbulence such as anisotropy, decay rates and correlation structures. Interfaces with similar properties of sharp gradients in velocity occur in many other kinds of flows, such as geophysical flows, free surface flows, and magnetohydrodynamics [6]. Dynamics and plasma flows were also presented.

In this review, it is shown how there are three types of fluctuating thin layers and interfaces in turbulent flows (with thickness $l$ and $l_I$, see figure 1):

— boundary layers and interfaces (BI) between the regions of turbulent and non-turbulent flows, which may have significant mean shear ($\Delta U_I \gtrsim u'$) across the interface or may be shear-free ($\Delta U_I \ll u'$), respectively, denoted by BIS, BISF, where $\Delta U_I$ is the mean velocity jump and $u'$ is the root mean square (RMS) turbulence at the interface;
— internal layers and interfaces (II) separating regions of turbulent flows, which again may or may not have significant shear (IIS, IISF; e.g. [7], Ishihara et al. 2010); and
— boundary or internal interfaces associated with discontinuities in density or density gradients (IB), where there are significant body forces.

Studies of such interfaces in experiments, numerical simulations and idealized theoretical models are helping to identify and answer some of the basic questions raised by these flows.

— What are the different thicknesses $l$ and $l_I$ of these thin layers and their interfaces, and the small scale of their interior eddy motions ($l_e$) [8,9] at high Reynolds number? What determines the fluctuations of the interface position and shape in different flows [10]? 
— How do local mechanisms interact? Local instabilities on the scales of $l$ or $l_e$ lead to self-induced transport of turbulence towards the low-energy region [4] or a distortion and folding of the layers by external large-scale eddies? Following Corrsin & Kistler [11], the former local or ‘nibbling’ process has recently been found to dominate the movement of the interface [3,12] but the scaling is uncertain. The latter engulfment process may be significant in certain flows with vigorous large-scale eddies, such as in mixing layers [13]. This mechanism is affected by how external eddies interact with thin interfacial layers, a process which is characterized as a combination of ‘blocking’ caused by normal fluctuations being damped at the interface and ‘sheltering’ when external fluctuations on one side are uncorrelated with fluctuations on the other side (e.g. [14]).
— How do boundary interfaces affect the large scale and mean properties of turbulent flows which are often characterized in terms of types of ‘entrainment’ velocities? These have been defined as the mean movement into non-turbulent flows (e.g. the outward boundary entrainment velocity, $E_b$), its effect on the mean velocity field (e.g. the inwards entrainment velocity, $E_v$) or on mean fluxes of scalars across the interface $E_I$ [15]. Altering the external and internal turbulence affects these entrainment velocities but the mechanisms are still unclear (e.g. [16]).
Figure 1. Schematic of the classifications of the interfacial processes, with (a) boundary layers and interfaces (BI), (b) internal layers and interfaces (II) and (c) interfaces associated with changes in density or density gradients (IB). Note the difference between the turbulence across and within the interface.
What roles do interfaces play in affecting the statistical–physical process in different types of turbulent flows, such as the mechanism, spread and transfer of energy between large- and small-scale eddy motions? The intermittency of energy dissipation, the rate at which large eddies grow and the extent of the influences of initial and boundary conditions remain controversial questions (e.g. [17]). In turbulent flows distorted by body forces the strength of these interfaces [18] and their spacing are affected, for example, in stably stratified flows. It is striking that these basic questions about turbulence have changed substantially since they were reviewed at the international programme at the Isaac Newton Institute in 1999 (Hunt et al. 2001) largely as a result of research projects reviewed here.

2. Boundary interface dynamics

Zaki et al. [19] analysed the penetration of vortical disturbances across interfaces from a uniform flow to a shear flow using matched asymptotic analysis [14]. The asymptotic limits were contrasted in the shear-dominated limit (shear-sheltering regime); the vortical modes possess a three-layer structure at the interface, decay exponentially and do not interact significantly with the mean shear. However, at a lower Reynolds number $Re \sim 400$, when viscosity becomes significant, vortical disturbances can penetrate the mean shear, and can subsequently lead to significant amplification of the perturbation energy. In an intermediate convective–diffusive regime, there is a balance between the accumulative sheltering effect of the shear and the influence of viscosity. Beyond the initial stage of vortical penetration into the shear, the evolution of interfacial disturbances is studied using the nonlinear parabolized stability equations (PSE) developed by Cheung & Zaki [20]. Unlike linear stability analysis, this PSE formulation accounts for non-parallel effects in developing boundary layers, and also finite deformation of the interface, energy exchange between instability waves and the distortion of the base flow owing to the non-linear disturbance field. The importance of nonlinear effects is demonstrated by contrasting the new PSE formulation with linear theory. The notion of modal competition was revisited, showing how nonlinear effects serve a dual purpose. It can lead to saturation, thus curbing the amplification of frequency in linearly stable modes. But when these fluctuations are large enough, the elongated streak eddy structures which are simulated in the boundary layer convert rapidly by nonlinear amplification into fully developed turbulent spots. The presence of a two-fluid interface introduces a scale conversion mechanism: the vortical mode changes wavenumber across the interface, which affects their penetration into the shear as well as their amplitude.

