Rubber tubes in the sea

BY F. J. M. FARLEY1,*, R. C. T. RAINEY1,2 AND J. R. CHAPLIN1

1School of Civil Engineering and the Environment, University of Southampton, Highfield, Southampton SO17 1BJ, UK
2Oil and Gas Division, Atkins Ltd, Woodcote Grove, Epsom, Surrey KT18 5BW, UK

A long tube with elastic walls containing water is immersed in the sea aligned in the direction of wave travel. The waves generate bulges that propagate at a speed determined by the distensibility of the tube. If the bulge speed is close to the phase velocity of the waves, there is a resonant transfer of energy from the sea wave to the bulge. At the end of the tube, useful energy can be extracted. This paper sets out the theory of bulge tubes in the sea, and describes some experiments on the model scale and practical problems. The potential of a full-scale device is assessed.

Keywords: distensible; distensibility; bulge wave; Anaconda; wave energy; capture width

1. Introduction

The idea of using a distensible tube in the sea originated with Rainey inspired by the book by Lighthill [1]. He predicted [2] that if the tube is aligned in the direction of wave travel and the velocity of the longitudinal pressure wave (called bulge wave) in the tube is the same as the velocity of the waves in the sea, there will be a resonant transfer of energy to the tube. Further mathematical analysis revealed that the sea will generate a bulge in the tube that propagates in front of the sea wave and grows progressively larger. After about one wavelength the energy density in the tube is about 10 times that in the surrounding sea and can in principle deliver useful power, already demonstrated in models.

Here we present the more or less chronological steps in our understanding of the process, experiments inspired by theory, followed by new calculations inspired by observations. This leads to measurements and theory in reasonable agreement, allowing predictions for a full-scale wave energy converter (the Anaconda) [3]. The detailed design of such a wave energy machine with a practical mechanism for converting the bulge power to useful power will not be addressed here. In a related paper [4], the mathematics is developed in another way, and wave tank measurements are presented in greater detail.

We take this opportunity to include information and theorems about bulge tubes that we have found useful and relevant. The analysis assumes small sea waves and small pressure changes in the tube around its static inflation pressure. Second-order and nonlinear terms are neglected.

*Author for correspondence (f.farley@soton.ac.uk).

One contribution of 18 to a Theo Murphy Meeting Issue ‘The peaks and troughs of wave energy: the dreams and the reality’.

381
Hysteresis in the rubber is significant: so we start with the equations for hysteresis in rubber, and then develop the mathematics of a bulge tube interacting with water waves.

2. Force in rubber with hysteresis

Consider a rubber tube with wall thickness $h$ stretched so that the strain in the rubber is $\epsilon$. It is found that the force required is more or less proportional to the strain $\epsilon$ but in addition there is a component proportional to the rate of strain $\dot{\epsilon}$. This term implies that work is done internally upon the rubber, and the corresponding energy loss gives rise to hysteresis. The circumferential force in the rubber per metre length of the tube is

$$F = Eh(\epsilon + \beta \dot{\epsilon}),$$

(2.1)

where $E$ is Young’s modulus. For a sinusoidal strain $\epsilon = \epsilon_0 \sin \omega t$,

$$F = Eh\epsilon_0(\sin \omega t + \beta \omega \cos \omega t) = Eh\epsilon_0\sqrt{1 + \beta^2 \omega^2} \sin (\omega t + \delta),$$

(2.2)

where $\delta$ is called the loss angle, often tabulated for various rubbers, and

$$\tan \delta = \beta \omega.$$  

(2.3)

The first term in (2.2) is a force in phase with the strain and gives the peak energy stored (half maximum force $\times$ maximum strain) $J_{\text{peak}} = Eh\epsilon_0^2/2$, while the second term with force in phase with velocity determines the energy lost.

Integrating $F \cdot d\epsilon$ over one cycle gives

$$\frac{\text{energy lost per cycle}}{\text{peak energy stored}} = 2\pi \beta \omega = 2\pi \tan \delta.$$  

(2.4)

For a tube of radius $r$, cross section $S$ stretched by an amount $\delta r$ (2.1) becomes

$$F = \frac{Eh}{r} (\delta r + \beta \dot{r}) = \frac{Eh}{2S} (\delta S + \dot{S}),$$

(2.5)

and so the pressure across the tube wall as a result of its distension (called the bulge pressure) is given by

$$p_b = \frac{F}{r} = \frac{1}{DS} (\delta S + \beta \dot{S}),$$

(2.6)

in which $D$ is the distensibility of the tube, which is defined as

$$D = \frac{1}{S} \frac{dS}{dp_b} = \frac{2r}{Eh}.$$  

(2.7)

The dominant first term in (2.6) tells us that $\delta S = DS p_b$; so if $\beta$ is small we can replace $\dot{S}$ in (2.6) by $DS \dot{p}_b$. This gives

$$\delta S = DS(p_b - \beta \dot{p}_b).$$  

(2.8)
3. Bulge wave

Suppose the bulge pressure in the tube (owing to the distension of the wall) is \( p_b \), and the pressure of the wave outside is \( p_w \). The total pressure inside is then \( p = p_b + p_w \). Let \( u \) be the particle velocity in the bulge wave at longitudinal coordinate \( x \). Then following Lighthill [1] with an incompressible fluid of density \( \rho \) inside the tube, the acceleration is driven by the gradient of the total pressure

\[
\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x}, \tag{3.1}
\]

while the change in tube cross section \( S \) due to the accumulation of fluid at \( x \) is

\[
\frac{\partial S}{\partial t} = -S \frac{\partial u}{\partial x}. \tag{3.2}
\]

