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Decoherence without dissipation

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That decoherence can take place in the presence of energy conservation seems to be a poorly known fact. That lack of knowledge has, for example, bedevilled the discussion of the 'black hole information' problem. I present a simple model that illustrates such energy-free decoherence.

It has become a truism in the discussions of the quantum behaviour of black holes that if black holes destroy the coherence of the quantum state outside the black hole, then that must lead to energy non-conservation. This argument was first expressed in the paper by Banks *et al.* [1] and has become a 'fact', leading to the regard of the so-called black hole information paradox as one of the greatest problems facing quantum gravity, despite the arguments of Unruh & Wald [2] that those arguments were suspect. In short, they said that black holes remember the amount of energy that went into their formation in the gravitational field surrounding the black hole. Mass, angular momentum and charge are all encoded in the fields surrounding the black hole, and as such can and will influence the radiation emitted via the Hawking process in black hole evaporation.

In this paper, I want to look at a very simple problem: the scattering of two particles off each other, in which, if one examines only the two particles, one finds that the scattering process preserves the energy after the scattering event (i.e. energy before equals energy afterwards), but the density matrix for the state of the two particles goes from a pure state to a mixed state after the scattering.

This demonstration will take place not in the context of quantum gravity, but in the context of ordinary quantum mechanics, in which case the only way that conversion can take place is by the system becoming correlated (entangled) with another 'hidden' quantum system. In the black hole case, this is the 'singularity' within the black hole that could equally well be regarded with no loss of generality as some sort of baby universe, or eventually disconnected space–time. In my model, I will use a 'spin-bath' as the hidden sector. By using 'spins', I mean any two-level quantum system that can always be modelled as if it were a three-dimensional spin 1/2 system. In what follows, I will refer to these as spins (without the quotes), although they should not necessarily be seen as real spins but as generic two-dimensional Hilbert-space objects. My model is closely related to the 'spin decoherence' emphasized by Prokof'ev & Stamp [3] and Stamp [4] as a dominant decoherence mechanism in some condensed matter systems.

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One contribution of 11 to a Theme Issue 'Decoherence'.

(In their case, the 'spins' are also real spins, but can often be any system with a finite dimensional Hilbert space.) They also found that their ideas met with initial disbelief in the condensed matter community, although the ideas have had experimental confirmation [5,6]. Their ideas have still not diffused into general knowledge in their community, just as Unruh and Wald's ideas have still resisted diffusion in the gravitational/particle physics community.

Consider two particles, which for sake of simplicity I will assume have the same mass, and live in a $1+1$ dimensional space–time. They interact only when in contact with each other, and their interaction is mediated by some hidden degrees of freedom that are represented by a number N of spin $1/2$ objects, with spin operators $\vec{S}_i$. The interaction Hamiltonian is assumed to be of the form $\delta(x_1 - x_2)\sum_i S_i^3$, where $S^3 = \frac{1}{2}\sigma^3$ is the third Pauli spin matrix, whereas the kinetic energy is the usual $(1/2m)(p_1^2. + p_2^2)$.

The spins alter the interaction potential of the two particles, making that potential a function of the spins. At the same time, the two objects, when located together, cause the spins to precess around the z-axis. It is the correlation introduced between the scattering of the particles and the direction of the spin vector that leads to the entanglement between the two systems, and the decoherence of the particle scattering.

Going into the centre of mass coordinates

$$Y = \frac{x_1 + x_2}{2}, \tag{1.1}$$

and

$$y = (x_1 - x_2), \tag{1.2}$$

the Schrödinger equation, becomes

$$i\partial_t \Psi(t, Y, y, \{s_i\}) = -\frac{1}{m}\partial_Y^2 \Psi - \frac{1}{2m}\partial_y^2 \Psi + \mu\delta(y)\left(\sum_i S_i^3\right)\Psi, \tag{1.3}$$

where μ is a coupling constant.

This system has interactions that are time-independent, and thus the energy is an exactly conserved quantity, and one can find the energy eigenstates of the system. The energy eigenstates of this model can be solved by the separation of variables

$$\Psi_{E+\epsilon} = \psi_E(Y)\phi_\epsilon(y, \{s_i^3\}), \tag{1.4}$$

where the s_i^3 are the eigenvalues of S_i^3,

$$-\partial_Y^2 \psi = mE\psi \tag{1.5}$$

and

$$-\frac{1}{2m}\partial_y^2 \phi + \mu\delta(y)\left(\sum_i s_i^3\right) = \epsilon\phi, \tag{1.6}$$

hence the total energy is $E + \epsilon$.