Westerweel et al. [21] reviewed experimental measurements of the velocity field near the turbulent/non-turbulent boundary interface of a jet characterized by typical RMS velocity $u'$. These interfaces are characterized by sharp jumps (discontinuities) in the conditional flow statistics relative to the interface, e.g. $\Delta U_1$ for the conditional ‘mean’ velocity jump. New experiments have been carried out to measure the conditional flow statistics of a non-isothermal jet, i.e. a jet that is cooled relative to the interface. These experiments are complementary to the
experiments reported by Westerweel et al. [3,22] for a jet at $Re \simeq 2000$. Here, the molecular diffusivity of heat ($\kappa$) is intermediate between that of momentum ($\nu$) and matter ($D$). The experimental method involves a combined laser-induced fluorescence/particle image velocimetry (PIV) method, where a temperature-sensitive fluorescent dye (Rhodamine 6G) is used to measure the instantaneous temperature fluctuations. The results show that the cooled jet behaves like a self-similar jet without any significant buoyancy effects. As in the previous experiments, the interface was detected using the instantaneous temperature fluctuations. Flow statistics reveal that the jump in the conditional vorticity, the temperature across the interface and the mean thickness of the momentum layer occur over a distance of the order of the Taylor microscale (i.e. $LRe^{-1/2}$), where $Re$ is the Reynolds number of the energy containing eddies, based on the RMS velocity $u'$ and the integral length scale $L$. The mean thickness of the thermal layer is of order $LRe^{-1/2}Pr^{-1/2}$ where $Pr = \nu/\kappa$. Note that these measurements could not resolve the fluctuations within the interface layer confirming the previous findings that the ratio of the jump in the mean scalar across the interface $\Delta \bar{c}$ to the large-scale fluctuations $C'$ within the jet was 4 or 5 times larger than the same ratio for the velocity field, i.e. $\Delta \bar{c}/C' \gg \Delta U_i/u'$.

Da Silva & Reis [8] have studied the internal and external flows near boundary interfaces of turbulent jets in terms of local conditional coordinates using direct numerical simulations [8,23–25]. They have shown that the overall structure of the interfaces in the simulation confirmed the experimental results of Westerweel et al. [3]. The mean thickness of the interface layer defined by a conditional mean profile (averaged over lengths of order $L$ along the interface) is of order $LRe^{-1/2}$ (i.e. the Taylor microscale) across which the velocity fluctuations are of order $u'_i$. This is why the local dissipation rate in the layer $\bar{\epsilon}_i$ is of the order of the mean dissipation $\bar{\epsilon}$ in the bulk of the turbulent jet (i.e. $u'^3/L$). Within the layer these elongated vortices (up to $L$ in length) develop with diameters of the order of the Kolmogorov microscale—defined as $\eta \sim (\nu^3/\bar{\epsilon})^{1/4} \sim (\nu^3/\bar{\epsilon}_i)^{1/4}$ with vorticity greatly in excess of the mean vorticity $u'/\eta$ in the interface layer. This is because these vortices are undergoing significant distortion [14] and are unlike equilibrium Burgers vortices which have been proposed as a generic model (e.g. [26]). The intense small-scale velocity fluctuations induced by the vortices are probably associated with the small-scale ‘nibbling process’ that determines the boundary entrainment velocity, $E_b$ [27].

An important fundamental and practical finding was that the mean scalar dissipation $\bar{\epsilon}_vl$ in the interface layer is significantly greater (by a factor of 5–10) than the mean scalar dissipation in the interior of the jet. This is consistent with the theoretical and experimental findings of Westerweel et al. [3] that, relatively, the mean temperature jump across the interface is much greater than the velocity jump. In the last 10 years there have been many studies of statistical relations between second and third moment of the velocity gradients, expressed as rates of strain $S_{ij} = \frac{1}{2}(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$ and vorticity $\Omega_i = \epsilon_{ijk} \partial u_k/\partial x_j$. The distributions of eddy structures and the paths of fluid parcels through these structures are indicated by the graphs of $Q = \frac{1}{4}(\Omega_i \Omega_i - 2S_{ij}S_{ij})$ against $R = \frac{1}{3}(S_{ik}S_{km}S_{mi} + \frac{2}{3}\Omega_i \Omega_k S_{ik})$ (see [17,28,29, p. 265]). Da Silva computed this relation for fluid elements outside the interfacial layer which showed a similar $Q-R$ plot implying random formation, straining and stretching of vortical structures. But inside the layer the $Q-R$ plot was markedly different, as it
was largely confined to $Q < 0$ and $|R|$, which was symmetrical and finite, correspond to the highly stretched vortices that are formed. This suggests that the error in using large eddy simulations (LESs) for turbulent boundary interfaces results from the failure to represent not only their internal dynamics, but also the impact of the interface on the larger-scale eddy motions. This raises the question of whether new algorithms of numerical simulations are needed for these regions in the flow (see [30]). Holzner et al. [27,31] described experimental measurements, as well as an analysis of the boundary interface where there is no mean shear (BISF). Approximate homogeneous turbulence driven by an oscillating grid on one side of the interface moves away from the region of turbulence with a mean velocity $E_i$. The flow field is measured with a three-dimensional particle tracking system [31]. The Reynolds number $Re_b$ is approximately 50 based on the Taylor microscale $\lambda \sim 50 \text{mm}$ and on the Kolmogorov length scale $\eta \sim 0.6 \text{mm}$. Focusing on the outer surface of the interface moving at a local velocity internal direction $\Delta v_I$, relative to the local velocity, Holzner et al. showed that the velocity of the interface $v_I$, at location $Y_I$ (both in fixed laboratory coordinates), can be expressed as $v_I = dY_I/dt = (\Delta v_I + u)(Y_I)$ where $\Delta v_I$ is of the order of the Kolmogorov microscale velocity $v_{kol} \sim (\nu \epsilon)^{1/4}$. The vorticity equation shows why viscous diffusion causes this outward movement. Discussions showed the mean normal velocity of the interface $E_b = \langle v_I \rangle \sim u(Y_I) \sim u'$ is much greater than the Kolmogorov velocity probably because the greater outward velocity of the bulges than the inward velocity of the cusps [32]. The curvature of the interface is at its maximum which, as the analysis of Holzner et al. [31] showed, increases the speed of the viscous diffusion.