By differentiating both equations (3.1) and (3.2) again, one can eliminate \( u \) and obtain

\[
\frac{\partial^2 S}{\partial t^2} = \frac{S}{\rho} \frac{\partial^2 p}{\partial x^2}. \tag{3.3}
\]

Differentiating (2.8) twice with respect to \( t \) gives

\[
\frac{\partial^2 S}{\partial t^2} = DS \left\{ \frac{\partial^2 p_b}{\partial t^2} - \beta \frac{\partial^3 S}{\partial t^3} \right\}. \tag{3.4}
\]

Combining (3.3) and (3.4) to eliminate \( \partial^2 S/\partial t^2 \) gives

\[
\frac{\partial^2 p_b}{\partial t^2} - \beta \frac{\partial^3 p_b}{\partial t^3} = \frac{1}{D\rho} \frac{\partial^2}{\partial x^2} \{p_b + p_w\}. \tag{3.5}
\]

This is the differential equation for the bulge wave, including loss in the rubber.

In the absence of hysteresis (\( \beta = 0 \)) and with no external wave, this reduces to a simple wave equation with bulge speed \( c_0 \) related to the distensibility by

\[
D = \frac{1}{\rho c_0^2}. \tag{3.6}
\]

In passing, it is interesting to note that this definition of the distensibility (3.6) has diverse applications. For example, the adiabatic change in a volume of air for an applied pressure \( dp \) is given by

\[
\frac{dV}{V} = -D \frac{dp}{\rho} = -\frac{dp}{\rho c^2}, \tag{3.7}
\]

in which \( c \) is the velocity of sound waves in air, \( 340 \text{ m s}^{-1} \). Substituting numerical values, one recovers the adiabatic gas law \( dp/p = -\gamma_a dV/V \) with \( \gamma_a = 1.4 \).

Or consider the vertical stretching of the vertical coordinate \( \eta \) in a water wave of amplitude \( A \) and wavenumber \( k \) with \( c^2 = g/k \)

\[
\frac{d\eta}{\eta} = D d\rho = \frac{\rho g A}{\rho c^2} = kA, \tag{3.8}
\]

which is a well-known result.
Combining (3.6) with (2.7) gives a useful design equation relating the bulge speed \( c_0 \) to the parameters of an all rubber tube:

\[
Eh = 2\pi \rho c_0^2. \tag{3.9}
\]

For a given bulge speed \( c_0 \) and given modulus \( E \), this implies that the volume of rubber in the tube wall (and the cost of rubber) is proportional to the enclosed volume of water.

Returning to the tube in calm water \( (p_w = 0) \), let us drive the tube at one end \( x = 0 \) with an oscillating pressure of amplitude \( \rho g B \) at angular frequency \( \omega \). Assume that the solution is a gradually attenuated wave propagating along the tube in the \( x \)-direction, given by

\[
p_b = \rho g B e^{-\gamma x} e^{i(\omega t - k_2 x)}. \tag{3.10}
\]

Substituting in (3.5) and equating the real parts on each side of the equation gives

\[
c_2^2 = \left( \frac{\omega}{k_2} \right)^2 = c_0^2 \left( 1 - \frac{\gamma^2}{k_2^2} \right), \tag{3.11}
\]

where \( c_2 \) is the bulge velocity and \( c_0 \) is the velocity with no hysteresis given by (3.6). The bulge velocity is slightly reduced by the hysteresis.

From the imaginary part of (3.5) with substitution from (3.10) and (2.3)

\[
\gamma = \frac{\omega^2 \beta \omega}{2k_2 c_0^2} \approx \frac{1}{2} k_2 \beta \omega = \frac{1}{2} k_2 \tan \delta; \tag{3.12}
\]

so the attenuation length of the bulge wave is \( \lambda / \pi \tan \delta \).

Equation (3.10) is the expression for the bulge wave with the tube in air or calm water. The real part of (3.10) is

\[
p_b = \rho g B e^{-\gamma x} \cos(\omega t - k_2 x). \tag{3.13}
\]

Following the usual convention for differential equations, we call it the complementary function (CF).

Using (3.1) and (3.10) and assuming \( \gamma \) small one finds that the horizontal particle velocity in the bulge wave is

\[
u = \frac{k_2}{\rho \omega} p_b = \frac{p_b}{\rho c_2} = \frac{g}{c_2} B, \tag{3.14}
\]

where \( u \) and \( p_b \) are in phase. The amplitude of the longitudinal displacement in the bulge wave is \( u/\omega = (g/\omega c_2) B = (c_1/c_2) B \), where \( c_1 = g/\omega \) is the velocity of a water wave in deep water with angular frequency \( \omega \). If the bulge speed is equal to the speed of the waves, the particle displacement in the tube is the same as the bulge pressure in metres head.

The oscillating volume flow in the tube is \( Su \). If one defines the bulge tube impedance as \( Z = \) pressure/flow (see appendix A), then \( Z = \rho c_2 / S \).

