**Constraints on gravity on cosmic scales with upcoming large-scale structure surveys**

**By Rachel Bean**

*Department of Astronomy, Cornell University, Ithaca, NY 14853, USA*

We consider how upcoming, prospective large-scale structure surveys, measuring galaxy weak lensing and position in tandem with the cosmic microwave background temperature anisotropies, constrain cosmic scale modifications to general relativity. In such theories, both the homogeneous expansion history and the growth of large-scale structure can have signatures of the modification. We consider an equation of state figure of merit parameter, and introduce an analogous figure of merit parameter for modified gravity, to quantify the relative constraints. We discuss how assumptions about the presence of astrophysical and instrumental systematics such as galaxy bias, intrinsic alignments, weak lensing shear calibration uncertainties and photometric redshift offsets can impact the prospective dark energy constraints.

**Keywords:** gravitation; general relativity; cosmology; dark energy; large-scale structure

1. Introduction

Cosmic scale deviations from general relativity (GR) have been proposed as an alternative to the cosmological constant ($\Lambda$) and scalar field quintessence models, to explain the observed, recent accelerative expansion of the universe.

The principal evidence for cosmic acceleration comes from distance measurements—the luminosity distance to type Ia supernovae (SN1a), the ratio of angular diameter and radial distances from baryon acoustic oscillations (BAOs) in the three-dimensional galaxy distribution and the angular diameter distance to last scattering of the cosmic microwave background (CMB). However, if acceleration arises from modifications to GR, then it could also have distinct implications for the growth of large-scale structure.

There has been much interest in understanding how large-scale structure surveys could constrain the properties of gravity through measuring the growth history including: multi-frequency imaging, galaxy and weak lensing measurements [1–16], spectroscopic surveys to measure the peculiar velocity distribution [10,11,17–20] and 21 cm intensity surveys [21].

*rbean@astro.cornell.edu*

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A number of large-scale structure surveys are underway or planned in the coming decade, promising an unprecedented opportunity to test the origin of cosmic acceleration: large-scale structure surveys, such as Dark Energy Survey, the Large Synoptic Survey Telescope (LSST), the prospective ESA EUCLID mission; BAO surveys, BOSS and the prospective BigBOSS; and the PLANCK CMB survey. Together, these combine multiple observables: galaxy position, weak lensing, peculiar velocity and integrated Sachs–Wolfe (ISW) measurements, across a spread of epochs, to constrain in tandem the cosmic expansion and growth histories.

In this paper, we discuss the potential constraints on cosmic scale gravity that such upcoming surveys could give. We first review the cosmological framework to describe the homogeneous expansion and the growth of inhomogeneities, and then outline how galaxy position, weak lensing, and CMB auto- and cross-correlations are calculated. We employ a Fisher matrix technique to quantify potential constraints and discuss the complementary constraints from the different datasets.

2. The cosmological model

It has been shown that some large-scale modifications to gravity, such as $f(R)$ gravity, can be tuned to mimic the background expansion history of $\Lambda$CDM. We consider a standard, simple parametrization for the homogeneous evolution, including photons, baryons, cold dark matter (CDM) and an additional dynamical component producing accelerative expansion with an effective time-evolving equation of state (EoS), $w(a) = w_0 + (1 - a)w_a$, where $a$ is the scale factor, normalized to be 1 today. The Friedmann equation determines the expansion rate

$$H^2 = a^2 H_0^2 \frac{\Omega_\tau}{a^4} \left( \frac{\Omega_\gamma}{a^3} + \frac{\Omega_c + \Omega_b}{a^3} + \frac{\Omega_{\text{de}}}{a^{3(1+w_0+w_a)}} e^{-3w_a(1-a)} \right),$$

(2.1)

where $H = d\ln a/d\tau$ is the conformal time Hubble factor, $\tau$ is conformal time, $\Omega_X = 8\pi G \rho_X / 3 H_0^2$ is the fractional energy density in component $X$ today and subscripts $\gamma$, c, b, de, respectively, denote relativistic, CDM, baryonic matter and dark energy. We assume a flat spatial geometry throughout.