The first equation has the usual plane wave states, $\mathrm{e}^{\mathrm{i}KY}$, as solutions with $K=\pm\sqrt{mE}$. The second equation has the scattering states

$$\phi_{\epsilon,\{s_i^3\},+}=(\mathrm{e}^{\mathrm{i}ky}+A_{k,s_i}\,\mathrm{e}^{-\mathrm{i}ky})\Theta(-y)+B_{k,s_i}\,\mathrm{e}^{\mathrm{i}ky}\Theta(y) \tag{1.7}$$

and

$$\phi_{\epsilon,\{s_i^3\},-}=(\mathrm{e}^{-\mathrm{i}ky}+A_{k,s_i}\,\mathrm{e}^{\mathrm{i}ky})\Theta(y)+-B_{k,s_i}\,\mathrm{e}^{-\mathrm{i}ky}\Theta(-y), \tag{1.8}$$

where $\Theta(y)$ is the Heaviside step function, and the $+$ and $-$ refer to incoming states from the left and right, respectively. (I will ignore the bound states, as the initial states, which I will choose, will not contain any bound state components.) The boundary conditions at $y=0$ give

$$1+A=B \tag{1.9}$$

and

$$ik((1-A)-B)=2m\mu\sum_i(s_i^3)B \tag{1.10}$$

(where I have suppressed the subscripts on the coefficients). The solution is

$$B=\frac{2ik}{2ik+2m\mu\sum(s_i^3)} \tag{1.11}$$

and

$$A=-\frac{2m\mu\sum(s_i^3)}{2ik+2m\mu\sum(s_i^3)}. \tag{1.12}$$

Now, let us assume that the initial state is two well-separated Gaussian packets for the two particles travelling towards each other, and that the state of each of the spins is the $+\frac{1}{2}$ eigenstates of the S_i^1 operators at times far in the past. We have that the state in the far distant past is given by $\lambda_1(x_1,t)\lambda_2(x_2,t)\prod_i|s_i^1$, where λ_1 and λ_2 are assumed to have support only for very large values of x_1 and $-x_2$, and such that the two particles are travelling towards each other. For the sake of argument, let me take

$$\lambda(x_1,0)=\frac{1}{\sqrt{\sigma\sqrt{2\pi}}}\mathrm{e}^{-x_1^2/4\sigma^2}\mathrm{e}^{\mathrm{i}k_0x_1} \tag{1.13}$$

and

$$\lambda(x_1,0)=\frac{1}{\sqrt{\sigma\sqrt{2\pi}}}\mathrm{e}^{-x_1^2/4\sigma^2}\mathrm{e}^{-\mathrm{i}k_0x_1} \tag{1.14}$$

where I have projected to time $t=0$ using the uncoupled equations. In the distant past ($t\ll 0$), these represent Gaussian packets travelling towards each other with momentum k_0. Expressing these in terms of the Y, y coordinates, we have

$$\lambda_1\lambda_2 = \frac{1}{\sigma\sqrt{2\pi}}\,\mathrm{e}^{-Y^2/2\sigma^2}\,\mathrm{e}^{-y^2/8\sigma^2+ik_0 y}. \tag{1.15}$$

Thus, long before the scattering, the state is assumed to be the product state of the $s_i^1 = +\frac{1}{2}$ eigenstates of the spin, the centre of mass state

$$\psi_{in}(t, Y) = \frac{1}{2\pi\sqrt{\sigma\sqrt{4\pi}}}\int \sqrt{\sigma}\,\mathrm{e}^{-\frac{(\sigma^2+2i)\frac{t}{m}}{2}K^2}\,\mathrm{e}^{\mathrm{i}KY}\,\mathrm{d}K \tag{1.16}$$

and the difference state

$$\phi_{in}(t, y) = \frac{1}{2\pi\sqrt{\sigma\sqrt{\pi}}}\int \sqrt{\sigma}\,\mathrm{e}^{-2\sigma^2(k-k_0)^2+i(t/2m)k^2}\,\mathrm{e}^{\mathrm{i}ky}\,\mathrm{d}k. \tag{1.17}$$

Long after the scattering ($t\to\infty$), the total state is

$$\Psi = \psi(t, Y)\frac{1}{2\pi\sqrt{\sigma\sqrt{\pi}}}\sum_{\{s_i^3\}}\int_0^\infty \left(A_{k,s_i^3}\mathrm{e}^{-iky} + B_{k,s_i^3}\mathrm{e}^{\mathrm{i}ky}\right)\mathrm{e}^{\mathrm{i}2mk^2t}\mathrm{e}^{-\sigma^2(k-k_0)^2}\mathrm{d}k\frac{1}{2^{N/2}}|\{s_i^3\}\} \tag{1.18}$$

Let me assume that $k_0\sigma \gg 0$, so that the behaviour only near k_0 need be considered.