4. Bulge wave excited by waves in the sea

To get some indication of how a bulge tube might perform in the sea, we try to solve the differential equation (3.5) in the presence of an external wave with
angular frequency $\omega$, wavenumber $k_1$, velocity $c_1 = \omega/k_1$ and pressure amplitude $\rho g A$. No doubt the excited bulge wave, if there is one, will radiate energy in the form of secondary waves; this is a refinement that should be included, but difficult to calculate, and so in the first pass it was omitted. This ‘radiation damping’ is computed later in the paper.

The external wave pressure is taken as

$$p_w = \rho g A \cos(\omega t - k_1 x),$$

and we look for a solution to (3.5) in which the bulge pressure is forced to have the same frequency and same wavenumber as the external wave but may have a complex amplitude $B$. Assume

$$p_b = \rho g B \cos(\omega t - k_1 x).$$

Using (3.11) and (3.12), this satisfies (3.5) if

$$B = \frac{k_1^2}{k_2^2 - \gamma^2 - k_1^2 - 2i k_2 \gamma} A.$$

Equation (4.2) is called the particular integral (PI). If $k_2$ is close to $k_1$, that is if the bulge speed is close to the phase velocity of the sea wave, the bulge amplitude $B$ becomes large: there is a kind of resonance.

Using (3.12) at the peak of the resonance, the pressure amplitude of the bulge wave is

$$B = i \frac{A}{\tan \delta}.$$

Note that the PI has the same wavenumber $k_1$ (and velocity) as the external wave. The CF has the same frequency, but its wavenumber $k_2$ (and velocity) is that of a free bulge wave.

To obtain the general solution of (3.5), we can add a CF (3.10) of any amplitude to the PI (4.2), because the combination still satisfies (3.5). Assuming the bulge pressure to be zero at the bow, $x = 0$, the desired solution is

$$p_b = \rho g B \left\{ \cos(\omega t - k_1 x) - e^{-\gamma x} \cos(\omega t - k_2 x) \right\}$$

with $B$ given by (4.3).

Combining (4.3) and (4.5) the pressure amplification factor $F = B/A$ is

$$F = \left| \frac{k_1^2}{k_2^2 - \gamma^2 - k_1^2 - 2i k_2 \gamma} \right| \cdot \left\{ \cos(\omega t - k_1 x) - e^{-\gamma x} \cos(\omega t - k_2 x) \right\}.$$  

$F$ gives the oscillating pressure inside the tube, compared with the oscillating pressure in the sea wave outside. We see that if $k_1^2 = k_2^2 - \gamma^2$, i.e if the incoming wave velocity matches the velocity of the free bulge wave, see equation (3.11), then $F$ is large and there is a resonant magnification of the bulge.
Figure 1. How the bulge wave works. The water wave is travelling to the right. The bulge leads the wave by $90^\circ$. Open arrows show the particle velocity in the bulge: forward when the pressure is high, backward when it is low, just as in the water wave. Black arrows show the movement of the tube wall. It is moving inwards when the wave pressure is high; so the wave does work on the tube. When the wave pressure is low, it is sucking the tube outwards, again doing work. So energy is transferred from the sea wave to the bulge wave.

If there is no hysteresis, $\gamma = 0$, (4.5) can be written as

$$p_b = -2 \rho g B \sin(\omega t - \bar{k}x) \sin \left( \frac{x \Delta k}{2} \right),$$  \hspace{1cm} (4.7)

where $\bar{k} = (k_1 + k_2) / 2$ and $\Delta k = k_2 - k_1$. This is a travelling wave with the mean wavenumber, and the pressure amplification factor $F$ is

$$F = \frac{k_1^2}{k \Delta k} \sin \left( \frac{x \Delta k}{2} \right) \approx \frac{1}{2} k_1 x.$$  \hspace{1cm} (4.8)

The approximation is valid if $x \Delta k$ is small; note that it does not matter if $\Delta k$ is positive or negative as the sign cancels.

We see from (4.8) that close to the resonance the bulge amplitude grows linearly along the tube: the bulge energy (see below) rises as $x^2$. But if the bulge wave velocity differs appreciably from the sea wave velocity, the beating between the CF and the PI in (4.5), which have different wavenumbers, causes the bulge amplitude to rise and fall along the tube.

If one takes a snapshot of the system at a moment when $\omega t = 2n\pi$, the bulge wave in (4.7) will look like $\sin(\bar{k}x)$, while the water wave looks like $\cos(k_1 x)$. The wave is propagating towards larger $x$; so we see that the bulge leads the wave by $90^\circ$: maximum bulge occurs where the water outside the tube is rising fastest. Thus the wave does work in lifting the bulge and this is not completely refunded where the wave is falling, because the bulge is then at its minimum: a net amount of energy is transferred to the bulge. In another sense, the bulge is surf-riding on the front of the wave, picking up energy as it does so. More generally, according to (4.4), on resonance the bulge leads the water wave with a phase shift of $90^\circ$ (figures 1 and 2).

*Phil. Trans. R. Soc. A* (2012)
Rubber tubes in the sea

Figure 2. If the tube snakes with the waves (travelling to the right), gravity drives the bulge forward along the tube. But the velocities match; so the wave keeps up with the bulge. In effect, the bulge is surfing in front of the wave, picking up energy and growing larger as it runs. Another point of view: in front of the wave the water is rising and lifts the bulge putting in energy; behind the wave the water is falling; so the tube is dropping losing energy, but at the waist there is less water to drop. (Online version in colour.)