To describe the growth of inhomogeneities, we use the conformal Newtonian gauge, with the two Newtonian potentials $\psi$ and $\phi$ describing the time–time and space–space components of the perturbed metric [22]:

$$ds^2 = -a(\tau)^2[1 + 2\psi(x, t)]d\tau^2 + a(\tau)^2[1 - 2\phi(x, t)]dx^2,$$

(2.2)

We assume that matter is minimally coupled to gravity (whether it is modified or not), so that fluid equations describing the evolution of density, $\delta_i$, the divergence of the peculiar velocity, $\theta_i$, and anisotropic shear stress, $\sigma_i$, fluctuations are unaltered from GR + $\Lambda$CDM:

$$\dot{\delta}_i = -(1 + w_i)(\theta_i - 3\dot{\phi}) - 3H(c^2_{\text{si}} - w_i)\delta_i$$

(2.3)

and

$$\dot{\theta}_i = -H(1 + 3w_i)\theta_i - \frac{\dot{w}_i}{1 + w_i} \theta_i + k^2 c^2_{\text{si}} \delta_i - k^2 \sigma_i + k^2 \psi,$$

(2.4)

where dots represent a derivative with respect to conformal time.
The perturbed Einstein equations describe how the metric and matter perturbations are related. We introduce two functions, \(Q(k, a)\) and \(R(k, a)\), to allow a phenomenological modification to GR,

\[
k^2 \phi = -4\pi G Q a^2 \sum_i \rho_i \left( \delta_i + 3H(1 + w_i)\frac{\theta_i}{k^2} \right)
\]

and

\[
\psi - R \phi = -12\pi G Q a^2 \sum_i \rho_i (1 + w_i)\frac{\sigma_i}{k^2}.
\]

The combination \(QG\) can be viewed as an effective, scale- and redshift-dependent gravitational constant, \(G_{\text{eff}}\). Measuring \(Q \neq 1\) would not \textit{a priori} be a definitive signature of a modification to gravity, as a growth or suppression in perturbations in a dark component, as could arise through non-minimal couplings between CDM and neutrinos, could lead to the same effect. Introducing a difference between the two metric potentials is far harder to do through modifications to the matter sector. \(R \neq 1\) at late times would be equivalent to the sustained presence of an anisotropic shear stress in one of the dominant matter components. Anisotropic shear stresses are only really sustained in relativistic species, and would be exceptionally hard to maintain in a fluid component with \(w < 0\). As such, measuring \(R \neq 1\) may therefore represent a smoking gun for a deviation from GR on large scales.

We consider a phenomenological model for the modification to gravity

\[
Q(a) = 1 + (Q_0 - 1)a^3 \tag{2.7}
\]

and

\[
R(a) = 1 + (R_0 - 1)a^3. \tag{2.8}
\]

This allows the modification to evolve monotonically at late times in an analogous fashion to dark energy’s modification of the expansion history.

3. Cosmological constraints

The CMB, galaxy lensing shear and position correlations can be calculated using a line of sight approach [23]. The angular correlation between two observables \(X\) and \(Y\), where \(X, Y = T, E, g, \epsilon\) for CMB temperature and polarization, galaxy position and lensing observables, respectively, can be written as

\[
C^{X_a Y_b}_\ell = 4\pi \int \frac{dk}{k} \Delta^2_{\ell}(k) I^{X_a}_\ell(k) I^{Y_b}_\ell(k),
\]

where \(X_a\) is the \(a\)th photometric redshift bin of observable \(X\) and \(\Delta^2_{\ell}(k)\) is the dimensional power spectrum of the primordial curvature perturbations arising from inflation. \(I^{X_a}_\ell(k)\) is the angular transfer function for the \(a\)th redshift bin of source \(X\). The angular transfer function is dependent upon a window function.
$W_{X_a}$ and source term $S_X$,

$$I^X_a(k) = \int_0^{z_a} d\chi W_{X_a}(\chi) j_\ell(k\chi) S_X(k, z).$$

(3.2)

The sources for the observables are

$$S_{\text{ISW}} = e^{-\tau_{\text{reion}}(\dot{\phi} + \dot{\psi})},$$

(3.3)

$$S_g(k, \chi) = \delta_c,$$

(3.4)

and

$$S_s(k, \chi) = -\frac{k^2}{2}(\phi + \psi),$$

(3.5)

and the window functions are given by

$$W_{T,E}(\chi) = 1,$$

(3.6)

$$W_{g_a}(\chi) = n_a(\chi)$$

(3.7)

and

$$W_{e_a}(\chi) = \chi \int_{\chi}^{\infty} d\chi' n_a(\chi') \left( 1 - \frac{\chi}{\chi'} \right),$$

(3.8)

where $n_a$ is the distribution of galaxy number in the $a$th redshift bin, normalized such that $\int_0^{\infty} d\chi n_a(\chi) = 1$.