Ouvrard et al. [30] addressed the question of how well current approximate simulations describe and predict the flow structure at boundary interfaces, in particular at the edges of boundary layers and wakes. LES models are available to represent flows and eddy motions down to the filter scale $\Delta$ (which should be significantly smaller than 1/10 of the integral scale $L$; e.g. [33]). However, this is not a practical method for calculating the flow over an aircraft wing or in geophysical flows where the integral scale of the turbulence is much less than the overall scale of the flow. Consequently, hybrid simulation methods (e.g. detached or organized eddy simulations) are used where the large-scale eddy $U$ motions are computed explicitly and the small scales $\tilde{u}_i$ are modelled statistically (e.g. as a Reynolds stress $\tilde{u}_i \tilde{u}_j$ acting on the large scales; e.g. [34]). This method needs modification to represent the high gradients and anisotropic characteristics of turbulence near boundary interfaces. A new approach is being developed to model the turbulence near the boundary interface. First, the location of the interface is calculated using a local criterion based on the mean square vorticity (e.g. [35]). The new concept is to introduce an intermediate scale velocity field near the interface $\tilde{u}(x,t)$, consisting of random modes corresponding to the general forms of the inhomogeneous field near an interface. The velocity is rotational inside the interface and irrotational outside it [36]. Introducing this field into the dynamical equation helps in maintaining a thin interface [32]. Using the results of direct numerical simulations near interfaces [4] and experiments [10] it is assumed that the intermediate local eddies $\tilde{u}$ are continuously generated by upscale energy transfer from $\hat{u}$, which is depleted, while at the same time receiving energy from gradients of $U$ and $\hat{u}$. Upscale or back scatter methods.
have previously been introduced into LES in turbulent boundary layers (e.g. Mason & Thomson [37]), but not as a systematic method for thin interfaces (see also [38]).

Wolf et al. [39] investigated the influence of shear and swirl on the properties of the turbulent/non-turbulent interface in a high Reynolds number \((Re = 10^4)\) axisymmetric flow using scanning PIV and three-dimensional particle tracking velocimetry. Swirl is used in many practical devices such as combustors and cyclone separators and it frequently occurs in natural flows like in tornadoes. It is well known that swirl added to an axisymmetric jet enhances the entrainment rate, i.e. the mean boundary entrainment velocity \(E_b\). But the mechanism causing this increase is not yet well understood. The addition of azimuthal momentum to a round jet flow produces additional strain near the interface which yields an enhanced growth rate. Initial PIV and particle tracking velocimetry measurements for the validation of the designed jet facility show the rapid growth of the jet width, as observed in many types of cyclones (e.g. [40]). However, it is not clear how these global changes of flow characteristics are connected to the small-scale dynamics occurring locally at the instantaneous entrainment interface. This is an example where body forces have a marked effect on the turbulence and entrainment processes at an interface.

3. Interfaces with turbulent flows

Kaneda et al. [41] analysed the velocity fields within and near internal interfaces in homogeneous and isotropic turbulence [4] using high resolution direct numerical simulations (DNS) with up to \(4096^3\) grid points and a Taylor scale Reynolds number \(Re_l\) up to approximately 1100. Comparisons have been made with interfaces across which the vorticity distribution changes sharply, which are often observed in various flows, including turbulent jets, mixing layers and boundary layers (figure 2).

A detailed study of these layers was undertaken in a subregion (with box size \(0.7L_m\), where \(L \sim 30\lambda \sim 2000\eta\)), where \(\eta\) is the Kolmogorov microscale \((\nu^3/\bar{c})^{1/4}\). The interfaces on either side of the layers were defined where the enstrophy \(\omega^2\) was greater than 8 times the average (following [3]). Typically \(\omega^2\) increases over a distance of order \(\eta\). Within the layers there are peaks of vorticity with typical diameter of order \(10\eta\) with peak velocity jumps of order \(u'\). These quantities have a probability distribution with a high flatness factor (normalized on the whole flow region) within, but not outside, the layer. These interfaces at \(n_1\) and \(n_2\) (defined in terms of a coordinate normal to the interface) are more sharply defined on the upstream face \((n = n_1)\) where the layers are generally moving relative to the large-scale flow (as with boundary interfaces [27]). The thickness of the layers is approximately \(4-5\lambda\). The typical distance over which these layers are coherent is about \(l\), but they tend to roll up at their ends into spiral structures so that their separation distance ranges from \(L\) down to about \(l_e\). Most often vortices have the same sign and are aligned approximately in the plane of the layer so that there is a net mean shear across the layer with a mean velocity jump \(\Delta U_1 = \int_{n_1}^{n_2} \omega dn\), where \(\Delta U_1\) (as with boundary interfaces) is about order \(u'\). This jump in the large-scale flow across the layer is clear from the change

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in direction of the streamlines. To test whether this thin shear layer acts as a barrier to the fluctuations outside the layer, conditional cross correlations of the velocity fluctuations were evaluated, showing high correlations between points on the same side and low correlations between points at different sides of the upstream interface of the layer. There is some indication that eddies are transported out of the layer on the less well-defined downstream face.

Computations of the energy transfer $T(x,k)$ between different length scales show [42] that upscale transfer (from small to large) is almost as large (within 10%) as the downscale transfer, which has generally been assumed to be dominant and the most important process governing the small-scale structure of turbulence [43,44]. New results show that $T(x,k)$ is very intermittent; its fluctuations being much larger than its mean value. Conditional analysis near the interfaces of $T(x,k)$ shows that $T$ is largest within the thin layers where the most intense vortices and dissipation occur, but is also very large just outside the layers (where viscous stresses are weak), by comparison with the average values of $T$ over the whole region. Since the viscous effects are large within the layers and small outside them, it follows that inviscid energy transfer, which is a characteristic of the inertial range in the spectrum—is dominated by the interaction just outside the interface layers. The conditional statistics show that the pdf of $T$ is significantly non-Gaussian with high extreme values, both outside and inside the edges of the interface. The relative contributions of large and small wavenumber components consist of one small and two large components which indicates mainly non-local wavenumber interactions, but there are also significant contributions from local wavenumber interactions as well.