5. Capture width

Using (3.14) the power in the bulge wave for a tube of area $S$ is

$$P_b = \frac{S u_{\text{max}} p_{b_{\text{max}}}}{2} = \frac{S \rho g^2 B^2}{2 c_2} = \frac{S \rho g^2 (AF)^2}{2 c_2}.$$  (5.1)

Compare this with the power per metre width in the sea wave of amplitude $A$

$$P_w = \frac{\rho g^2 A^2}{4 \sigma}.$$  (5.2)

Vertically, this power is attenuated exponentially with characteristic distance $1/2k_1$, so the power density in the sea per square metre at the surface is $2k_1 P_w$. Comparing this with (5.1), we find that the ratio of power per square metre in the tube to the power per square metre in the sea is $F^2$. Typically, for a tube one wavelength long, (4.8) implies $F = \pi$; so the power density in the bulge at the stern is about 10 times the power density in the sea.

Dividing (5.1) by (5.2) we find the capture width (CW)

$$CW = 2k_2 SF^2.$$  (5.3)

The CF is attenuated and will die away slowly: at the end of a very long tube we will be left with the PI only. Equation (4.7) shows that if the velocities do
not match, the bulge amplitude will oscillate as we progress along the tube. This oscillation fades as the CF dies away.

CWs obtained from (4.6) and (5.3) are plotted versus wave period in figure 3 for a tube 7 m diameter tuned to 10 waves for various values of the loss angle \( \delta \) and two lengths 100 m and 300 m. The multiple zeroes at small periods are due to the beating of the CF with the PI. One sees an impressively large CW and broad response. The longer tube has much larger peak CW, but the response is narrower because the beating of the PI with the CF has more time to take effect.

These predictions stimulated optimistic dreams and further study. It was realized that the bulge would radiate new waves into the sea and this would limit its growth, particularly for tubes longer than one wavelength, but it was not clear how this could be calculated. The large CWs predicted by (4.6) and (5.3) at the shorter wavelengths and plotted in figure 3 were not confirmed by...
experiments. Wave radiation by the bulge will be estimated below and turns out to be very important, especially for the shorter waves.

6. Verification

Figure 4 is a photograph of a tube being tested in a wave tank. One sees the bulges running in front of the water wave, as predicted. A video of these tests can be seen at http://www.bulgewave.com. Quantitative verification, including wave radiation by the bulge, is discussed shortly. Before that, we address some practical questions concerning real bulge tubes in the water.

7. Bulge size

Using (3.6)

\[ \frac{dS}{S} = \frac{dp_b}{\rho c_2^2}, \]

so the strain at the end of a bulge tube of radius \( r \) with an incident wave of amplitude \( A \) is

\[ \frac{dr}{r} = \frac{dS}{2S} = \frac{1}{2} k_2 AF. \]

For example, on resonance with a tube one wavelength long, (4.8) shows that \( F = \pi \), and so a wave of amplitude \( A = 2 \text{ m} \) would give a pressure amplitude of \( AF \approx 6 \text{ m head} \) at the end of the tube, while the wavenumber \( k_2 \) is typically \( 1/24 \text{ m}^{-1} \), and so the strain in the tube would be about \( \pm 12 \) per cent. If the tube is pressurized so that the pressure never drops below zero, the maximum circumferential strain would be 25 per cent. This is quite small for natural rubber.
It is apparent that most of the circumference could be made of inextensible fabric with a small elastic section comprising rubber.

8. Stored energy

In all waves and oscillating systems, there is a cyclic interchange of energy, usually between kinetic and potential energy. In the bulge wave, the potential energy is stored in the elastic wall of the tube. From (5.1) and (7.1), the energy to be stored per unit length is

\[ J_b = \frac{1}{2} p_{b\text{max}} d S_{\text{max}} = \frac{S p_{b\text{max}}^2}{2 \rho c_2^2} = \frac{P_b}{c_2} = \frac{P_b T}{\lambda} = \frac{\text{energy out per cycle}}{\text{wavelength}}. \]  

(8.1)

The energy is stored in the rubber before being delivered at the stern. But as the bulge is increasing along the tube, this energy density is only required near the stern. If the tube is inflated to a mean pressure equal to \( p_{b\text{max}} \), so that the internal pressure never swings negative, then the peak energy stored in the rubber will be four times larger.

Let us compare this with the energy that can be stored in rubber. With modulus \( E \) at strain \( \epsilon \), the energy stored per cubic metre is \( E \epsilon^2 / 2 \). So for tube radius \( r \) with wall thickness \( h \), the energy stored per metre length would be

\[ J_r = \pi r E h \epsilon^2 = 2 \pi r^2 \rho c_2^2 \epsilon^2, \]  

(8.2)

where we have taken the value of \( Eh \) from (3.9). Equating this to the requirement (8.1) and allowing a factor of 4 because of the static pre-inflation gives the bulge power capability of the rubber tube

\[ P_b = \frac{\pi r^2 \rho c_2^2 c_3^3}{2}. \]  

(8.3)

It is interesting to find that this is independent of the modulus and wall thickness, provided they satisfy (3.9). Furthermore, \( P_b \) is proportional to the tube cross section. The CW (5.3) is also proportional to the cross section; so we have a good match at all tube diameters.

