Decoherence and its role in the modern measurement problem

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Decoherence is widely felt to have something to do with the quantum measurement problem, but getting clear on just what is made difficult by the fact that the ‘measurement problem’, as traditionally presented in foundational and philosophical discussions, has become somewhat disconnected from the conceptual problems posed by real physics. This, in turn, is because quantum mechanics as discussed in textbooks and in foundational discussions has become somewhat removed from scientific practice, especially where the analysis of measurement is concerned. This paper has two goals: firstly (§§1–2), to present an account of how quantum measurements are actually dealt with in modern physics (hint: it does not involve a collapse of the wave function) and to state the measurement problem from the perspective of that account; and secondly (§§3–4), to clarify what role decoherence plays in modern measurement theory and what effect it has on the various strategies that have been proposed to solve the measurement problem.

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1. The myth of the ‘conventional interpretation’

Foundational discussions of quantum theory have a tendency to talk about the ‘conventional’ or ‘orthodox’ or ‘standard’ interpretation of quantum mechanics. It is not generally very clear what is meant by this, but a rough stab might be that it consists of these two principles.

— The Measurement Algorithm. Observable quantities are represented by self-adjoint operators; the possible outcomes of a measurement of some observable are the eigenvalues of the associated operator; the probability of a given measurement outcome obtaining is given by the usual (Born) probability rule.

— The Projection Postulate. While in the absence of measurement, a system evolves unitarily and deterministically, according to the Schrödinger equation, when a measurement is made the system evolves stochastically, with its state vector being projected onto the eigensubspace corresponding to the actual measurement outcome. As such, the dynamics of quantum theory have a dual nature, with one evolution rule for non-measurement situations and one for measurement situations.

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One contribution of 11 to a Theme Issue ‘Decoherence’. 
Often (including by Dirac [1]), the second is derived from the first by consideration of repeat measurements: presumably (so goes the reasoning), two measurements in succession had better give the same result, and this can only be guaranteed if the Projection Postulate holds.

It is generally held that this ‘conventional interpretation’ is profoundly unsatisfactory conceptually and philosophically, essentially because it treats ‘measurement’ as a primitive term. Measurement, so the argument goes, is a physical process like any other, so (i) our theory should not contain assumptions that can be stated only by treating measurement as some primitive process and (ii) whether a quantum system evolves according to one rule or another should not depend on whether we classify a physical process as a ‘measurement’ or not.

My concern, however, is not what makes the orthodox interpretation unsatisfactory; it is what makes it orthodox. To be sure: the orthodox interpretation is pretty much what we find in von Neumann’s and Dirac’s original presentations of the subject [1,2]. To be sure: it is pretty much what we find in most textbooks on quantum mechanics.1 And to be sure: it is pretty much what we find in most discussions of the quantum measurement problem. But it is not something that we actually find in use, much or at all, in mainstream applications of quantum theory.

We can see this pretty straightforwardly in the case of the Projection Postulate. The postulate tells us how a quantum system evolves during measurement, and this tells us immediately that it can only play a role in applications of quantum physics in situations where we want to analyse repeated measurements. If all we care about is the outcome of single measurements, the Measurement Algorithm tells us all we need to know.

But if the point of the Projection Postulate is to analyse repeated measurements, there is an embarrassing problem: the Postulate tells us successive measurements of the same quantity always give the same result, and this is straightforwardly false in many—perhaps most—experimental contexts. (We can see this particularly dramatically in quantum optics: when we make a measurement of a photon, the photon is typically gone at the end of the measurement.)2

The case of continuous measurements (like those performed by a Geiger counter) also demonstrates that the Projection Postulate is unacceptable as an analysis of quantum dynamics during the measurement process. The only way to analyse continuous measurements via the Postulate would be to treat them as the limiting case of increasingly frequent projective measurements. But the quantum Zeno effect ([11]; see [10] for discussion) tells us that the result of any such limiting-case measurement is to freeze the system’s evolution altogether, in flat contradiction of what we actually observe in continuous measurements. (This was the original reason that Misra and Sudarshan, who did analyse continuous measurement in exactly this way, called the quantum Zeno effect a ‘paradox’.)

