Drag reduction in turbulent boundary layers by in-plane wall motion

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Drag-reduction techniques capable of reducing the level of turbulent friction through wall-parallel movement of the wall are described, with special emphasis placed on spanwise movement. The discussion is confined to active open-loop control strategies, although feedback control is briefly mentioned with regard to peculiarities of spanwise sensing and/or actuation. Theoretical considerations are first given to explain why spanwise motion is expected to be particularly effective in skin-friction drag reduction. A review of the spanwise oscillating-wall technique is given next, with discussion of recent results and prospects. Last, waves of spanwise velocity are addressed, either spanwise- or streamwise-travelling. The latter include the oscillating wall as a special case. The generalized Stokes layer—i.e. the laminar, transverse oscillating boundary layer that develops under the action of the streamwise-travelling waves—is described, and its importance in determining turbulent drag reduction discussed. Finally, open issues like energetic efficiency and its dependence on Reynolds number are addressed.

Keywords: turbulent wall flows; drag reduction; spanwise wall forcing

1. Introduction

Among the techniques currently under active study that aim at reducing skin-friction drag in fully developed turbulent wall flows, this paper addresses those based on the movement of the wall, either the entire wall surface or a portion of it. Although, strictly speaking, wall movement does not include the use of body forces like the Lorentz force, volume forcing is not totally excluded from the picture. The considered techniques are active (i.e. require energy to work) and open-loop (no need for sensors), with other contributions in this Theme Issue addressing passive techniques. Other papers describe recent developments and applications in terms of sensors and actuators, so that the technological aspect is not addressed in detail here. Closed-loop (feedback) techniques, a vast subject by itself (see, for example, [1], for a recent review) are touched upon briefly, to highlight the importance of the forcing direction for this class of techniques.

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non-isotropic flows. Emphasis is given to spanwise wall forcing, and the paper aims at recollecting and illustrating the main results achieved so far after the review by Karniadakis & Choi [2].

The flows considered in this paper will be simple turbulent wall flows, either parallel flows (plane channel and circular pipe) or turbulent boundary layers. The universal nature of the viscous cycle that regenerates turbulence near the wall makes the results quite general. The very existence of this cycle motivates control techniques that target the crowd of coherent structures populating the near-wall region. On the other hand, Kang et al. [3] have discussed the relevance of a possible interaction between this cycle and the outer region of the turbulent boundary layer for active control of turbulence.

(a) Importance of energetic performance of the forcing

The choice of addressing open-loop techniques renders the energy effectiveness of the control particularly important. Indeed, the weak point of the open-loop approach is that forcing at finite amplitudes is employed, and thus a considerable amount of energy is required with respect to feedback techniques. This energy must be compared with the potential savings to establish the energetic performance of the technique. We follow the notation proposed by Kasagi et al. [4] for the quantities describing the energetic performance of the control, and indicate with $R$ the drag-reduction rate, which in a constant-rate flow is equivalent to the reduction of the pumping power $P$, i.e. $R \equiv (P_0 - P)/P_0$, where the subscript 0 refers to the uncontrolled flow. The net energy saving $S \equiv (P_0 - (P + P_{in}))/P_0$ accounts additionally for the power $P_{in}$ required to enforce the control action, although $P_{in}$ is often a lower bound that does not consider the actual efficiency of the actuator, but only the efficiency of the process by which an ideal actuator interacts with the flow. Finally, a third parameter linked to the efficiency of the control algorithm is its gain $G \equiv (P_0 - P)/P_{in}$, which expresses the yield of the control in terms of net saved energy per unit of energy input. Its importance is that $1/G$ is the minimum efficiency required by a physical actuator to achieve a net saving, i.e. $S > 0$.