For a 10 s wave with a pressurized tube and maximum strain \( \epsilon = 50\% \), then \( P_b = d^2 \times 0.38 \text{MW} \), where \( d \) is the diameter of the tube. For tube of diameter 7 m, the bulge power capability would be 19 MW!! This is much larger than our requirement.

This leads to the conclusion that we should use less rubber and work it harder. The circumference of the tube can be partly rubber with the rest comprised of inextensible watertight fabric. If only a fraction \( f \) of the unstretched circumference is rubber, for the same change in radius the strain in the rubber is increased by the factor \( 1/f \); so to exert the same force the wall thickness should be reduced by \( f \); the volume of rubber required is reduced by \( f^2 \) with corresponding savings in cost. The stored energy per unit volume of rubber is increased by \( 1/f^2 \). The design equation (3.9) becomes

\[ Eh = 2 \pi \rho c_2^2 f. \]  

(8.4)
Values of $f$ of order 0.25 are acceptable with the strain in the rubber increasing from 50 to 200 per cent. In this case, only one sixteenth of the amount of rubber is required.

A useful analogy between bulge tubes and electrical transmission lines is given in appendix A.

9. Aneurisms

As soon as one starts to experiment with inflated rubber tubes, one gets a nasty surprise... aneurisms, illustrated in figure 5. Beyond a critical pressure, the tube becomes unstable and bulges uncontrollably. This can be understood with reference to figure 6, which shows the tension $T$ in the wall of the tube versus radius $r$: the nonlinear curve (1) is the characteristic of the rubber; the modulus drops off as the rubber is stretched because the section gets thinner. The linear relationship (2), $T = pr$, is the tension required to balance the internal pressure $p$. For small pressures the lines intersect at A, a stable equilibrium point. As the pressure increases, the slope of line (2) increases and the equilibrium point moves upwards. Eventually, the equilibrium point jumps from B to C with a sudden increase in radius: there is an aneurism. With a further increase in pressure, the tube will break.

As one approaches the aneurism, the tube is becoming unstable; with less restoring force, the bulge speed drops. One can think of the aneurism as a bulge wave with zero velocity.

Figure 5. Aneurisms in rubber tubes and in a toy balloon. (Online version in colour.)
wall tension, $T = pr$

Figure 6. Wall tension $T$ versus radius $r$ for an all rubber tube. The required tension (line 2) intersects the rubber characteristic (line 1) at equilibrium point A. As pressure $p$ increases the equilibrium moves up to B, then jumps to C, creating the aneurism.

$T = p(a + x)$

Figure 7. Wall tension $T$ versus rubber elongation $x$ if tube circumference comprises $a$ metres of fabric for each metre of unstretched rubber. The origin of line 2 is displaced to the left. Intersection point A of lines 1 and 2 becomes stable, no aneurism.

When the circumference is only partly rubber, figure 7 applies. If the circumference comprises $a$ metres of fabric for each metre of unstretched rubber, the tube radius increases in proportion to $(a + x)$, where $x$ is the elongation of the rubber; so the required tension in the wall is proportional to $T = p(x + a)$, and the origin of the straight line (2) is displaced to the left. The tendency to make an aneurism is reduced and may be eliminated altogether if the fraction of rubber is small. We have found that with rubber fraction $f \approx 0.25$, there are no aneurisms. Moreover, as explained already, this uses less rubber and is more cost-effective.

10. Bulge speed versus pressure

Because of the nonlinearity of the rubber, the bulge speed varies according to the static pressure inside the tube. This can be calculated by extending the concepts
described earlier, using a polynomial fit to the stress–strain properties of the rubber. Figure 8 shows a theoretical prediction compared with measurements (made by S. Rimmer, Checkmate Seaenergy Ltd) on a composite tube intended for model tests. The agreement gives some confidence that we can design tubes for the full scale.

11. Power take-off

Many concepts for converting the bulge power into useful power have been explored. One of the simplest in principle is illustrated in figure 9. When a positive bulge pressure arrives at the stern, it drives water through a one-way valve into the high pressure accumulator. Similarly, during the low pressure phase of the bulge wave, water is sucked into the tube from the low pressure accumulator via a one-way valve. The head difference between the accumulators generates a smoothed flow through the turbine. Energy is stored in the accumulators, either by means of gravity, the compression of air or in rubber. Figure 10 shows experimental gravitational accumulators fitted to the tube of figure 8 with a level difference generated by the bulge wave. The flow between the accumulators was used to measure the bulge power.

More generally, similar one-way valves may be fitted along the length of the bulge tube, connected to long tubular accumulators that run parallel to the tube, with the effect that power is transferred from the bulge progressively as it travels. This distributed power take-off protects the tube from excessively large bulges that can build up during a storm, endangering the rubber. As we will see later, it can also contribute to larger CWs.

12. Wave radiation by the bulge

To compute the power captured by a bulge tube in a general case, with power take-off (PTO) at the stern or with the distributed PTO described earlier, we
all rubber PTO

one-way valves

turbine

storage volume
high pressure

storage volume
low pressure

storage may use
rubber
air pressure
or gravity

gravity has no hysteresis

Figure 9. Hydraulic PTO principle.

Figure 10. Model scale PTO. One-way valves let water enter the right-hand tank and draw water from the left-hand tank. A leak between the tanks measures the power produced. (Online version in colour.)

use a time domain simulation. The tube is divided into short segments and the fundamental equation (3.5) is followed step by step for each segment, typically with steps in time of about 1 ms. The simulation programme includes the wave radiated by the bulge, which tends to cancel the incoming wave. This is estimated as follows.