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Worth & Nickels [45] described high Reynolds number turbulence results from a recently constructed experimental apparatus. This is a 2 m diameter water tank with a height of 2 m, with two contra-rotating impellers rotating with a frequency of 0.5 Hz. Since the turbulence was generated by the mean shear, the large-scale eddy motions were not the same as in the inhomogeneous isotropic turbulence simulations, the value of $Re_l$ was comparable at approximately 150–200. In a small volume of size 5 cm (approx. equal to $L$), three-dimensional measurements of the velocity field were made with a spatial resolution of $500 \times 500 \times 200$ elements of dimension $(1–3)\eta$ and a temporal resolution of about $\tau_\eta$ (8 s) (where $\tau_\eta = (\nu/\overline{\epsilon})^{1/2}$). Thus, spatial and (unlike DNS results) temporal rotational energy (enstrophy) and turbulent energy dissipation intermittency could both be resolved in agreement with previous studies of Ishihara et al. [4] and Kaneda et al. [41]. Highly intermittent behaviour is observed with intense coherent flow structures clustering in thin layers which were located near the periphery of larger vortices. The space/time measurements enabled the effects of advection to be separated from other effects, elucidating not only their topology but also the evolution of these intense ‘rod-like’ vortices within the layers in which there was found to be a local balance of straining and diffusion. Movies of high regions of vorticity in the flow field showed that as the structures moved they retained their layer-like form for a period of the order of $\lambda/u'$, but also began to roll up. This time scale is consistent with the dynamics of a rolling up vortex sheet in a straining flow (e.g. [46]). There was no evidence in these data or those of Kaneda et al. of Kelvin–Helmholtz billows forming as internal layers—as has been noted in simulations at low Reynolds number flow by Ruetsch & Maxey [47] or on certain boundary interfaces. Rather as the pdf of strain–vorticity diagrams demonstrate, the vortices were generated by large-scale straining motions as also observed in DNS.

The findings of the thickness of the layers and the vortices are similar to those of Ishihara et al. [4] and contribute towards a better understanding of the intermittency phenomenon. This should pave the way for more accurate computational fluid dynamics and LES models of the small-scale motions based on an understanding of the underlying flow physics.

Hunt [48] presented a combined dynamical and statistical model of a high Reynolds number turbulence in terms of thin layer interactions with impinging eddies. The analysis of experiments and numerical simulations of these flows show how thin shear layers tend to form between regions of high and low levels of turbulence energy (with velocity and integral length scales $u_0$ and $L$) and regions of high and low mean velocity on either sides of the layer. A new rapid distortion analysis was presented for large-scale eddies, with scale $k_{ij}^{-1}$, which impact and are blocked by these layers. Blocking leads to a straining motion which distorts smaller-scale eddies. Progressively higher wavenumber fluctuations are generated near the interface, with a self-similar Fourier transform for the vorticity $\tilde{\omega} \sim k^{-p}$ (taken over a scale $L$). As large eddies move away from the interface, there is a local upscale transfer of small-scale fluctuations in separate regions of intermittent down and upscale transfers of small-scale fluctuations as are found in numerical simulations [4,41]. These quasi-linear interactions lead to an energy spectra $E(k) \sim k^{-2p}$ where $2p$ lies between 3/2 and 5/2, depending on the form of the large-scale strain between the axisymmetric plane and on how shear along the outside of the interface affects length scales, rather than on the structure of the small-scale eddies far from the interface. Since a slowly varying
stable spatial structure of the distorted eddy field exists relative to the interface (on a time scale greater than that of the small eddies), there has to be a steady flux of energy $\varepsilon_f$ per unit volume into the thin shear layer where the energy is dissipated at an average rate $\varepsilon$ over the whole volume analogous to wave motion being dissipated at a beach or critical layer. Thus $\varepsilon_f \approx \varepsilon$. Since the eddies are decreasing in scale as the normal distance $n$ from the interface edge decreases, there has to be a downscale transport of energy into the smallest eddies, which are independent of $k$ and $n$. In other words, there is a close connection in the inertial wavenumber range between energy transfer in wavenumber and physical space.

Applying Townsend’s [10] homogeneous model for the nonlinear interaction of these distorted eddies shows a local steady state can only occur if $2p = 5/3$ (i.e. $E(k) \sim \varepsilon^{2/3}k^{-5/3}$) within the thin layer. The impinging ‘inertial range’ eddies stimulate (e.g. [19]) the random growth of microscale vortices (with a mean thickness of the Kolmogorov microscale) $\eta = LRe^{-3/4}$ within the layer. The magnitude of the velocity fluctuations around these very small-scale vortices is of the order of the RMS velocity of large-scale eddies—quite different from the usual concept of small velocities from small scales of turbulence. Conditional analysis of experiments (see [45]) and numerical simulations (by Ishihara & Kaneda 2010) are consistent with this model.

It is argued that these local distortion mechanisms combined with the thin layer dynamics explain why fully developed turbulence can be generated quite fast (in a time scale $L/u'$) in very inhomogeneous and unsteady flows, neither of which can be entirely explained by the non-intermittent, slow cascade model of Richardson & Kolmogorov. When the model is extended to scalar fluctuations with length scale $L_S$, it shown that similar scalar interfaces and thin layers are coincident with the velocity layers, but their internal structure is different [8]. The inertial range form of the scalar spectrum can differ from the velocity field depending on the ratio of $L_S/L$ (as suggested by Warhaft 1993). This research is based on the joint work with Eames, Westerweel, Fernando, Braza, Davidson, Voropayev, Kaneda and Ishihara and a preliminary version was published by Hunt et al. [49,50].