(b) Importance of forcing direction

All attempts carried out in the recent past towards achieving a reduction of turbulent friction apparently have been somewhat heuristic in choosing the direction in which the wall forcing is applied. At the wall, however, the flow is highly anisotropic, and forcing in each direction brings about its own peculiarities. Forcing along the streamwise direction is usually thought to be a less effective means to affect the flow. On intuitive grounds, forcing in the wall-normal direction is usually thought of as the ‘best’ way of forcing, but it disrupts the natural state of turbulence significantly, at the finite amplitudes typically required by an open-loop control. Finally, forcing in the spanwise direction is found to be quite effective, and this work addresses several open-loop techniques based on this method. The idea that spanwise forcing is at least as effective as wall-normal forcing is not well assessed, though. This is probably the reason why most work dealing with distributed feedback control is limited to wall-normal blowing and suction, although the interest for spanwise forcing is recently growing (e.g. [5]).
Recent results are placing the effectiveness of spanwise forcing on a firmer ground. The review paper by Karniadakis & Choi [2] discusses physical aspects of the interaction of near-wall spanwise forcing with vortical structures, connecting the effectiveness of such a forcing strategy with the disruption of the near-wall viscous cycle. More on a statistical description of the flow, Jovanović & Bamieh [6] carried out an input–output analysis of the linearized Navier–Stokes equations that is tantamount to computing the frequency response of a sub-critical laminar flow to infinitesimal disturbances made by a distributed body force. Their results highlight the property of the system to actively respond to spanwise and wall-normal forcing, the response being larger in the streamwise velocity component. Jovanović & Bamieh [6]’s results, however, have no direct quantitative relevance to a true turbulent flow, for which the very concept of impulse response needs to be translated in a stochastic environment. The mean linear response of the entire turbulent flow to an impulsive forcing applied at the wall has been defined and then measured via direct numerical simulation (DNS) in a channel flow by Luchini et al. [7]. Such a mean response includes the average effects of turbulent diffusion and quantifies the relative effectiveness of forcing at the wall in all possible directions, in terms of both the amplitude of the response and how far in space and time the forcing can reach. Not unexpectedly, the quantitative picture that emerges from their work is that spanwise forcing is orders of magnitude more effective than streamwise forcing, while being minimally disruptive of the natural turbulent flow when the forcing is exerted at finite intensities.

In this paper, $x$, $y$ and $z$ will be used to indicate the streamwise, wall-normal and spanwise coordinates; the respective velocity components are $u$, $v$ and $w$. When not otherwise indicated, quantities are intended to have physical dimensions. A superscript $+$ indicates non-dimensionalization with the fluid viscosity and the friction velocity $u_t$ (cf. §6 for a discussion of this choice). When using units, the length scale is the channel half-width $h$, and the velocity scale is the centreline velocity $U_P$ of a laminar Poiseuille flow with the same flow rate. The structure of this paper is as follows. The oscillating-wall technique will be addressed in §2. Then, in §3, techniques that involve a spanwise forcing distributed along the spanwise direction will be discussed, while the more recent option of distributing the spanwise forcing along the streamwise direction will be addressed in §4. Available experimental realizations will be described in §5. Finally, §6 discusses the most important open issues to be addressed in the near future.

2. Spanwise-oscillating wall

The first demonstration of spanwise wall forcing to achieve a significant turbulent drag reduction dates back almost 20 years, with the papers by Jung et al. [8] and Akhavan et al. [9]. They leveraged the idea [10] that, in the context of three-dimensional boundary layers, a sudden spanwise pressure gradient generates a transient drop in turbulent friction, eventually followed by a recovery and a realignment of the near-wall flow to the new oblique direction. Making the spanwise pressure gradient harmonic in time, and suitably adjusting its frequency, yields a sustained reduction of drag. In their original paper, Jung et al. [8] carried out a numerical DNS study, and assessed the equivalence between an alternating
pressure gradient and a harmonic oscillation of the wall, which leads us to the first (and simplest) form of spanwise wall forcing considered in this paper:

\[ w = A \sin(\omega t), \]

where \( A \) is the forcing amplitude, and \( T = 2\pi/\omega \) is the forcing period.

