PREFACE

Nonlinear phenomena, optical and quantum solitons

Research in the theory of solitons and its applications has undergone impressive development in recent years and has affected a broad range of fields, including theory of special functions, quantum integrable systems, numerical analysis, cellular automata, representations of quantum groups and discrete geometry among others.

Robin K. Bullough (21 November 1929–30 August 2008) was a British mathematical physicist famous for his contributions to the theory of solitons. In particular, he is known for his role in the development of the theory of the optical soliton, a term coined by him, and now commonly used, for example, in the theory of propagation of electromagnetic pulses in optical fibres. Bullough is also known for deriving exact solutions to nonlinear equations describing these solitons and for associated work on integrable systems, infinite-dimensional Hamiltonian systems (both classical and quantum) and the statistical mechanics of these systems. His famous Grand Synthesis of Soliton Theory, a huge diagram containing a vast array of areas in mathematics and mathematical physics, with connections (solid lines when established; dotted when expected), portrays the state of the art for about 10 years ago of the theory, and at the same time one of its strengths, namely its interdisciplinary character. Indeed, the diagram shows remarkable connections between seemingly disparate subjects. The diagram is a legacy of Robin: it perpetually evolves as dotted connections become solid and as new subject areas with corresponding links are added so that the diagram becomes increasingly more complex.

Bullough obtained his first academic position in the Mathematics Department at UMIST in Manchester in 1960, and was appointed chair of Mathematical Physics in 1973, where he remained until his retirement in 1995. He was an Emeritus Professor at UMIST until 2004 and, following the merger with the Victoria University, in the School of Mathematics at the University of Manchester. He was active in research until the day he died.

This Theme Issue has been put together as a special tribute to Robin K. Bullough. The selection of research papers brings together recent developments and findings, written by people with links to Robin, as colleagues, as students, as research assistants, as academic collaborators or simply as admirers of Robin’s many research achievements.

One contribution of 13 to a Theme Issue ‘Nonlinear phenomena, optical and quantum solitons’.
The Theme Issue begins with four review papers, as follows.

**Isochronous dynamical systems**

F. Calogero reviews recent results on isochronous dynamical systems, showing that to manufacture them is remarkably straightforward—the author’s *leit motiv* is that *isochronous dynamical systems are not rare*. The author calls for criteria for a measure of complexity in the behaviour of dynamical systems over a finite time, as opposed to the current, standard division into integrable and chaotic dynamical systems based on their behaviour over an unlimited span of time.

**The Dicke model in quantum optics: Dicke model revisited**

B. M. Garraway presents a survey of recent applications of the Dicke model in quantum optics, and considers in particular a zero temperature cavity in the rotating wave approximation. The author discusses new developments in topics such as spectroscopic structures, entanglement and phase transitions.

**Extreme events in solutions of hydrostatic and non-hydrostatic climate models**

J. D. Gibbon and D. D. Holm consider the hydrostatic (HPE) and non-hydrostatic (NPE) primitive equations, used extensively in numerical weather prediction and climate modelling. Analytical methods are used to demonstrate that, notwithstanding the requirement for regularity of the solutions of the HPE, the possibility of very extreme conditions, localized in space and time, is not ruled out.

**Nonlinear theory of slow light**

A. Rybin and J. Timonen review the properties of the one-soliton solution of the integrable three-level Maxwell–Bloch system on a non-vanishing background. They use the Darboux–Backlund transform to build the soliton solution of this system and present a set of interesting results on how to manipulate such a solution by stopping and reviving it, so that it can be used to store readable information. Despite the mathematical technicalities the review never loses contact with the underlying physical questions, a wonderful aspect of this paper.

Other contributions follow: they include a mixture of review papers and regular contributions which summarize the state of play in several topics important in Robin’s scientific career, related to nonlinear (quantum) optics and exactly solvable and integrable classical and quantum systems.

The paper by P. Caudrey, Robin’s co-author of *Solitons*, the first real textbook on soliton theory, is a description of how soliton solutions were derived in Manchester in the early 1970s.

P. L. Christiansen, H. Arnbak and Y. B. Gaididei consider relativistic and non-relativistic scattering by short-range potentials. They consider in particular application of a regularization technique to such potentials that contain a derivative of the Dirac $\delta$ function.

J. C. Eilbeck, S. Matsutani and Y. Ônishi discuss a family of multi-term addition formulae for Weierstrass functions on specialized curves of low genus with many automorphisms. The authors analyse the cases of genus 1 and 2 in detail; in the former, they find addition formulae for the equianharmonic and lemniscate cases, while in the latter they derive new addition formulae for various curves.

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A. P. Fordy considers a class of maps, recently derived in the context of cluster mutation. The author provides a brief review of the quiver context and focuses on a related Poisson bracket, along with the Poisson algebra, of a special family of functions associated with these maps; he uses them to prove complete integrability.

M. Lakshmanan presents a self-contained review of the mathematical properties and exact solutions of variants of the Landau–Lifshitz–Gilbert equation. The equation can admit many dynamical structures, including spin waves, elliptic function waves, solitons, dromions, vortices, spatio-temporal patterns and chaos depending on the physical and spin dimensions and the nature of interactions. The author focuses on the various versions, which can be treated by exact mathematical techniques.

R. Sasaki introduces the notion of ‘discrete’ quantum mechanics and suggests a simple recipe to construct exactly and quasi-exactly solvable Hamiltonians in one-dimensional discrete quantum mechanics. He discusses issues such as the intertwining relations, shape invariance and dynamical symmetry algebras including the q-oscillator algebra and the Askey–Wilson algebra.

J. Timonen and N. Bogoliubov discuss a quantum phase model that appears in the limit of very strong interactions from a strongly correlated q-boson hopping model. The authors review the phase model and its solution, and consider then its temperature-dependent correlation functions in a system of finite size. They find exact expressions for these correlation functions in determinant form, and determine their asymptotics, the scaling exponents and the amplitudes, in the low-temperature limit. Their amplitudes are shown to be related to the number of plane partitions.

Th. Th. Voronov and H. M. Khudaverdian give a direct proof of a theorem of Buchstaber and Rees concerning the symmetric powers of algebras and spaces. The proof is based on the notion of a formal characteristic function of a linear map of algebras and it is inspired by the authors’ studies in supermanifold geometry.

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