Chernyshenko [38] presented a general theoretical study of low-speed streaks (i.e. well-defined elongated regions parallel to the mean velocity) in turbulent shear flows bounded by sharp interfaces. Streaks are observed in the interior of all well-developed turbulent shear flows that are associated with the highly distorted eddy motion, and passive scalars like temperature also form organized streaks in these shear flows. A new theoretical approach developed here requires knowledge of the mean profile shape, $Re$, transverse and wall-normal Reynolds stresses to derive the spanwise streak spacing by evaluating the optimal perturbation. Comparisons were made over an order-of-magnitude range of variation of the predicted quantity. The theory is free of any adjustable parameters given the assumption. Conceptually the generalized optimal perturbation (GOP) approach is based on the idea that given almost any broadband wall-normal velocity the mechanism of lift-up of the mean profile combined with mean shear and viscous diffusion has a pattern-forming property responsible for streak formation as an input, this pattern-forming property working like a filter passing through a streaky pattern with certain spanwise scales only. This approach differs from others where the pattern of wall-normal motions has to be defined or computed.

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The GOP concept provides a new explanation as to why linearized solutions (e.g. rapid distortion theory \[51\]) can describe these significant nonlinear turbulence phenomena. GOP can also predict the large-scale motions, although the predicted scales differ somewhat from those observed in experiments.

Yakovenko et al. \[52\] presented direct numerical simulations of a breaking internal gravity wave which forms an isolated region or ‘patch’ of turbulence surrounded by a sharp interface. The wave was forced by the imposition of an appropriate bottom boundary shape (a two-dimensional cosine hill, length \(L_h\) and height \(h\)) within a density stratified domain having a uniform upstream velocity \(U\) and Froude number \(U/Nh \approx 0.6\), where \(N\) is the buoyancy frequency. Typical Reynolds numbers based on the vertical scale of the breaking region and the upstream velocity were around 6000—an order of magnitude larger than any previous DNS of such a flow. The computations required over \(10^9\) mesh points and yielded sufficient resolution to capture the fine-scale transition process (namely hydrostatic instabilities with typical Rayleigh–Taylor ‘mushroom’ eddy structures), which evolved into a full-developed turbulent patch. The turbulence statistics within the patch region reached a steady state within a time scale of about \(10N^{-1}\). These patches greatly influence the mean flow over the hill and determine its drag \[53\]. The patch remains approximately fixed in space because of the mean recirculating flow within it, being of order \(U\)—the flow is analogous to a pair of line vortices of width \(L_p\) \[54\].

There is an overall balance in the turbulent patch, in the production, dissipation and transport processes for turbulent kinetic energy, so that the patch remains quasi-steady over a significant time. Most of the turbulence generation occurs near the upstream bottom part of the patch where the mean velocity shear is particularly large, and a sharp turbulent interface term (e.g. \[41\]). On the downstream side vorticity is shed and a turbulent wake is formed that has similar properties to a classical two-dimensional wake \[55\].

Catalano & Moeng \[56\] presented results of a LES simulation of thermally driven valley flows \[57\] using a three-dimensional meteorological model WRF \[58\]. This is derived from a fully compressible non-hydrostatic set of Navier–Stokes equations that includes a terrain following vertical hydrostatic pressure coordinate with mesh refinement close to the ground, together with a modified subgrid model that takes into account the effects of the anisotropic grid of smaller scale problems. The focus was on the fine structures of the anabatic and katabatic winds. The thermal forcing was applied with a sinusoidal time dependence for the surface temperature anomaly, \(\Delta \theta_S = \Delta \theta_{S,\text{max}} \sin(2\pi t/T)\), where \(\Delta \theta_{S,\text{max}}\) is the amplitude, \(T = 24\) h is the period and \(t\) is the time. A surface layer parametrization, based on the Monin–Obukhov similarity theory, was used as a wall layer model. The valley geometry is symmetric and two-dimensional. Initially, there is a stably stratified capping inversion over the valley. During the day, the anabatic current up the slopes extends from the ground up to about 200 m and interacts with convective roll structures at the top of the valley. This flow can be considered another example of a strong internal interface within a turbulent flow. The horizontal breeze associated with the roll structure determines the presence of a relative maximum of turbulent kinetic energy over the upper slopes. Waves are identified on the interface between the upslope current and the horizontal breeze.

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In the presence of significant along-valley geostrophic wind, the cross-wind horizontal breeze is weaker, due to changes in strength and height of the capping inversion over the valley. The night time circulation reveals the downslope gravity current impacting the quiescent air of the developing cold pool at the bottom of the valley. This leads to a local vortex form with an associated updraft current quite similar to a hydraulic jump: a significant positive vertical velocity can be observed in the middle of the instability region as well as stronger values of turbulent kinetic energy. In agreement with Yu & Cai [59], waves are observed downstream of the jump (see also [52]).

Espa et al. [60] reported on a study of elongated flow structures in ocean flows with sharp north–south gradients and flows in the east–west direction. Like streaks in boundary layers these (and analogous structures in Jupiter’s atmosphere) zonostrophical structures have sharp boundaries. As in the scaling arguments of Rhines [61], the velocity energy spectrum has the form $E(k) \sim \beta^2 k^{-5}$, where $\beta$ is the gradient of the Coriolis parameter. There are discontinuities in velocity gradients but not vortex sheets at the edges of these large structures. Their formation results from upscale transport of energy—characteristic of two-dimensional turbulence—which is eventually limited by rotational forces [62]. Espa et al. [63] studied the effects of dissipation caused by frictional effects, which showed different forms of the spectrum because of large upscale problems. These two-dimensional structures may also be different in the atmosphere and in the ocean. Laboratory experiments in rotational tanks (with sloping bottom to simulate the $\beta$-plane effect) confirm these concepts. They showed how, as with other sheared interfaces, these synoptic scale interfaces act as ‘potential vorticity’ or shear sheltering [19] barriers—blocking and decorrelating motion on either side (cf. [6]), but dynamically evolving ‘barriers’ enable particles to pass from one side to another and for mixing to occur.