The reported performance of the oscillating-wall technique is interesting, with drag reduction of the order of 50 per cent at the low values of Reynolds number \( Re \) considered in that early DNS. Experimental confirmation of the drag-reduction effect appeared soon thereafter, but the next result that raised the interest of the community was the outcome of the DNS study by Baron & Quadrio [11], where it was demonstrated that the energy saving \( R \) related to the reduced friction level may offset the energy loss due to the oscillating wall moving against the viscous resistance of the fluid, and thus that a net energy saving \( S \approx 0.07 \) is possible, with \( G = 1.5 \). This typically means working at low \( A \), because the energy cost \( P_{\text{in}} \) of moving the wall increases rapidly with \( A \).

Optimizing the oscillating-wall parameters is an obvious problem that did not find a quick solution. It must be recognized that the control law (2.1) implicitly defines a third parameter, the maximum displacement of the wall during the oscillation, given by \( D_m^+ = AT^+/\pi \). This means that searching for the true optimum combination of \( \omega \) and \( A \) must be done in such a way that \( D_m^+ \) is left free to change accordingly. This is often not the case in laboratory experiments, where a mechanical oscillating device is typically crank-driven, which of course imposes a fixed \( D_m^+ \), while in DNS this limitation does not apply. This is the reason why several authors, based on experimental work, proposed \( R \) scaling with \( A \), whereas DNS studies suggested the existence of an optimal oscillation period at all amplitudes, given by \( T^+_{\text{opt}} \approx 100 \). Quadrio & Ricco [12] suggested the existence of two optimal periods, i.e. the optimal period at fixed \( D_m^+ \) and the optimal period at fixed \( A^+ \), and this suggestion was later confirmed by Ricco & Quadrio [13]. \( R \) is found to increase monotonically with \( A^+ \) if \( D_m^+ \) is left free to change; at a given \( A^+ \), maximum drag reduction is always obtained at \( T^+ = T^+_{\text{opt}} \).

A further step towards understanding the working mechanism of the oscillating wall is to relate the amount of drag reduction to a function of the oscillation parameters. The key observation here [14] was that the oscillations induce a spanwise alternating flow that, once space-averaged, follows the analytical solution of the laminar Stokes layer. This analytical solution enabled Choi et al. [15] to suggest \( R \) being proportional to the product of the maximum acceleration of the Stokes layer during the cycle and the wall-normal distance at which this maximum takes place. This route was further pursued by Quadrio & Ricco [12], who were able to prove that the proportionality relation is linear. Finally, consensus exists on a qualitative explanation of how oscillations interact with the near-wall turbulence. The thickness \( \delta \) of the Stokes layer when oscillations are effective is of the order of a few wall units. In particular, the laminar Stokes solution yields \( \delta^+ \approx 6 \) for \( T^+_{\text{opt}} = 100 \). This points to an interaction between the Stokes layer and the near-wall structures, which can be described in terms of either a phase shift between the low-speed streaks and the quasi-streamwise vortical structures [11], or the creation of negative spanwise vorticity during the oscillation cycle [16]. Duggleby et al. [17] observe that velocity fluctuations present an outward shift of the peak value of their root mean square (r.m.s.) profiles,
and infer an outward shift of the turbulence structures, which thus convect at a faster speed, possibly explaining the shorter duration of sweeps and bursts events measured in experiments.

(a) From time to space dependence

It is worth mentioning here, as a preliminary step towards introducing the streamwise-travelling waves in §4, that the oscillating wall has a spatial counterpart. It is possible to get rid of the unsteadiness in the control law (2.1), on account of the fact that the flow itself is unsteady and convective in nature. Near the wall, where the forcing is located, the true convective velocity scale $C$ differs substantially from the (vanishing) mean velocity [18]. The profile $C(y)$ of the convection velocity is indeed similar to the mean velocity profile far from the wall, but assumes a nearly constant value $C_\infty^+ = 10$ as the wall is approached.