Phil. Trans. R. Soc. A (2012)
Rubber tubes in the sea

The power radiated by a shallow buoy with water plane cross section $S_b$, heaving with amplitude $B$, is

$$J = \frac{\rho k \omega^3 (BS_b)^2}{4}. \quad (12.1)$$

If the radiated wave amplitude at distance $r$ from the buoy is $\eta$ then

$$J = \frac{2\pi r \times \rho g^2 \eta^2}{4\omega}. \quad (12.2)$$

Therefore,

$$\eta = \sqrt{\frac{k^3}{2\pi r}} \cdot BS_b, \quad (12.3)$$

which agrees with [5, eqn (1)].

Assume that the radiation by a section of bulge tube of radius $r_0$ is the same as that of a shallow buoy displacing the same external volume. Suppose the bulge tube has cross section $S$ and bulge pressure amplitude $p(x)$ at distance $x$ along the tube and the bulge speed is $c$. From (3.6), the distensibility is $1/\rho c^2$. The volume displaced by an element of length $dx$ replaces $BS_b$ in (12.3). We add a factor $e^{-kr_0}$ because the tube is at depth $r_0$ below the surface on average and this will reduce the radiated amplitude.

$$BS_b = \frac{Sp(x)dx}{\rho c^2} \cdot e^{-kr_0}. \quad (12.4)$$

Substituting this in (12.3) gives the amplitude of the wavelet arriving at distance $x$ along the tube from an element of length $dx$ nearer the bow:

$$d\eta = \sqrt{\frac{k^3}{2\pi}} \cdot e^{-kr_0} \cdot S \cdot \frac{p(x)dx}{\rho c^2 \cdot \sqrt{x}}. \quad (12.5)$$

This formula applies if the source pressure $p(x)$ is harmonic. But in the more general case, the wave radiation is due to $dp/dt$ that is available in the programme; so we use

$$d\eta = \sqrt{\frac{k^3}{2\pi}} \cdot e^{-kr_0} \cdot S \cdot \frac{dp/dt}{\rho c^2 \cdot \omega \sqrt{x}} dx. \quad (12.6)$$

A wavelet from a segment near the bow takes time to propagate to the segment of interest. If the incoming wave is periodic, all motions will be at the same frequency and will propagate in the direction of wave motion at the normal sea wave velocity. To do this, we keep a record of $dp/dt$ all along the tube at all useful time steps in a circular stack. Then pull out the values we need in computing for a particular segment. This is not strictly accurate for a mixed sea for which we assume that the forward propagation is at the wave velocity corresponding to the peak of the Pierson–Moskowitz spectrum. Backward propagation is neglected, because the wavelets will arrive at any point with a mixture of phases and will cancel each other: a travelling wave only radiates in the forward direction.

The total radiated wave amplitude is subtracted for the incoming wave before calculating its driving effect on the bulge. Both are reduced by the factor $e^{-kr_0}$ because the centre of the tube is below the surface.
13. Radiation damping

In radiating an external wave, the bulge tube does work. This gives rise to a reaction pressure resisting the expansion of the tube. This pressure is proportional to the rate of change of bulge pressure \( \frac{dp}{dt} \), and so has an effect similar to hysteresis.

For the heaving buoy, suppose the reaction pressure is \( p_{\text{damp}} \). The buoy velocity amplitude is \( v = \omega B \); so the power radiated should be \( J = \omega B S_b p_{\text{damp}} \). Equating this to (12.1) gives

\[
p_{\text{damp}} = \frac{1}{2} k \omega \rho S_b v. \tag{13.1}
\]

To apply this to the bulge tube segment of length \( dx \), put \( S_b = 2\pi r dx \) and radial velocity \( v_r = \left( \frac{r}{2pc^2} \right) \omega p(x) \) and add the factor \( e^{-kn_0} \) because the radiation is reduced by the tube depth. The result is proportional to the length of the section \( dx \) considered (probably because there is interaction between neighbouring parts of the tube via the radiated wave). It is hard to know what value to put in for \( dx \). We put it equal to \( 1/k \), and the final result for the radiation damping pressure is

\[
p_{\text{damp}} = \left( \frac{\pi}{2\omega} \right) (kn_0)^2 \cdot e^{-kn_0} \left( \frac{dp}{dt} \right). \tag{13.2}
\]

This is in quadrature with the bulge pressure and adds to the hysteresis. Smaller values of \( dx \) will reduce the radiation damping in proportion. The value adopted seems to be validated by the agreement with experiment that we discuss next.

14. Theory versus experiment

The simulation was run for an experimental tube 6.8 m long, 0.266 m in diameter, with bulge speed tuned to 1.8 s waves, with a matched PTO at the stern. The CW is shown versus wave period \( T \) in figure 11 together with measurements carried out by the University of Southampton and reported in the associated paper [4]. The prediction of the simple theory (equations (4.6) and (5.3)), without wave radiation and without the radiation damping pressure, is also plotted and one sees that it was wildly optimistic, especially at short wave periods. The new theory fits the data quite well.