Busse & Sandham [64] presented a new model to stimulate the effects of wall roughness elements on turbulent channel flow. In general it is not practical to fully resolve the details of flow around the roughness elements, but to account for their dominant effects on the outer region of the flow, such as the shift in the mean velocity profile [65,66].

The effects of the roughness elements are modelled by adding an extra forcing term to the r.h.s. of the Navier–Stokes equations. The forcing term $f_i = -\alpha_i g_i(z, k) |u_i| u_i$ is composed of a roughness amplitude $\alpha_i$ and a shape function $g_i(z, k)$ where $z$ is the wall normal coordinate and $k$ the roughness height parameter. The forcing term is chosen to be quadratic in the corresponding velocity component $u_i$ and the form $u_i |u_i|$ ensures that the forcing term always has a damping effect on the respective velocity component. A series of simulations of turbulent channel flows at a Reynolds number $Re = 180$ has been conducted in order to explore the effects of different roughness heights, amplitudes and shape functions. The roughness term is applied symmetrically to both the upper and lower wall of the channel. As expected the streamwise roughness term $f_1$ has by far the most significant effect. The turbulent drag is increased in all cases and for a sufficiently high roughness amplitude or roughness height the turbulent flow becomes fully rough. If the roughness amplitude $\alpha_1$ for the streamwise component of the velocity is set to zero, the spanwise roughness term $f_2$ becomes dominant. In this case, there is a reduction in drag and an upwards shift in the velocity profile can be observed, as in the flow over riblet surfaces [67]. The shape function

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regulates the form of the velocity profile across the channel. The effects of different shape functions emerge clearly in the profiles of the RMS velocity fluctuations whereas the shape of the profile of the mean streamwise velocity shows—for the shape functions investigated so far—a relatively weak dependence on the shape function.

4. Turbulence dynamics and interfaces

Davidson [68] presented a theoretical description of the long-range interactions in turbulence and their effect on the rate of decay of energy in homogeneous flows. He also considered the effect of body forces following Batchelor & Proudman [69] and Saffman [70], and it was shown that the decay of homogeneous turbulence depends on how it is initially generated and how the long-range interactions affect its evolution as a consequence of the Biot–Savart law. He noted that, somewhat surprisingly, these long-range correlations are very weak in decaying isotropic $E(k) \sim k^4$ turbulence. This should also be true for magnetohydrodynamic (MHD), rotating and stratified turbulence. These interactions are determined by distant velocity fields induced by the vorticity of eddies (i.e. the Biot–Savart law); but they can be limited by the vorticity of other eddies (i.e. a shielding effect, Ruelle 1990) or by vortical interfaces (shear sheltering). As the vorticity fields evolve, it was shown from numerical simulations that the long-range correlation over a distance $r$ decays to zero faster than $r^{-5}$ in $E(k) \sim k^4$ turbulence. Effectively as the large-scale turbulence consists of quasi-independent ‘lumps’ on scale $L$ and velocity $u'$ the angular momentum of which is conserved statistically, this leads to invariant integrals (i.e. $u'^2L^5 \sim \text{constant}$). With the usual assumption of an inviscid transportation of energy to small scales, the Kolmogorov [71] results are obtained, i.e. $u'^2 \sim t^{-10/7}$ and $L \sim t^{2/7}$ (while the Reynolds number of the turbulence remains large, i.e. $u'L/v \gg 1$). Where there are anisotropic body forces the turbulence is distorted and thus tends to induce anisotropic wave motion (if $f$ is not locally proportional to $u'$) as occurs with buoyancy forces, large-scale rotation [72] or magnetic fields (at high magnetic Reynolds number $R_m$). At low $R_m$, where $f \sim u'$, MHD forces do not produce wave motion. Typically, the length scales and velocity components are different in directions parallel and perpendicular to the forcing resulting in anisotropic eddy structures and invariants. But the isotropic methods can be adapted to these flows because certain features of the eddy structure are maintained. In the case of stably stratified vertical density gradients, after the turbulence has had time to evolve into a fully developed state, sharp density and velocity interface forms above and below the ‘pancake’ structures in which there is a balance between kinetic and potential energy. This determines the smallest integral length scales and the rate of decay of the turbulence [73,74]. In the case of MHD turbulence, the energy is dissipated through electrical currents as well as viscous stresses [75]. At high $R_m$, these will be concentrated in interfacial layers as in anisotropic processes.

In general, turbulence that is initially homogeneous and isotropic develops a new wave-dominated anisotropic structure that is still three dimensional on a time scale longer than the local wave dominated time scale, i.e. $L_0/c > L_c$, provided the wave speed associated with the rotational body force $c$ satisfies $c < u'$. If $c \gg u'$ the body force produces a strong and rapid distortion of the turbulence with a
quite different structure so that the decay process also differs (e.g. [76]). For weak body forces, the coherent structures have a finite scale \( L(t) \) which evolves because the waves convert kinetic energy into potential energy. The kinetic energy of the turbulence decreases and can reduce the rate of growth of the minimum \( L \) so that dissipation increases faster with these kinds of body forces.