Following Smail et al. [24], for a flux-limited survey, we assume that the galaxies per unit redshift per square arcminute are distributed in redshift as

$$n_{\text{tot}}(z) \propto z^2 \exp \left( -\frac{z}{z_0} \right)^{1.5}.$$

(3.9)

We consider photometrically obtained objects for the lensing surveys, where there is an inherent uncertainty in an object’s true redshift. We assume that there are negligible catastrophic failures in how the photometric redshifts are related to their true redshift, $z$, and model the probability of an object having a photometric redshift $z_{\text{ph}}$ as a Gaussian conditional probability, with width $\sigma(z) = \sigma_{\text{ph}}(1 + z)$,

$$P(z_{\text{ph}}|z) = \frac{1}{\sqrt{2\pi}\sigma(z)} \exp \left( -\frac{(z - z_{\text{ph}})^2}{2\sigma(z)^2} \right).$$

(3.10)

The true distribution of galaxies $n_a(z)$ that falls into the $a$th photometric redshift bin, with $z_{\text{ph}}^{(a)} < z < z_{\text{ph}}^{(a+1)}$, is given by

$$n_a(z) = \int_{z_{\text{ph}}^{(a)}}^{z_{\text{ph}}^{(a+1)}} n_{\text{tot}}(z) P(z_{\text{ph}}|z) dz_{\text{ph}},$$

(3.11)

which for the conditional probability distribution (3.10) gives

$$n_a(z) = \frac{1}{2} n_{\text{tot}}(z) \left[ \text{erf}(x_{a+1}) - \text{erf}(x_a) \right],$$

where $x_a \equiv (z_{\text{ph}}^a - z - \Delta_z^a)/\sqrt{2}\sigma(z)$. We have allowed for a systematic offset, $\Delta_z^a$, in each photometric redshift bin.
When considering cross-correlations of multiple observables across multiple redshift bins, calculating the full $C_\ell$ becomes computationally onerous. For expediency, we instead impose the Limber approximation and approximate the full $C_\ell$ using the two-dimensional angular correlation function

$$C_{\ell}^{X_aY_b} = \int_0^{\infty} \frac{d\chi}{\chi^2} W_{X_a}(\chi) W_{Y_b}(\chi) S_X(k_{\ell}, \chi) S_Y(k_{\ell}, \chi),$$

where the source functions are evaluated at $k_{\ell} = \ell/\chi$.

We consider a prospective 20000 square degree large-scale structure survey configuration for a DETF stage IV type survey, emulating a EUCLID-like or LSST-like survey. The survey spans a range $0.001 < z < 3$, is split into $N_{\text{ph}} = 10$ photometric redshift bins and has a projected density of galaxies $n_g = 35$ per square arcminute of sky. The redshift bin boundaries are chosen so that the total number of observed galaxies in each survey is equally distributed between the bins. We assume an r.m.s. shear accuracy $\gamma_{\text{r.m.s.}} = 0.35$ and photometric redshift accuracy $\sigma_{\text{ph}} = 0.05$.

We treat nonlinear scales simplistically, in the context of modifications to gravity, and use the Smith et al. [25] prescription, calibrated off GR simulations, to boost galaxy transfer functions, $S_g$ and $S_e$, on nonlinear scales. If the effect of modification to gravity on the peculiar motion and overdensity follows the Zel’dovich approximation, then the Smith et al. prescription should still hold in the nonlinear regime [26,27]; however, we recognize that there are modified gravity models, where deviations on nonlinear scales can deviate from this [28].

We parametrize our ignorance about the underlying galaxy bias model by including scale- and redshift-dependent bias factor, $b_g$, interpolated over a $5 \times 5$ grid in scale and redshift. We allow the cross-spectra to be biased differently from auto-spectra through the introduction of an analogous grid for the correlation coefficients, $r_g$ [29].

We exclude galaxy position auto- and cross-correlations on nonlinear scales from the analysis by incorporating a cutoff for the $a$th photo-z bin, with median redshift $z_a$ of $z_{\text{max},a} = 0.132 z_a h \text{Mpc}^{-1} (\bar{z}_a)$ [29,30].

For our analysis, we consider constraints on standard cosmological parameters $\Omega_m h^2, \Omega_k, \Omega_b, \tau, n_s, \ln(10^{10} A_s)$, dark energy EoS parameters $w_0, w_a$, and modified gravity parameters $Q_0$ and $Q_0(1 + R_0)/2$. This combination of modified gravity parameters is chosen because galaxy position observables primarily measure $\psi$, and hence are sensitive to the combination $QR$, weak lensing and CMB ISW measurements, dependent on the geodesic path of photons are sensitive to the combination of the potentials $\phi + \psi$ and hence $Q(1 + R)/2$.

The large number of parameters, $p_i$, especially when using the full-bias model favours the use of a Fisher matrix approach; for $n$ parameters, only $n + 1$ samples
are required to estimate the parameter of Fisher matrix,

$$F_{ij} = \frac{\partial t_a}{\partial p_i} \text{Cov}^{-1} \frac{\partial t_b}{\partial p_j},$$  \hspace{1cm} (3.15)$$

where \( t_a \) are the set observables \( \{C_{\ell}^{\text{CMB}}, C_{\ell}^{gg}, C_{\ell}^{ge}, C_{\ell}^{ee}\} \) across all redshift bin combinations and \( \text{Cov} \) is the observational covariance matrix between each observable, including both statistical and systematic uncertainties. We have modified the publicly available CosmoMC and CAMB [31] codes to calculate the Fisher matrix and the correlation functions for the future survey specifications, in light of the intrinsic alignment (IA) and modified gravity models.