So if the only way the Projection Postulate could figure into the practice of physics is via the case of repeated measurements, and if it is patently unacceptable

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1See, for instance, [3–6]; [7] is an interesting exception.
2Margenau [8] pointed this out almost at the dawn of quantum theory, and it has been extensively discussed since; see, for instance, [9,10] and references therein.

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for that task, then there can be no practical role for the Postulate in physics.\textsuperscript{3} And for what it is worth, crude sociological measures seem to back this up: a search for ‘projection postulate’, ‘wave-function collapse’ and the like in the archives of Physical Review turns up only a few hundred references, nearly all of which turn out to be (i) foundational discussions, (ii) discussions of proposed alternatives to quantum theory, or (iii) theoretical quantum-computation discussions, where ‘measurement’ does indeed get treated as primitive. (For comparison, searches for terms like ‘state vector’ or ‘Hilbert space’ or ‘Schrodinger equation’ typically turn up several tens of thousands of references.) It is also notable that in relativistic quantum mechanics, even defining the Projection Postulate is a delicate matter because of relativity of simultaneity. One might then expect that textbooks on quantum field theory would need to address this point, but no such textbook of which I am aware does so; we can infer that the Projection Postulate is not much missed by practicing particle physicists or quantum field theorists.

However, I do not wish to suggest that we have no effective way to analyse the process of repeated measurement. Actually, we can analyse it just fine (where here ‘just fine’ means ‘just fine if our goal is getting the empirically observed answer’, not necessarily ‘just fine conceptually speaking’). Doing so, though, requires us to drop the idea that measurement, as conceived in the Measurement Algorithm, is a primitive process.

This should have been obvious in any case. Treating ‘measurement’ as a primitive term is not just philosophically unsatisfactory: it is a non-starter if you actually want to build and use a measurement device. And since we do actually build and use measurement devices, we cannot just be treating measurement processes as primitive!

To expand: textbook and foundational discussions of quantum mechanics often give the impression that measurement devices are inexplicable black boxes that we find scattered across the desert, each one stamped with a self-adjoint operator and equipped with a funnel to insert quantum systems and a dial displaying the outcomes. But, of course, real measurement devices are designed very carefully by experimental physicists. It is not a primitive feature of them that they measure whatever it is that they measure: it is a deliberate design feature. Perhaps at one time that design proceeded by handwaving appeals to quantum-classical correspondence, but in modern experimental physics, the design of a quantum-mechanical measurement device relies extensively on quantum theory itself.

How can this be done? The answer, pretty much invariably, is that we apply quantum theory—unitary quantum theory, with no Projection Postulate—to the measurement process itself, and then apply the Measurement Algorithm to the combined system of original-system-plus-measuring-device. This might

\textsuperscript{3}An apparent counter-example might be repeated measurements of neutron spin or photon polarization via differently oriented filters. I am inclined to agree with Margenau [8] that such processes are not really examples of repeated measurements, since no outcome is recorded until the whole process terminates. A similar criticism applies to use of repeated ‘measurement’ in measurement-based quantum computation. The thesis of this paper is not in any case appreciably affected if ‘no role’ becomes ‘virtually no role’: the processes described here can be perfectly well accounted for in the general framework I discuss later. (I am grateful to an anonymous referee for the ideas behind this footnote.)
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seem like an infinite regress—does not applying the Algorithm presuppose yet one more measurement?—but in practice, the regress can be ended when the system is macroscopically large, so that we can treat its macroscopic features—say, its centre-of-mass position or momentum—as straightforwardly observable properties.