Now, we want to look at the reduced density matrix for the two particles. The centre of mass motion $\psi(Y, t)$ is unaffected by anything, and will simply factor out of the equations. Thus, we need look only at the density matrix for the relative coordinate y. This is most easily carried out in the momentum representation, so that

$$\begin{aligned}\rho(k, k') = \frac{1}{2\pi\sigma\sqrt{\pi}}\,\mathrm{e}^{\mathrm{i}2m(k^2-k'^2)t}\frac{1}{2^N}\sum_{s_i^3}\Big[&A^*_{k',s_i^3}A_{k,s_i^3}\,\mathrm{e}^{\mathrm{i}2m(k^2-k'^2)t}\Theta(-k)\Theta(-k')\\ &+ B^*_{k',s_i^3}B_{k,s_i^3}\Theta(k')\Theta(k) + B^*_{k',s_i^3}A_{k,s_i^3}\Theta(k')\Theta(-k)\\ &+ A^*_{k',s_i^3}B_{k,s_i^3}\Theta(-k)\Theta(k')\Big],\end{aligned} \tag{1.19}$$

where the assumption that the N spins are all in the S_i^1 eigenstates is contained in the factor of $1/2^N$ and the equal sum over all of the s_i^3 eigenstates.

Let us assume that the Gaussian is very narrow in momentum space ($1/\sigma < 2m\mu$), so that we can take $k = k' = k_0$. The on-diagonal terms are

$$\rho(-k_0, -k_0) \propto \frac{1}{2^N}\sum_{s_i^3}\frac{(2m\mu)^2(\sum_i s_i^3)^2}{(2m\mu(\sum_i s_i^3)^2 + k_0^2)}, \tag{1.20}$$

$$\rho(k_0, k_0) \propto \frac{1}{2^N}\sum_{s_i^3}\frac{k_0^2}{((2m\mu(\sum_i s_i^3))^2 + k_0^2)} \tag{1.21}$$

and

$$\rho(-k_0, k_0) = \rho(k_0, -k_0) = 0. \tag{1.22}$$

The last term comes about because these terms are proportional to

$$\sum_{s_i^3} \frac{\sum_i s_i^3}{(2m\mu \sum_i s_i^3)^2 + k_0^2} \tag{1.23}$$

and this term flips sign if we take all $s_i^3 \to -s_i^3$. Because both the positive and negative signs have equal probability under the assumption that we have made about the spin states, these cancel. This reduced density matrix for the scattering loses its off-diagonal terms in the momentum representation when one reduces over the spin space.

This system clearly conserves energy in the particle sector. Long before and long after the interaction, the spins all have zero eigenstate of energy, and the energy is all in the two particles. The total energy is conserved. But while the initial state is a pure state, the final density matrix for the particles on their own is not. It has non-zero entropy. For a broader initial state, one also gets decoherence between the various k values as well, caused by the different scatterings by the various spin states.

Note that, because of the form of the interaction (which depends only on the difference of the two particle coordinates), this system also trivially conserves momentum.

While the above-mentioned example has a minimal loss of coherence (entropy increase of $\ln(2)$ if k_0 has value $2m\mu\,\mathrm{N}$), this arises because of the assumption of a very narrow packet in phase space. If the packet has a broad width of the order of $2m\mu\,\mathrm{N}$, then there will also be loss of coherence between the energy eigenstates. Essentially, the decoherence is caused by a rotation of the spin vector. Because, for a total spin of S, the spin states can be distinguished if they rotate by about $\Delta S = 1$. Thus, the states of various k_0 for which the spin rotations differ by unity will be decohered by the scattering.