Viotti et al. [19] applied a stationary wall forcing of the kind

$$w = A \sin(\kappa_x x), \quad (2.2)$$

and verified that at fixed $A^+$ the wavelength $\lambda_x = 2\pi/\kappa_x$ has an optimal value for drag reduction, in analogy to the period $T = 2\pi/\omega$ of the oscillating wall. Moreover, the two optima turn out to be identical upon space–time conversion through the wall value $C_w$ of the convection velocity. The general picture in terms of drag reduction does not change, although the energetic budget is more favourable for the stationary waves than for the oscillating wall, with up to $S = 0.23$ and $G = 2.2$.

3. Spanwise-travelling waves

A spatially non-uniform spanwise forcing was first proposed by Du & Karniadakis [20] and Du et al. [21]: they simulated via DNS the forcing of the flow with a spanwise-oriented body force as follows:

$$F_z = I e^{-y/\Delta} \sin(\kappa_z z - \omega t), \quad (3.1)$$

where the forcing, with intensity $I$ and exponentially decreasing away from the wall on a length scale $\Delta$, is modulated in time and in the spanwise direction to form a harmonic wave with wavelength $2\pi/\kappa_z$ that travels along the $z$ direction. It must be said that forcing (3.1), being a body force, cannot represent an actual wall movement. However, when $\Delta$ is very small, except for the very first wall units of distance from the wall, the spanwise velocity profile induced by the body force, at least for $\kappa_z = 0$, resembles the oscillating one of the alternate spanwise Stokes layer. Du et al. [21] report drag reductions up to $R = 0.3$ at $Re_x = 150$, and claim that high $R$ can be achieved with good overall energetic performance when the product $IT\Delta$ remains constant. The effect of varying the wavelength is examined by giving $\kappa_z$ three different values, with the other parameters fixed, and concluding that $R$ increases with decreasing $\kappa_z$, the largest tested spanwise wavelength being 840 wall units.

Although non-conclusive because of the limited number of parameter combinations, their study [21] is, however, important since the flow visualizations reported therein clearly point to a significant modification of the near-wall turbulence cycle, with the meandering low-speed streaks almost totally...
disappearing and a wide, straight ribbon of low-speed fluid appearing in turn. This is contrasted with the flow modifications above the oscillating wall, which do not appear to alter the nature of turbulence regeneration significantly, apart from a periodic left/right tilting of the streaks and the obvious change in $Re$. More studies have followed, and, in particular, an experimental study is discussed in this Theme Issue.

Important to the focus of the present paper is the work by Zhao et al. [22]. They translated the spanwise-travelling wave of body force into a space–time distribution of wall acceleration. This is a considerable step forward, since one important parameter exits the picture, making it potentially simpler to determine the general dependence of $R$ on the wave parameters. If one wants to write the wall velocity forcing equivalent to equation (3.1) with $A = 0$ in terms of velocity waves, i.e.

$$w = A \sin(\kappa_x z - \omega t), \quad (3.2)$$

it turns out that a second harmonic component appears. Zhao et al. [22] indeed found very similar results between wall travelling waves and waves of body force in terms of drag reduction and flow statistics, but with disappointing results in terms of energetics, with negative $S$ for all the parameters tested. However, they too carried out a limited number of simulations, so that the general dependence of flow energetics on the wave parameters is an issue that deserves further consideration.

### 4. Streamwise-travelling waves

The spanwise-travelling waves considered by Zhao et al. [22] are a peculiar type of forcing where the spanwise velocity component is spatially modulated in the spanwise direction to form waves travelling in the spanwise direction. We have already discussed why forcing in the spanwise direction may be advantageous. It remains to be seen whether or not the other two ‘spanwise’ are essential, or even optimal, in terms of drag reduction.