In the particular case of repeated measurements, a multiply conducted measurement will need multiple readout displays. The measurement algorithm, applied to the combined original-system-plus-apparatus quantum system, delivers a determinate probability prediction over those displays. In some situations, it predicts that the displays are perfectly correlated; in these situations, we could have got away with using the Projection Postulate. In most situations, the correlation will be either imperfect or absent altogether. Similarly, in the case of continuous measurements, analysing the actual measurement dynamics—using unitary quantum mechanics—and then applying the Measurement Algorithm will give us the probability of a decay (say) being registered as a function of time (and will, in general, tell us that the decay rate is not especially sensitive to how closely the system is monitored, except in extreme cases).

We are left with an understanding of the measurement problem that is a better fit to contemporary scientific practice. There is no conventional interpretation of quantum theory. What there is, is the formalism of unitary quantum theory, shorn of any interpretation. We go from quantum theory to empirical predictions, not by understanding unitary quantum theory as a description of physical reality, but by applying a Measurement Algorithm, in principle at the level of microphysics but, when pushed, ultimately at the level of macroscopic, straightforwardly observable, facts. But that Algorithm (i) seems ill-defined and shot through with terms that do not belong in fundamental physics (‘macroscopic’ and ‘straightforwardly observable’ are only slightly more satisfactory than ‘measurement’ as primitives), (ii) relatedly, runs into the problem that even macroscopic observation processes are just more physical processes, and so should not have special status, and (iii) seems to block any attempt to understand quantum theory as a theory that tells us about Nature, and not just about what the dials and computer screens in our laboratory are going to do in any given situation.

From this point of view, it is also reasonably clear what would count as solving the problem. We either need to find some way of understanding unitary quantum theory so that it can give experimental predictions directly and not via the unsatisfactory Measurement Algorithm—or we need to modify or supplement unitary quantum theory so as to obtain a new theory that does not need the Measurement Algorithm.

2. Two concepts of state space

The account of the previous section may look unfamiliar. Normally (at least in foundational and philosophical discussions), the argument for a measurement problem is more direct: unitary quantum theory predicts macroscopic superpositions, which are not observed, so unitary quantum theory must fail when applied to macroscopic systems (and presumably something like a Projection Postulate must be applied, albeit perhaps at the macroscopic level rather than for microscopic systems).
The basic assumption here is that Schrödinger-cat states like

\[ \alpha |\text{cat alive}\rangle + \beta |\text{cat dead}\rangle \] (2.1)

have to be understood as somehow describing a cat in an indefinite state of aliveness—a cat that somehow is alive and dead at the same time. But why are we required to think this way?

In the philosophy literature, this reading of (2.1) is often justified by appeal to the so-called eigenvalue–eigenvector link (E–E link).

**Definition 2.1 (E–E link).** A quantum system possesses a definite value of an observable if and only if its state is an eigenstate of the associated operator; if so, the definite possessed value is the eigenvalue associated with that eigenstate.

According to the E–E link, the state I wrote schematically as |cat alive\rangle can be understood as a state of a live cat because it lies in some subspace of the cat’s Hilbert space corresponding to the ‘aliveness = 1’ subspace of the ‘aliveness operator’; similarly, the state |cat dead\rangle is in the ‘aliveness=0’ subspace and so is the state of a cat that is determinately not alive. The superposition of the two, lying in neither subspace, represents a cat that is neither alive nor dead.

This argument should not worry us though because the E–E link has no place in any serious discussion of the measurement problem. Like the Projection Postulate, it plays basically no role in physical practice (searching the archives of Physical Review for the phrase and its variants gives precisely two hits); unlike the Projection Postulate, it seems to be purely an invention of philosophers, which does not appear in any quantum physics textbook of which I am aware. And this is not surprising, since in certain contexts it is fairly absurd, as can be seen easily by considering measurements of position. No realistic quantum wave packet will remain in a bounded region of space for any non-zero length of time, unless kept there by infinite (and hence unphysical) potentials; in any physically realistic situation, some small section of the wave function will tunnel to arbitrarily large distances, arbitrarily quickly. So realistic quantum wave functions are spread right across space. Of course, macroscopic bodies can have wave functions that are, and that remain, extremely small outside some narrow region; wave functions like these are straightforwardly used in physics to represent localized bodies. But the fact remains that no such state lies in the eigenspace corresponding to any outcome of a position measurement, however coarse grained.\(^4\) So according to the E–E link, all realistic quantum systems have completely indefinite positions. If the E–E link were an inextricable part of quantum physics, this would count as a *reductio ad absurdum* of quantum theory; as it is, it just counts as a *reductio ad absurdum* of the E–E link.

