A gravitational puzzle

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The challenge to understand the physical origin of the cosmic acceleration is framed as a problem of gravitation. Specifically, does the relationship between stress–energy and space–time curvature differ on large scales from the predictions of general relativity. In this article, we describe efforts to model and test a generalized relationship between the matter and the metric using cosmological observations. Late-time tracers of large-scale structure, including the cosmic microwave background, weak gravitational lensing, and clustering are shown to provide good tests of the proposed solution. Current data are very close to proving a critical test, leaving only a small window in parameter space in the case that the generalized relationship is scale free above galactic scales.

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We have a puzzle before us: the Universe does not behave as it ought. The intensity of light rays travelling over cosmological distances appears to be weaker than expected. A simple interpretation that the distances are greater owing to an acceleration of the cosmic expansion merely deepens the mystery. The acceleration is widely attributed to dark energy, a heretofore unknown entity that pervades the cosmos. However, the simple interpretation contains numerous, implicit and explicit assumptions that may not be fully justified: that the motivating data are free of confusing, systematic effects of astrophysical origin; that our Universe is well described by the Robertson–Walker metric on large length and time scales; that light travels on geodesics of the Robertson–Walker metric; that the laws of gravitation are determined by general relativity (GR); and so on. We cosmologists have made exhaustive lists of these assumptions and tested one by one whether suspension of any one of these items might explain the behaviour or lead to a deeper understanding of the Universe. To date, it appears entirely reasonable that the observations may be explained by new laws of gravitation.

The solution to the puzzle, then, may be of gravitational origin. It seems like a small step to speculate that perhaps GR yields to a different theory on cosmological scales, whereby normal matter and radiation are sufficient to fuel an accelerated expansion. But what theory? We already have a perfectly good theory and yet we intend to supersede it on the basis of observations of a single phenomenon! There is little comfort to be gained by imagining that Einstein’s creation of GR was motivated by a single phenomenon, the perihelion precession.

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of Mercury. We know that Einstein had a higher motivation, being the natural extension of the principles of relativity and equivalence to the laws of gravitation. What principles do we apply?

It is useful to take a step back and consider what the evidence seems to suggest, and what GR assumes. For example, our puzzle suggests an imbalance in Einstein’s field equations: \( R_{\mu\nu} - (1/2)g_{\mu\nu}R \neq \kappa T_{\mu\nu} \). Specifically, the Ricci scalar curvature exceeds \( \kappa \) times the negative trace of the stress–energy tensor \( \tilde{R} = 6(\dot{H} + 2H^2) > \kappa(\rho - 3p) \), and moreover, the expansion scalar \( \Theta^2 \) exceeds \( 3\kappa \) times the energy density, \( \Theta^2 = 9H^2 > 3\kappa \rho \). In each case, however, GR predicts an equality. Dark energy, we know, is hypothesized to provide an additional source of energy density with sufficiently negative pressure to balance both of these inequalities. Perhaps, the observational data are evidence of a secular drift in the metric response to matter and radiation, so that no new energy and pressure are required. Instead, the cosmic acceleration is a manifestation of a change in the amount of space–time curvature produced per unit mass on cosmological scales. One can imagine that this might occur in a theory in which the source of curvature in the gravitational field equations is not merely the stress–energy tensor of gravitating matter and radiation, but instead some meta-function \( \Sigma \),

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}R = \kappa \Sigma_{\mu\nu}(T).
\]

If something like the above is at work, as occurs in a variety of scalar–tensor theories of gravity, then we may expect an enhanced response of metric perturbations to inhomogeneities of the matter. How do we test this idea? Recall that in GR, static Newtonian and longitudinal potentials owing to an ideal fluid source are determined as

\[
\psi(x) = -\frac{\kappa}{8\pi} \int d^3x' \frac{\rho + 3p}{|x - x'|} \quad \text{and} \quad \phi(x) = -\frac{\kappa}{8\pi} \int d^3x' \frac{\rho - p}{|x - x'|},
\]

where we use the conventions of Ma & Bertschinger [1]. In the cosmological context, if the source energy density and pressure in the above integral equations are replaced by the appropriate meta-functions,

\[
\psi(x) = -\frac{\kappa}{8\pi} \int d^3x' \frac{2\delta \Sigma_{\mu}(T)}{|x - x'|} \quad \text{and} \quad \phi(x) = -\frac{\kappa}{8\pi} \int d^3x' \frac{2\delta \Sigma_{ij}(T)}{|x - x'|},
\]

then we might reasonably expect the onset of cosmic acceleration to coincide with a sign change in the response of \( \psi \), since its source is proportional to the (de)acceleration parameter \( q \). We may also expect an enhancement in \( \phi \) as its source must also grow in order to balance the expansion and acceleration equations implied by GR. Together, these equations imply a change to the Poisson equation and a relationship between the potentials that depends not merely on the material source but also on new, implicit degrees of freedom that control the gravitational response to matter and radiation.

The consequences of this idea have been explored in Daniel et al. [2] and elsewhere. (See references therein for context and a longer bibliography.) There, it is conjectured that the gravitational response is such that the background evolution is indistinguishable from that predicted in the cosmological constant plus cold dark matter scenario, but the fluctuation response is different. A new
relationship between the Newtonian and longitudinal potentials, a ‘gravitational
slip’ or broken equivalence between $\phi$ and $\psi$ in the presence of non-relativistic
matter, arises as does a new Poisson-like equation,

$$\psi = [1 + \sigma(\tau, k)]\phi \quad \text{and} \quad \nabla^2 \phi = \mu(\tau, k)4\pi G a^2 \rho_m \Delta_m.$$ 

How big should we expect $\sigma$ and $\mu$ to be? Since the meta-functions must provide
a sufficiently large response to make up for the gap between the matter energy
density and the expansion scalar, different by a factor of nearly 3, then we may
expect $\mu$ as large as approximately 3 by today. And based on the relationship
between $\phi$ and $\psi$, parametrized by the effective pressure that the meta-functions
must provide, we may expect $\sigma$ as large as approximately $-1.5$ by today. Of
course values of $\mu$, $\sigma$ much closer to the GR values are condoned. But this rough
estimate gives a feeling for the degree of departure from GR that we may expect.
These expressions, along with conservation of the standard matter and radiation stress–energy tensor, are sufficient to specify the linear perturbation evolution. In the case that the post-GR effects develop at late times, with a strength that scales as the size of the mismatch between the left-hand side and right-hand side of Einstein’s equations, then $\sigma = \sigma_0 a^3$ and $\mu = 1 + \mu_0 a^3$. The consequences for the cosmic microwave background (CMB), for example, are shown in figure 1 for different values of the parameters. A more detailed analysis, presented in Daniel et al. [3], shows that the integrated Sachs–Wolfe (ISW) contribution to the temperature anisotropy varies monotonically with these parameters, nearly cancelling with the Sachs–Wolfe contribution at one point. However, the angular power spectrum is quadratic in the temperature anisotropy, so that ultimately, extreme values of $\mu$ and $\sigma$ yield large anisotropy. As with the ISW effect, observations such as weak lensing that depend on the combination $\phi + \psi$ will display a degeneracy in parameter sensitivity along the curve $\mu = 2/(2 + \sigma)$. The observational constraints to $\mu_0$ and $\sigma_0$ as of 2010 are shown in figure 2. The expected values $\mu_0 \sim 2$, $\sigma_0 \sim -1.5$ are just consistent with current data and may be put to a critical test in the near future.

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References


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