For a more accurate comparison, we measured the loss angle of the rubber for a sample of the rubber from the tube, and the experimental value \( \delta = 5.8^\circ \) was used in the theory. The data are plotted on an expanded scale in figure 12. The theory is slightly pessimistic and has the peak response at slightly lower wave period than the data. It agrees with an independent prediction developed by John Chaplin [4]. This suggests that the simulation is satisfactory.

To see what would happen in the real sea, the simulation was run using a Pierson–Moskowitz spectrum peaking at 0.125 Hz, power level 50 kW m\(^{-1}\), for a variety of tubes all tuned to 8 s waves, loss angle \( \delta = 2^\circ \), (i) with output from a matched load at the stern and (ii) with hydraulic PTO via one-way valves at 11 positions equally spaced along the tube feeding hydraulic accumulators, computing the power from the flow between the accumulators. The flow resistance and accumulator time constant were re-optimized for each point. The results are plotted in figure 13. One sees that the CW at first increases linearly with tube length but then flattens off owing to wave radiation. The radiation effect is more
important for the fatter tubes. If one compares the increase in cost (see (3.9)) with the increase in CW, it is apparent they are not cost-effective. They use more rubber but radiate so much that the CW does not rise in proportion. With the single matched load at the stern, it is very difficult to get above $CW = 7-8$ m. For diameter 2.5 m, the radiated wave kills the response for long tubes and it is not worth increasing the length much beyond 200 m. For fatter tubes, the useful length is about 100 m.

With the distributed feed to the accumulators, one does a good deal better because we pull off power before the decrease due to beating and before the radiated wave has much effect. With diameter 2.5 m, we get $CW = 10$ m for length.
Figure 13. CW for 8s waves, tubes of various lengths and diameters. Simulation results for a Pierson–Moskowitz spectrum with peak frequency at 0.125 Hz, power level 50 kW m$^{-1}$. The length of the tube is along the horizontal axis. The open points (circles, diameter 2.5 m; diamonds, diameter 5 m; triangles, diameter 8 m) are for tubes with a matched load at the stern. The solid points (circles, diameter 2.5 m; triangles, diameter 4 m) are for the distributed PTO with 11 pairs of one-way valves equally spaced along the length. Because the waves are random the CW has some scatter.

400 m rising to 15 m for length 750 m. For thicker tubes, you can get CW up to 20 m, though this may not be worth the expense. Note that this is hydraulic power from the accumulators, so it already includes the inefficiency of the accumulator process, which gives some mismatch and reflection and we are computing the power averaged over the Pierson–Moskowitz spectrum. For all these tests, the accumulator outflow and time constant were separately optimized to get the best CW in each condition.

With a distributed PTO, the CW for 2.5 m diameter tubes continues to rise even for lengths up to 750 m. For fatter tubes it plateaus earlier. Apparently the most cost-effective tube would be of diameter about 3 m, length 400–600 m and CWs up to 20 m hydraulic power averaged over the Pierson–Moskowitz spectrum appear possible.

15. Fatigue life of rubber

We have seen above in equation (8.1) that the energy delivered by the bulge must first be stored in the tube. How much rubber do we need for this and what is the optimum range of strain to use? By choosing the parameter $f$ in (8.4), that is the fraction of rubber in the circumference, we can decide how much stress there will be in the rubber.

The best data on the fatigue life of natural rubber are from Cadwell et al. [6, fig. 8] reproduced in figure 14. Although this is an old reference, we have been told by rubber experts (A. Muhr 2009, Tun Abdul Razak Research Centre, Phil. Trans. R. Soc. A (2012))
Malaysian Rubber Board, personal communication) that it is the best information available as no one else has cycled rubber so many times. It appears that for optimum life the rubber should never be allowed to relax to small strains; ideally the strain should not drop below 200 per cent (elongation factor 3) and very small strains should be rigorously excluded. With special arrangements, this can be achieved. Referring to figure 14, cycling between elongation 3 and 4 (strain 200–300%) would give a fatigue life of 100 million cycles, corresponding to 25 years of 8 s waves. So we should operate at a mean strain of 2.5 with a strain range $\Delta \varepsilon = 0.5$.

Data on natural rubber show that the effective modulus for small changes around strain 2.5 is $E_{\text{eff}} = 0.83 \text{MPa}$; so the energy stored in the tube would be $0.5E_{\text{eff}}\Delta \varepsilon^2 = 104 \text{kJ per tonne.}$ As an example, compare this with the energy storage requirement for a 1 MW bulge wave with 8 s period and wavelength 100 m. Equation (8.1) shows that we would need to store $80 \text{kJ m}^{-1}$ in the tube wall. This would require $80/104 = 0.77$ tonnes of rubber per metre length. The tube might be 200 m long with the same amount of rubber throughout; this would need 154 tonnes of rubber, costing about £5000 per tonne fabricated, a total cost of about £770,000. Assume that the conversion efficiency from bulge power to electricity is 50 per cent. Then in its 25 year expected life, the device would deliver 100 million kWh, and the cost of rubber per kilowatt hour comes to 0.8 p. A very low figure for the main structural cost.

This calculation is of course optimistic. It assumes that throughout its life the rubber is being optimally stressed. In practice, for a lot of the time the stress range (and therefore the stored energy) will be lower; so one needs to do a more
detailed calculation by taking into account the fluctuations in the wave climate. And one needs to protect the tube from larger strains, for example by using the distributed PTO mentioned above, and prevent the strain from going below about 100 per cent. And, while the cost of rubber is dominant, there are other costs to be included. So this calculation, based on the energy stored in the rubber, just gives a general order of magnitude estimate, which could be seen as encouraging.