Quadrio et al. [23] have tried a deceptively simple modification of the forcing described in equation (3.2), namely

$$w = A \sin(\kappa_x x - \omega t), \quad (4.1)$$

where the forcing still acts along $z$, but is modulated along $x$, so that the waves may travel in the streamwise direction with phase speed $c = \omega / \kappa_x$, the sign of which discriminates between forward- and backward-travelling waves. (An effect of the phase speed on the drag reduction, although the waves were waves of wall-normal velocity, was previously reported, for example, in [24]...) The waves (4.1) contain both the oscillating wall (2.1) and the stationary waves (2.2) as limiting cases, for $\kappa_x = 0$ and $\omega = 0$, respectively. Quadrio et al. [23] carried out a large DNS-based parametric study, exploring several hundred parameter combinations, the results of which are summarized in figure 1, where the contour plot shows how friction drag varies with $\kappa_x$ and $\omega$, at fixed $A^+ = 12$ and Reynolds number set at $Re = 200$.

While the dependences of drag reduction at fixed amplitude on $\omega$ at $\kappa_x = 0$ (oscillating wall) and on $\kappa_x$ at $\omega = 0$ (stationary waves) were known to be quite similar, the in-depth exploration of the entire $\kappa_x, \omega$ plane reveals an interesting yet complicated picture. Evident is a triangular region in the first quadrant (forward...
waves), delimited by the two thick neutral lines of zero drag reduction, where drag increases up to more than 20 per cent. Perhaps more interesting is the fact that the maxima of $R$ apparently reside on a quite well-defined crest line, not far from the vertical axis. In a $\kappa_x, \omega$ plane, the slope of a straight line from the origin identifies a velocity scale, and that identified by the drag-increasing triangular region corresponds to the value $C_w$, thus supporting the view that an increase in drag takes place when the waves travel at the same speed as the near-wall turbulence structures. However, the velocity scale related to the line of maximum $R$ still lacks an obvious explanation. The absolute maximum of drag reduction for $A^+ = 12$ is found to be $R = 0.48$, which is not far from relaminarization. (At this value of $Re$, about one-quarter of the friction drag is the laminar contribution, which has been demonstrated by Bewley [25] to be the true minimum one can reach. At lower $Re$, waves at $A^+ = 12$ have been found to relaminarize the flow. At the higher $Re$, waves at $A^+ = 12$ have been found to relaminarize the flow. At the higher $Re$, waves at $A^+ = 12$ have been found to relaminarize the flow.

Although the study was focused on finding the best performance in terms of $R$ at given $A^+$, a few calculations have highlighted the extremely good energetic characteristics of the waves (4.1), owing to the notable circumstance that the parameters that yield maximum $R$ are also those that guarantee almost minimal $P_{in}$. In the paper [23], a case is documented where $G = 12$ with $S > 0.1$, but it is believed that much better performances are possible. $G = 12$, however, is a high enough gain that a device with an intrinsic efficiency of as low as 8 per cent could still achieve a net saving.

Figure 1. Map of skin-friction changes, quantified as $100R$, induced by the streamwise-travelling waves, at $Re = 200$ and $A^+ = 12$. Dots indicate where a DNS has computed a change in friction drag. Thick lines indicate zero level, contours are spaced by 5 and negative contours (drag increase) are dashed. The short (long) arrow indicates regions of maximum drag reduction (increase). Maximum drag reduction is $R = 0.48$. Adapted from Quadrio et al. [23].
One step further has been achieved recently by Quadrio & Ricco [26]. Motivated by the insight gained through a similar analysis for the oscillating wall, as discussed in §2, they considered the travelling waves applied at the wall of a laminar Poiseuille flow, and were able to find an analytical expression for the transverse boundary layer created by the waves, called the generalized Stokes layer (GSL). The flow in this boundary layer depends on $x$, $y$ and $t$, and is described by the following formula, where $C$ is a complex constant and $\text{Ai}$ indicates the Airy function:

$$w(x, y, t) = A \Re \left\{ Ce^{2\pi i(x-ct)/\lambda} \text{Ai} \left[ \frac{\pi i/6}{\lambda v} \left( \frac{2\pi u_y w}{\lambda v} \right)^{1/3} \left( y - \frac{c}{u_y w} \right) \right] \right\}.$$  \hspace{1cm} (4.2)