We consider the survey’s ability to constrain dark energy EoS and modified gravity parameters by calculating figures of merit (FoM). This is the det \([C(p_1, p_2)]^{-1/2}\) of the \(2 \times 2\) submatrix, for the pairs of parameters \(\{w_0, w_a\}\) and \(\{Q_0, Q_0(1 + R_0)/2\}\), taken from the covariance matrix marginalized over all other parameters, \(C_{ij} = F_{ij}^{-1}\).

We find that FoMs for a prospective survey are highly sensitive to the assumptions made about systematic errors, both instrumental and astrophysical. Here, we quantify the impact of such uncertainties by looking at the change in dark energy FoM when one marginalizes over a key astrophysical systematic, IAs or instrumental systematics such as weak lensing shear calibration and photometric redshift offsets. We marginalize over IAs following an earlier approach [29,32,33]. Photometric redshift offsets, \(\Delta z_i\), alter the galaxy distribution inferred from observations as in equation (3.12). When systematic offsets are considered, we model them following [34]: we allow independent offsets in each photometric redshift bin and impose \textit{a priori} on these offsets of \(\sigma(\Delta z_i) = 0.002\). We model shear calibration offsets by altering the measured shear correlation

$$C_{\ell}^{\epsilon_i \epsilon_j, \text{offset}} = (1 + \Delta m_i)(1 + \Delta m_j) C_{\ell}^{\epsilon_i \epsilon_j}$$  \hspace{1cm} (3.16)$$

and

$$C_{\ell}^{n_i \epsilon_j, \text{offset}} = (1 + \Delta m_j) C_{\ell}^{n_i \epsilon_j}$$  \hspace{1cm} (3.17)$$

and impose \textit{a priori} of \(\sigma(\Delta m_i) = 0.001 \sqrt{N_{\text{ph}}}\) in each bin, again following the study of Albrecht \textit{et al.} [34].

In figure 1, we show the prospective constraints on the dark energy parameters for the upcoming survey when CMB, lensing and galaxy position data are used independently, and in combination. For the large-scale structure surveys, the inclusion of ingalaxy position–shear cross-correlations when marginalizing over systematic uncertainties is comparable with those when purely considering lensing constraints alone with no systematic errors. Adding in the CMB constrains the other cosmological parameters and significantly improves the FoM. This is primarily because the CMB gives a distance measurement that is complementary to the galaxy and lensing distance and growth constraints. However, the CMB ISW signal also provides a bias-independent measure of the evolution of the potentials and therefore a complementary constraint on the modified gravity parameters. Adding in cross-correlations between CMB and large-scale structure observations improves constraints over and above just co-adding the individual datasets; it allows one to constrain some of the bias parameters introduced to describe uncertainties in the galaxy bias and IA models. Hence, the future
Figure 1. A comparison of constraints on the (a) equation of state (EoS) and (b) modified gravity (MG) parameters showing the predicted figures of merit (FoM), defined as $\text{EoS FoM} = \det\left[\text{Cov}(w_0, w_a)\right]^{-1/2}$ and $\text{MG FoM} = \det\left[\text{Cov}(Q_0, Q_0(1 + R_0)/2)\right]^{-1/2}$. We consider the constraints for galaxy position auto-correlations ‘gg’, lensing auto-correlations ‘ee’ and their cross-correlations ‘ge’ and when the CMB is included, both as purely CMB correlations, $TT$, $TE$ and $EE$ and when galaxy–CMB cross-correlations are included ‘all’. For the EoS constraints, we show the impact on the FoM of generalizing the growth history of large-scale structure, by comparing the constraints when general relativity is assumed $Q = R = 1$, and when the modified gravity parameters are allowed to vary. FoM with no systematic uncertainties are shown, in addition to those when uncertainties in IA modelling and systematic offsets are included. (a) Black triangles with solid line, GR: excluding IA; black squares with solid line, GR: including IA; brown triangles with dotted line, MG: including IA; and blue squares with dashed line, MG: including IA and systematic offset. (b) Black triangles with solid line, excluding IA; brown squares with solid line, including IA; and blue crosses with solid line, including IA and systematic offset. (Online version on colour.)
prospects for constraining dark energy hinge both on having a portfolio of complementary observations and in the tight control of systematic uncertainties, both astrophysical and instrumental.

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References


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