So much for the E–E link. But even if *that* argument for the unacceptable weirdness of Schrödinger-cat states does not go through, there might perfectly well be *other* arguments to the same conclusion. To get more perspective on states like (2.1), it helps us to think a bit about the concept of a state space in general.

\(^4\)In my view, failure to appreciate this point has caused much confusion in discussions of the so-called ‘problem of tails’ in the Ghirardi–Rimini–Weber (GRW) dynamical-collapse theory ([12]; cf. [13] and references therein for discussion): the ‘problem’, as generally discussed, has little to do with the GRW theory *per se*, but is a general feature of quantum mechanics.
The paradigm example of a state space is classical phase space (I could tell essentially the same story with configuration space). States—points in phase space—represent the physical features of the system in question; different points in phase space represent physically distinct systems. But this is not the only kind of state space definable in classical physics: the space of probability functions on phase space can also be treated as a state space (call it distribution space). Mathematically, phase space and distribution space are just two sets of abstract mathematical objects with deterministic evolution equations: Hamilton’s equations in the one case, Liouville’s equations in the other. The central difference is conceptual: distinct points in distribution space do not represent systems in physically distinct states, but distinct probability distributions over physical states. In other words, there are two conceptions of state space available: physical or probabilistic.5

One way to understand the difficulty of making sense of quantum states is that we seem to shift between these two conceptions. When dealing with states like 2.1 in practice, we treat them as probabilistic, conveying nothing more or less than ‘the cat is either alive or dead, and it has chance $|\alpha|^2$ of being in the first state, and $|\beta|^2$ of being in the second’. On this reading of the quantum state, there is nothing at all mysterious about (2.1), and no need to invoke the Projection Postulate. Indeed, something like the Projection Postulate emerges spontaneously from the ordinary calculus of probability: it is commonplace that probability distributions evolve both under the ordinary dynamics of the system, and via probabilistic updating when we acquire more information. From that perspective, the ‘collapse’ of (2.1) onto either $|\text{alive cat}\rangle$ or $|\text{dead cat}\rangle$ upon measurement is no more mysterious than the collapse of the classical probability distribution ‘heads with probability 1/2, tails with probability 1/2’ onto ‘heads’ or ‘tails’ upon measurement.

When dealing with the quantum states of microscopic systems, however, this straightforward probabilistic reading of the quantum state breaks down. Partly, this is because the democracy of bases in Hilbert space allow a multitude of ways of expressing a state as a superposition of eigenstates: is the spin state

\[ \alpha |+z\rangle + \beta |-z\rangle \]  

(2.2)
to be interpreted as a probabilistic mixture of a particle with $z$-spin up and a particle with $z$-spin down, or as a different probabilistic mixture of $x$-spin up and $x$-spin down? (Or, somehow, mysteriously, as both at once?) But more crucially, the probabilistic reading simply fails to make sense of interference phenomena. Suppose that we set up a Mach–Zender interferometer, in which a beam of (say) photons is split into left (L) and right (R) beams by a half-silvered mirror and then re-interfered by means of another such mirror, and the resultant beams are fed into two detectors, $A$ and $B$. We can easily establish, by blocking the R beam, that if the photon is originally in the L beam, it has a 50 per cent chance of ending