This highlights the dependence of the solution on $u_y w$, the $y$ derivative at the wall of the streamwise velocity profile. This formula holds under the assumption that the thickness $\delta$ of the GSL is much smaller than the channel half-height $h$, so that the streamwise velocity profile can be considered linear for $y < \delta$. Within the limits discussed below, which apply to the oscillating wall and the Stokes solution too, the laminar GSL solution is found to agree with the turbulent space-averaged spanwise flow and to possess good predictive capabilities for turbulent drag reduction.

Figure 2 is a map showing how the GSL thickness, determined from the analytical solution (4.2), varies in the $\kappa_x, \omega$ plane. If an exception is made for the region very close to the origin, where the GSL thickness goes to infinity, reflecting the violation of the assumption mentioned above, the general appearance of this plot resembles the map of drag reduction of figure 1. The link between $\delta$ and turbulent drag reduction is evident through figure 3, which supports a clear relationship between $R$ and $\delta$, at least for a subset of the available data points, drawn with a dark colour.

The subset of dark points comprises all those points satisfying the following two conditions: (i) the phase speed of the waves is sufficiently different from the near-wall turbulent convection velocity $C_w$, and (ii) the waves act on a time scale that is significantly smaller than the lifetime of the turbulent structures. The first condition on the wave speed requires the forcing to be unsteady as seen by the
convecting turbulent structures, which might otherwise lock in to the waves and cause turbulent drag to increase. The condition on the wave time scale puts a limit on the maximum effectiveness of the drag-reduction mechanism. As in the conventional Stokes layer, the GLS thickness grows as the forcing become slower, and its interaction with the turbulent flows becomes more effective. However, the forcing time scale must remain smaller than the typical lifetime of the near-wall turbulence, otherwise the average structure sees an essentially steady, hence ineffective, forcing during its lifetime. An important observation, already put forward by Quadrio et al. [23], is that the forcing time scale is not the oscillation period of the waves, but the following quantity:

$$T \equiv \frac{\lambda_x}{(c - C_w)}.$$ (4.3)

In words, $T$ is the period of oscillation as seen by an observer moving at the same speed $C_w$ as the turbulence fluctuations, and reduces to the conventional oscillation period $T$ for the space-uniform oscillating wall.

5. Experimental realizations

A number of experimental realizations exist of the drag-reduction techniques discussed in the previous sections. Many of them, however, were designed to further our physical understanding, and did not employ technology suitable to be transferred to applications. However, recent achievements in actuator technology, as well as the promising energetic performances revealed by the travelling waves, are raising the interest for further developments.

(a) The oscillating wall

Early experimental verifications of the oscillating-wall concept are those by Laadhari et al. [27] in the geometry of a boundary layer and by Choi & Graham [28] and Choi [29] in the circular pipe. Several other studies, most of them
mentioned by Karniadakis & Choi [2], have further extended and generalized such results. Virtually all of them are low-Re set-ups where the wall oscillation is implemented through mechanical vibrating devices, and the measurements often involve putting a probe near the moving wall. A notable exception is that by Pang & Choi [30], who employed Lorentz forcing. Only recently, with energy efficiency coming into focus, the development of suitable actuators that can be deployed in applications and guarantee good energetic performance has seen progress, examples being the plasma dielectric barrier discharge (DBD) actuator and electroactive polymers. Other papers in this Theme Issue will address recent achievements in actuator technology.