Valente & Vassilicos [77] presented measurements of the decay of homogeneous, quasi-isotropic turbulence generated by low-blockage, space-filling fractal or multiscale grids. The experimental work of Mazellier & Vassilicos [78] was extended by increasing the length of the wind tunnel where the turbulence decays. Measurements showed that the turbulence intensity first increases as the wakes of the different size and spacing of the bars interact. It was shown that the measured one-dimensional energy spectra could be reasonably collapsed, as in Townsend [10], using the local RMS velocity and integral length scale \( L_x \). Unlike other single scale uniform grid experiments where \( L_x \) increases, here \( L_x \) did not vary with \( x \). It was suggested that this is caused through the small-scale structures and internal interfaces being energized and entrained into dissipative layers of larger scales. This result for \( L_x \) is consistent with equilibrium similarity based on a hypothesis of George & Wang [79] and Mazellier & Vassilicos [78]. It was noted in a discussion that the fast decay rates of the RMS turbulence (\( u' \)) meant that the Lagrangian time scale (or memory time) \( T_L \sim L_x/u' \) was increasing with \( x \) faster than the travel/distortion time \( T_D \sim x/U \). Usually \( T_L \sim T_D \) in classical self-preserving decaying turbulent flows which means that there is always some sensitivity to the initial condition. In this case, the turbulence structure remains more sensitive to the initial condition. This is consistent with the anisotropy ratio remaining constant as the turbulence decayed, \( v'/u' \sim 1.2 \) (even more pronounced than in Townsend’s [10] anisotropic grid).

It is also suggested that the weak anisotropy of the flow can be accounted for by computing the three-dimensional spectrum function from velocity signals with two separate components. Further checks on homogeneity and isotropy were also presented as well as measurements with a regular grid indicating that the single-length scale locking is neither an artefact of inhomogeneity nor an effect of the wind-tunnel walls.

Clegg & Halse [80] presented a theoretical analysis and numerical simulations of upper ocean dynamics and their connections between sea-surface temperature (SST), microgradients and winds over mesoscale horizontal distances (10–1000 km). The study focused on the remarkably strong relationships between the surface wind-current stress and the horizontal gradient of the SST. At scales larger than 1000 km, the effects of SST-induced wind perturbations are not significant and the ocean becomes a more passive recipient to atmospheric forcing, producing a negative correlation [81]. Current research is also focusing on mesoscale inter-relationships which might extend to smaller scales (less than 10 km) where the SST heterogeneities are complex. In this study an LES model was used for the upper ocean to examine various stability and other environmental conditions, such as wind waves. In LES of the upper ocean, the surface stress changes because of the evolution of currents relative to fixed wind and owing to an assumed ‘drag’ coefficient. The latter was allowed to respond locally to the difference between surface wind stress and ocean current. It also allows for the evolution of SST relative to a fixed air temperature. These results are driven by initial test runs of the LESNIC code (Nansen Environmental and Remote Sensing Center [82]) which
has been modified to allow it to be used in the ocean setting. The runs provide curl/div diagnostic data for cases where the relative importance of the dominant types of coherent structures in the upper ocean is caused by shear-turbulence, Langmuir circulation and convection. These mechanisms produce roll structures with significant internal interfaces [49,50]. This is studied by changing the Stokes drift velocity \( v_s \) in relation to the shear stress velocity \( u_x \) (i.e. \( v_s/u_x \)) and to the buoyancy driven \( w_* \) (i.e. \( V_S/w_* \)). The surface data are found to exhibit weak but systematic organization (extracted after the code has evolved from random initial conditions). An objective method for analysing these roll structures was presented which was based on relating the horizontal surface wind stress (\( \tau_{xy} \)) to the downwind gradient of SST (\( \partial \theta/\partial x \)) (e.g. associated with a front), and the curl of \( \tau \) to the lateral gradient, i.e. \( \partial \tau_{yc}/\partial y \) and \( \partial \theta/\partial y \).

Redford & Coleman [83] studied the flow in initially axisymmetric turbulent wakes in neutrally and stably stratified flows using DNS at a mean Reynolds number of about 3000. For computational simplicity the flow develops in time. Different types of initial conditions are considered consisting of a series of vortex rings (case VR) and small-amplitude disturbances superimposed upon the mean-velocity profile from the vortex-ring initialization (case VRX). When there is no stratification, both types of initialization produce axisymmetric flows that eventually exhibit self-similarity, in terms of the mean velocity deficit \( \Delta u \), non-dimensional spreading rates of the mean width, Reynolds stresses and turbulent kinetic energy budgets. But the forms of their profiles and their instantaneous structure converge to distinctly different states with either a continuous or a lumpy interface with larger initial disturbances (as in [35]). This demonstrates the permanent dependence on initial conditions first observed experimentally by Bevilaqua & Lykoudis [84] and explained in terms of memory (\( T_L/T_D \)) by Hunt [85] and self-similarity by Johansson et al. [86]. In a stably stratified flow, there is a large change in the structure of the wake; its thickness \( d \) decreases as it develops into sheared sheets and laminae. It spreads laterally in mushroom vortices (when \( Nt \sim 10^2 \), where \( N \) is the buoyancy frequency). Internal waves on the edge of the wake are significant over a certain range of the flow where \( N\delta/\Delta u \sim 1 \).

5. Interfaces in MHDs and wave motion

Thyagaraja [87] considered the conservation properties of inviscid, incompressible hydrodynamics and incompressible ideal MHDs in order to examine how these systems in three dimensions might lead to singular solutions (involving vortex and current sheets). These ideal systems have many properties, such as time-reversal invariance of equations, conservation laws and certain topological features [88,89]. However, numerical models of ideal flows involve small levels of dissipation of energy and practical situations for the appropriate forms of the model equations are uncertain. This is why understanding general properties of the system is, if possible, highly desirable. For example, while dissipative (visco-resistive) effects may modify the equations, so that solutions to the initial-boundary value are bounded, the time reversal symmetry and associated conservation properties are certainly destroyed. There is an analogy with (and suggested by) the Korteweg–de Vries regularization of the one-dimensional, nonlinear kinematic wave equation, in which a new term is introduced into the model equation. The objective is
to find regularizations of the original ideal equations that retain conservation properties and the symmetries of the original equations. Integral invariants for the original systems are shown to imply bounded vorticity and enstrophy. When adapted to the corresponding dissipative models (such as the Navier–Stokes equations and the viscos-resistive MHD equations) interesting general properties for the solutions of the latter are also obtained. The regularized models thus have intrinsic mathematical interest as well as possible applications to large-scale numerical simulations in systems where dissipative effects are extremely small or even absent.