I am not arguing that the internal states of the black hole are such spin states. However, I am arguing that they share with these spin degrees of freedom the feature that after the black hole has evaporated they have zero energy. The energy is all outside the black hole—initially in the energies of the particles that create the black hole, and finally in the particles that are evaporated from the black hole. As mentioned, they could be in some sort of baby universe that split off from the interior of the black hole, or they could simply represent the degrees of freedom that fell off the edge of space–time at the singularity within the black hole. Gravity, unlike flat space–time in which the intuition of most physicists was honed, has the feature that time can have an end. Degrees of freedom can disappear by falling off the edge of the universe, or they can appear from things such as the initial singularity of the universe.

Physicists are familiar with the loss of entropy in electromagnetic radiation travelling to infinity, a loss that certainly does not entail any crisis for physics. Now, that entropy is often associated with the loss of energy as well, which is probably the origin of the intuition that there is some relation between energy dissipation and entropy production. However, the necessity of that dissipation is simply not there. One can carry off entropy in internal spin degrees of freedom. If we imagine that there exists some particle with an internal spin of value $\mathrm{e}^{10^{10}}$. Then, while the kinetic energy of that particle can be small, its entropy can be huge (up to 10^{10}).

It is important to note that, while those two particles are interacting with each other, the total energy of the system is NOT completely contained in those two particles. The presence of the particles at $y=0$ causes the spins to precess, a precession that requires the spins to have energy, i.e. during the interaction, the total energy is shared between the spins and the particles. The spin–particle interaction energy is proportional to $\sum \sigma_i^3$. While its expectation value is zero in the chosen state, the expectation value of this operator squared is non-zero, i.e. the interaction energy has zero expectation value of energy during the collision, but a large value of the energy squared (a large energy fluctuation). However, after the interaction has taken place, the system is in an eigenstate 0 of the spin–particle interaction energy.

This explicit counterexample to the claimed theorem of Banks *et al.* raises the question of where the problem in their proof occurred. Their master equation for the density matrix is precisely of the Lindblad form [7,8], an equation that makes the Markovian assumption that the loss of coherence is completely memoryless. The change in the density matrix is without memory, and depends only on the current state. This, of course, means that it does not remember how much energy has been emitted, and thus has no means of conserving energy (part of the energy being hidden behind the horizon and inaccessible to the outside world). While a reasonable approximation for many systems, the earlier mentioned counterexample shows that it is not always a good approximation.

The main lesson is that decoherence and energy need not be linked. Decoherence can occur without energy loss. This is true of condensed matter systems and possibly also for gravitational black hole systems. Whether or not this actually occurs for black holes is of course a fascinating and still open question. But there is no crisis for physics, if black holes do create loss of coherence for the quantum fields outside the black hole.

References

1 Banks, T., Susskind, L. & Peskin, M. E. 1984 Difficulties for the evolution of pure states into mixed states. *Nucl. Phys. B* **244**, 125–134. (doi:10.1016/0550-3213(84)90184-6)

2 Unruh, W. G. & Wald, R. M. 1995 Evolution laws taking pure states to mixed states in quantum field theory. *Phys. Rev. D* **52**, 2176–2182. (doi:10.1103/PhysRevD.52.2176)

3 Prokof'ev, N. V. & Stamp, P. C. E. 2000 Theory of the spin bath. *Rep. Prog. Phys.* **63**, 669–726. (doi:10.1088/0034-4885/63/4/204)

4 Stamp, P. C. E. 2006 The decoherence puzzle. *Stud. Hist. Phil. Mod. Phys.* **37**, 467–497. (doi:10.1016/j.shpsb.2006.04.003)

5 Morello, A., Stamp, P. C. E. & Tupitsyn, I. S. 2006 Pairwise decoherence in coupled spin qubit networks (theoretical predictions). *Phys. Rev. Lett.* **97**, 207206. (doi:10.1103/PhysRevLett.97.207206)

6 Takahashi, S., Tupitsyn, I. S., van Tol, J., Beedle, C. C., Hendrickson, D. N. & Stamp, P. C. E. 2011 Decoherence in crystals of quantum molecular magnets (with experimental confirmation). *Nature* **476**, 76–79. (doi:10.1038/nature10314)

7 Kossakowski, A. 1972 On quantum statistical mechanics of non-Hamiltonian systems. *Rep. Math. Phys.* **3**, 247–274. (doi:10.1016/0034-4877(72)90010-9)

8 Lindblad, G. 1976 On the generators of quantum dynamical semigroups. *Commun. Math. Phys.* **48**, 119–130. (doi:10.1007/BF01608499)