To varying degrees, all the experimental works have had to deal with the problem of the spatial transient. As discussed by Quadrio & Ricco [12], when a finite portion of the wall is oscillated, the local friction decreases to its long-term reduced level only gradually with the streamwise distance from the leading edge. Such an adaptation length appears to increase with $A$ and can be quite long, a few thousands of wall units. To date, besides the space–time analogy that is possible in DNS, only a few laboratory experiments have addressed this problem, sometimes with contrasting results (see, for example, [16] versus [31]). The spatial transient is definitely something to be taken into account in those experiments where friction drag is evaluated through an integral effect, such as when the pressure drop across two duct sections is measured.

(b) Spanwise-travelling waves

There are, to the best of my knowledge, no experimental realizations of the spanwise-travelling wall forcing applied at the wall, whereas a few experiments have implemented the concept through the Lorentz body force, such as those carried out by Breuer et al. [32].

(c) Streamwise-travelling waves

The only laboratory test of the drag-reducing effect of the streamwise-travelling waves reported to date is that by Auteri et al. [33]. They work in the more experimentally friendly geometry of the circular pipe, where the naturally periodic spanwise (azimuthal) direction makes the implementation of the travelling waves easier. The spatio-temporal variations required to enforce the waves are obtained through a time- and space-varying azimuthal (rotational) speed of the pipe wall. While the harmonic dependence on time is easily implemented, the sinusoidal variation along the streamwise direction is discretized by imposing different rotation rates to different thin longitudinal slabs of the pipe, as illustrated by figure 4. Besides confirming the DNS results, this study demonstrates through a Fourier analysis the need to account for the unavoidable discretization of the ideal sinusoidal spatial waveform.

6. Future issues

We discuss briefly in this section the main open issues that, in the author’s opinion, need to be addressed in the next research steps.
Figure 4. Experimental realization by Auteri et al. [33] of the travelling-wave concept: the desired space–time variation of the transverse wall velocity is achieved through independent alternate motion of adjacent pipe slabs.

(a) Understanding the Reynolds number effect

This is obviously a fundamental question, as the values of $Re$ of applicative interest are typically very high; answering this question, either numerically or experimentally, is a true challenge and would have significant consequences. The question can be framed in the more general problem of describing the interaction between the inner and the outer motions in a turbulent boundary layer.

All the available information on spanwise wall forcing to date concerns flow at very low values of $Re$, with most of the DNS being at $Re_{τ} = 100–200$ and only a few reaching $Re_{τ} = 400$, whereas experiments reach perhaps three times higher. The current evidence is that performance seems to be mildly affected by increasing $Re$. However, the highest explored $Re$ is still far from those typical of applications. In that regime, two views are possible, both compatible with the available evidence. One possibility is that drag reduction eventually drops before reaching application-level $Re$. The alternative is that the observed weak decrease in performance is a low-$Re$ effect, which is deemed to disappear at reasonably high values of $Re$, say $Re_{τ} > 1000$. This view is supported by the numerical experiments of Iwamoto et al. [34], who considered a virtual active control system capable of suppressing the near-wall velocity fluctuations in a thin near-wall layer whose thickness $y_{d}^+$ is kept constant while increasing $Re$. The outcome is that $R$ indeed decreases at low $Re$ but then becomes almost constant—or, put another way, the thickness $y_{d}^+$ needed to achieve the same $R$ is almost constant, showing only a slow logarithmic growth, $y_{d}^+ \sim \log Re_{τ}$.

The physics of drag reduction still escapes our modelling ability, so that DNS is almost mandatory for numerical simulations, and it is well known that its computational cost increases rapidly with $Re$. Added to this is the fact that it is
not enough to carry out a single high-$Re$ simulation, since the very assumption of the optimum point scaling in wall units needs to be checked with parametric studies. On the other hand, a well-designed DNS enables us to measure friction with the reliability required to appreciate the small tendencies expected here. On the other hand, experiments can access higher values of $Re$, but at the price of somewhat more difficult measurements. There is a need for experiments especially tailored to high $Re$, as well as numerical approaches (like high-resolution large eddy simulation, LES) capable of reducing the computational cost of a full DNS.