Barnes [90] presented a theoretical analysis of how mean shearing motions affect transport properties in MHD plasma flows, characteristic of the toroidal shape of a tokamak fusion system. As in hydrodynamic flows considered at this conference, it was pointed out how intensely sheared regions could simultaneously be barriers, while also generating local instabilities—in this case hydrostatic or Rayleigh–Taylor instabilities. Although the plasma pressure is high, the mean free path in these flows is comparable with the scale of the flow, some parts of the plasma turbulence being similar to that of inhomogeneous large-scale shear turbulence. The shear restricts the growth of these scales which therefore limits transport across and out of the plasma. But in some cases the flow bifurcates and a new flow/transport pattern is set up. The modelling and understanding of these flows is clearly even more difficult than for homogeneous fluids.

Klettner [91] presented numerical studies focused on waves interacting with obstacles in which coherent structures are formed and diffused into the fluid domain. The conservation of integral invariants is critical to understanding these flows. In this study, solitary shoaling waves and leading depression waves interacting with a bottom seated semi-cylinder, with a Reynolds number in the range of 500–1000, were considered. A purposely written arbitrary Lagrangian–Eulerian finite element code to solve the two-dimensional Navier–Stokes equation was developed with approximately 10 million grid points. With this detail, it was possible to study the global measures of momentum and energy which require evaluation of sensitive integrals over the whole domain and boundary. Physical scaling models (e.g. [92]) based on conservation quantities can be tested. Flow diagnostics included plots of the rate of strain and vorticity fields and by appropriately normalizing these measures, it was possible to distinguish between regions in the flow which are irrotational, shearing and vortical. The solitary wave results were compared with analytical predictions. A leading depression wave such as those of certain tsunamis was studied as there has been little numerical work performed on these waves, which can be compared with recent laboratory experiments.

6. Discussion on further work and applications

Some general conclusions emerged from discussions at this and a related meeting at Imperial College London [93] about newer developments in understanding interfaces and turbulence. These suggest fruitful lines of further research and applications. There was a consensus about the existence over significant distances of continuous interfaces at boundaries and within turbulent flows; they have their own characteristic velocity fields and have significant influences on the flow.
However, it was also agreed that more research is needed to answer the basic questions reviewed in §1 and addressed in this paper. Firstly, it is essential to define more rigorously and describe more comprehensively the kinematic and geometrical conditional properties of quasi-continuous interfaces, noting how these properties change with the length scales when inspected at the Taylor microscales or on the smallest wrinkles on the surface at the Kolmogorov microscale. Another key question about interface geometry is whether the layers are space filling or not; some evidence from experiments and simulations shows how they roll up and therefore occupy a greater proportion of the flow than $l/L$. Recent research (see [41,68]) has identified more clearly the dynamic influence of interfaces on the external eddies. This has a significant effect on the upscale as well as downscale transport and on the large-scale eddy motion; their role is far from passive. The small-scale dynamics within the bounding or internal layers show that they are quite different from the inertial range straining of vorticity outside them. It appears that these intense vortices are triggered by external fluctuations rather than instabilities, but this is not confirmed quantitatively. One suggestion for further research was to study an idealized motion of these layers in isolation in the same way that the effects of turbulence on wall boundary layers have been studied (e.g. [14]). Idealized calculations are also needed of intense individual elongated vortices on the Kolmogorov microscale that form within these layers, in order to understand how they develop and decay—or do they only decay when the thin layers themselves break up, e.g. by the effects of varying changing external strain, by collision or by roll up. Studying these internal/external dynamics will also determine the Reynolds number at which their structure changes, which may explain the large-scale transitions in turbulent shear flows (e.g. [94]) and the sensitivity of these layers to additional external forcing (e.g. novel types of grid [77], free stream turbulence outside or mean shear (e.g. [38]) boundary interfaces [40], or body forces [68]). Such studies focused on the interface layers in conjunction with the dynamics of large-scale coherent structures [95] using the full range of research techniques to identify the universal features of microscale turbulence, and its sensitivity to many kinds of external influences including interactions with non-universal large-scale turbulence.

There are many applications for a better understanding of these interfacial layers. The fact that they have very high gradients and move and distort means that they generate wave, noise and produce sudden loads when they impact into an aircraft or fixed obstacle (e.g. [49,50]). A researcher from a chemical engineering company noted that in the boundary interface layers outside liquid jets, droplets are broken up more intensively than elsewhere; the vortices at interfaces can concentrate inertial droplets in gas flows [96] and cause collision. The highly localized processes are not well described by general statistical theories or numerical simulations that do not resolve the interface layers. One approach being explored is to artificially introduce large-scale straining (via upscale transport) located at the approximate position of the random interface (see [30]). Kinematic simulations might be constructed, e.g. for homogeneous or sheared turbulence, with random distributions of sheared interfaces with internal vortices and Fourier components outside them. This could be an improvement physically on most stochastic models (e.g. [97]); because of their resistance to small level fluctuations they can act to limit external influence below some
threshold (or order $\Delta U_i$) and then quite suddenly break down when influences exceed the threshold. This understanding could assist modelling of dispersion in the environment or flames in turbulent flows.

An advantage of analysing flows in terms of interfaces is that it represents correctly at high Reynolds number how gradients of velocity, scalar and advected vector fields (e.g. magnetic fields) all occur at the same location, which is not true for their Reynolds average statistics expressed in coordinates that do not move with the interfaces (e.g. usually fixed in space or relative to the mean flow). However, the detailed variations of the velocity and scalar fields inside and outside the interface are not the same (see [8,98]) which perhaps explains how the forms of the spectra, transport and mixing properties of these field can be similar or different. Current models for the transmission of signals through fluctuating media do not properly account for the boundary and internal interface. This could be an important application.

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