16. Summary and conclusions

We have described a completely new type of wave energy converter (the Anaconda) using a distensible tube aligned in the direction of wave travel. The tube can carry pressure waves, called bulge waves, associated with longitudinal oscillations of the fluid inside (generally similar to sound waves). Rather surprisingly, if the velocity of the bulge wave is close to the phase velocity of the waves in the sea, energy is transferred from the sea, and the bulge grows more or less linearly with distance. This theoretical prediction has been confirmed by tests in wave tanks. After one wavelength the energy density in the bulge is about ten times higher than the energy density in the sea wave outside. CWs are estimated for a number of possible tube diameters and lengths. Various methods, not described here, can be used to convert the bulge energy into useful power.

In summary, it seems that rubber tubes in the sea may be able to compete with other wave energy converters. Being flexible, with no hard limits, they should be good survivors.

This work was supported in part by the EPSRC (grant no. EP/F030975/1).

Appendix A

(a) Electrical equivalent circuits

Lighthill [1] treats the bulge tube as a transmission line and introduces the concept of fluid impedance defined in general for waves in pipes as \( Z = p / V \), where \( p \) is the pressure amplitude and \( V \) is the amplitude of the oscillating volume flow. These concepts are generally useful in designing systems associated with a bulge tube and are summarized here for convenience.

Lighthill shows that for a bulge tube, the characteristic impedance is

\[
Z_0 = \frac{\rho c}{S}, \tag{A 1}
\]

where \( c \) is the bulge wave velocity and \( S \) is the cross-sectional area of the pipe. \( Z \) is the characteristic impedance of the tube treated as a transmission line. For a bulge tube 5 m in diameter, tuned to 10 s waves, \( Z_0 = 795 \). (All units are MKS.)

(b) Inductance

Continuing the electrical analogy, the definition of inductance is

\[
L = \frac{p}{V}. \tag{A 2}
\]
A good example is a length of pipe with no viscous losses. If the flow is accelerating, some pressure gradient is required to drive it; so the pipe presents an inductance. For a rigid pipe of length \( x \), cross section \( S \), one finds immediately

\[
L = \frac{\rho x}{S}.
\]  

(A 3)

For example, the inductance of a rigid pipe of 5 m diameter is 51 per metre length. The usual rules apply; so for an oscillating pipe of period 10 s with \( \omega = 0.6 \), the impedance \( Z = \omega L = 30 \) per metre length. We see that this is low compared with the impedance \( Z_0 \) of the bulge tube calculated earlier; we would need about 25 m of pipe (490 tonnes of water) to have a reactance equal to \( Z_0 \). This means that a short section of rigid pipe can be attached to the end of a bulge tube with no serious impediment.

If a pipe of area \( S \) ends in a mass \( M \), it is equivalent to a length of pipe \( x = M/\rho S \); so the mass is equivalent to an inductance \( L = M/S^2 \).

(c) Capacitance

A storage reservoir has fluid capacitance, defined by

\[
C = \frac{\text{volume}}{p} = \int \frac{V \cdot \text{d}t}{p}.
\]  

(A 4)

For example, consider a storage accumulator that—when expanded to a volume of 600 m\(^3\)—develops a pressure of 1 bar. This could be either a water tank of horizontal section 60 m\(^2\) or some distensible container made of rubber. Its capacitance would be 0.006. For 10 s waves, its reactance would be \( Z = 1/\omega C = 280 \). Combined with the tube impedance calculated earlier, the time constant would be \( Z C = 4.8 \) s.

It is not always obvious how to construct the equivalent circuit of a complex fluid system; are two impedances in series or parallel? The essential criterion is how the individual pressures and volume flows combine. If the volume flow is the same in each element but the pressures add, then they are in series. If however they both experience the same pressure but the flow is divided between them, then they are in parallel.

(d) Bulge tube as transmission line

For an electrical transmission line with inductance \( L \) and capacitance \( C \) both per unit length, the line impedance is \( Z_0 = \sqrt{L/C} \) and the wave velocity is \( c = 1/\sqrt{LC} \). To apply these formulae to the bulge tube, use (A 3) and \( C = dS/dp = DS \), where \( D \) is the distensibility of the tube. This gives \( c^2 = 1/\rho D \) and \( Z_0 = \rho c/S \) both in perfect agreement with Lighthill [1], who uses a completely different derivation.

The bulge tube can be represented by a transmission line with the correct characteristic impedance, but the end conditions are counterintuitive. If the end of the tube is completely free to move longitudinally (large volume flow), this is equivalent to an electrical short circuit. If, on the other hand, the end of the tube is fixed (zero flow, maximum pressure), it is equivalent to an open-ended transmission line. If a mass \( M \) is attached to the end of the bulge tube, the

Phil. Trans. R. Soc. A (2012)
equivalent circuit is a line terminated by an inductance, calculated as above. The higher the mass (inductance), the smaller the movement and the closer the approach to an open ended line.

References

5 Rainey, R. C. T. 2001 The Pelamis wave energy converter: it may be jolly good in practice, but will it work in theory? In 16th Int. Workshop on Water Waves and Floating Bodies, Hiroshima, Japan. See http://www.iwwwfb.org/.