Finally, let us mention in passing that there is a second Reynolds number to watch while the flow Reynolds number increases, and this is the Reynolds number of the transverse oscillating layer, which includes its thickness as the proper length scale. The drag-reducing properties of the Stokes layer above its critical Reynolds number are unknown.

(b) Understanding the scaling

In the present author’s opinion, our understanding of the drag-reduction mechanism is still unsatisfactory. In this respect, recent progress towards a unifying view of spanwise- and streamwise-travelling waves, and the inclusion of the oscillating wall as a special case of the latter, looks promising. The picture, however, is still blurred: we have reasonable qualitative explanations in terms of structure dynamics, but an agreed-upon description of the phenomenon in statistical terms is still lacking.

One potentially very important issue is the scaling of the optimal forcing parameters. Quite often flow statistics are plotted in ‘wall units’, but the employed friction velocity is that of the reference flow, which has the same nominal value as the Reynolds number. In this way, such a wall unit scaling simply becomes a disguised outer scaling. One has then to be careful to correctly interpret changes in statistical quantities, as they come from two flows (the reference one and the drag-reduced one) at quite different values of $Re$. This point was already emphasized years ago in the framework of feedback control. In the present context it might become critical: besides the flow statistics, the correct scaling of the optimal forcing parameters must be properly determined. In DNS, a way to enforce a well-defined scaling is to carry out the simulation at constant mean pressure gradient; in this way $Re$ remains constant, and instead of drag reduction an increase in flow rate is observed. Based on this unambiguous wall scaling, it can be observed, for example, that profiles of r.m.s. values of velocity and vorticity fluctuations do not show any attenuation of their peaks. On the contrary, besides an outward shift comparable to the thickness of the GSL, some other quantities, like the streamwise vorticity, do show an increase of their peak value.

The use of proper scaling is key to drawing a correct picture of the energy budget of mean and turbulent kinetic energies. Figure 5 is a graphical representation of such a budget for an oscillating-wall case, computed with DNS at constant pressure gradient at $T^+ = 100$ and $A^+ = 12$ ($R = 0.32$), compared with a reference fixed-wall flow. Time and volume integrating the mean and the turbulent kinetic energy equations makes it possible to describe the energy budget of the system compactly. When the wall oscillates, pumping energy is more efficiently converted by the system into a larger flow rate $U_b$, but an additional energy $S_w$ is spent to move the wall. Transfer between mean and turbulent parts occurs.
The simulations are carried out at constant pressure gradient to define an unambiguous wall scaling. The numbers indicate the energy fluxes made non-dimensional in wall units. The arrow lengths are approximately proportional to the energy fluxes; light grey indicates the reference case, and black refers to the added contribution in the oscillating-wall case.

through the turbulent kinetic energy production terms $P_{uw}$ and $P_{vw}$, for which the changes induced by the oscillations are minimal. At this low $Re$, dissipation from the mean profiles still overwhelms turbulent dissipation. Most of the changes in dissipation due to the oscillations occur as dissipation $D_W$ of the Stokes layer. The mean streamwise profile increases its quota $D_U$ of total dissipation, owing to the larger flow rate, and turbulent dissipation $D_T$ increases but only a little.

Although changes in friction drag must necessarily alter the way energy is taken from the mean flow and eventually dissipated into heat, the picture above is not conclusive in explaining what lies at the root of the drag-reduction process. However, only the correct scaling can put such a picture in the right perspective. We believe that looking at flow energetics in more detail has the potential to further our understanding of spanwise forcing and, more generally, to be a useful framework to study the phenomenon of turbulent skin-friction control.

### 7. Conclusions

This paper has reviewed the state of the art of spanwise wall forcing for reducing turbulent skin-friction drag, focusing, in particular, on recent achievements with the streamwise-travelling waves. The resulting view is that the field has recently made significant progress since the early discovery of the oscillating wall as a drag-reduction technique. However, much work remains to be done, and several new questions need to be answered, while we approach applications.

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