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5 An alternative terminology [14] calls the physical conception ontic and the probabilistic conception epistemic. (In the case of the quantum state, this becomes ‘$\psi$-ontic’ and ‘$\psi$’-epistemic—hence Chris Fuch’s wonderfully cruel term (in conversation) for those who take the ontic view: ‘$\psi$-ontologists’.) This terminology, though, suggests a particular reading of probability as a quantification of our ignorance, which I want to avoid being committed to.
up at detector \( A \) after the second half-silvered mirror, and a 50 per cent chance of ending up at detector \( B \). We can easily establish that the same is true if it is originally in the \( R \) beam. But in that case, which beam it is in makes no difference to the outcome probabilities, so any probabilistic mixture of the two cases should lead to a 50% chance of each of the two results. And of course, this is not what happens: depending on the relative phase of the two beams, any outcome from 100 per cent \( A \) to 100 per cent \( B \) is possible.\(^6\)

The two objections are related, of course. Interference phenomena can occur in quantum mechanics precisely because amplitudes have phases as well as magnitude. The probabilistic reading of a quantum state like (2.1) or (2.2) interprets the magnitudes of the amplitudes \( \alpha \) and \( \beta \) as probabilities, but provides no interpretation of their phases. On this reading, replacing \( \alpha \) with \( \alpha \exp(-i\theta) \) should have no physical significance—but of course, doing so not only has ‘significance’, it has empirically detectable consequences.

This provides us with an alternative way to state the measurement problem:

We cannot consistently understand the state space of quantum theory either as a space of physical states, or as a space of probability distributions. Instead, we have to use one interpretation for microscopic physics and another for macroscopic physics. Furthermore, both the point at which we have to transition between the physical and probabilistic interpretation, and the basis with respect to which the probabilistic interpretation is to be specified, are defined only in an approximate, rough-and-ready way, which seems to make essential use of terms like ‘macroscopic’, which have no place in a fundamental physical theory.

It also provides a ready way to understand the various strategies that have been proposed to resolve the measurement problem. Firstly, there are the attempts to solve the problem, by replacing quantum theory by a new theory that can be understood in a consistent way. There are two basic strategies.

**Definition 2.2 (Hidden-variable theories).** These theories hold on to the dynamics of unitary quantum theory, but remove the probabilistic interpretation of the quantum state. Instead, some new physical entities are introduced, which are dynamically influenced by the quantum state but not vice versa. Probability is then introduced via a probability distribution over these ‘hidden variables’. The paradigm example is the de Broglie–Bohm pilot-wave theory, also called Bohmian mechanics,\(^7\) in which the quantum state is supplemented by a collection of point particles whose collective evolution is determined by the quantum state. (So-called ‘modal interpretations’\(^8\) also fit this category.)

\(^6\)Nor will it do to say ‘quantum probabilities just behave differently from classical ones’. The point of the argument is that we cannot explain the observed phenomena allowing the two ‘possibilities’ to influence one another. But if something that can have a dynamical effect on a real thing does not thereby count as real, we lose our grip on reality (cf. discussions in [15, ch. 4], [16, ch. 10], or [17].)

The bottom line is that a microscopic superposition cannot be understood as merely probabilistic: both terms in the superposition represent physical features of the world. But if the same is true for the Schrödinger-cat state, the paradoxical nature of that state is not alleviated by the supposedly ‘probabilistic’ reading.

\(^7\)See [18] and references therein.

\(^8\)See [19] and references therein.
Definition 2.3 (Dynamical-collapse theories). These theories\(^9\) try to make sense of the transition from a physical to a probabilistic reading of the quantum state, not as an interpretative shift but as an objective physical process. This is generally done by introducing some version of the Projection Postulate that objectively removes macroscopic superpositions, and does so in a probabilistic manner. According to dynamical-collapse theories, the quantum state is always to be interpreted physically, and its stochastic evolution deviates from the deterministic Schrödinger equation in certain circumstances—but it does so in such a way that the resultant probabilities are very close to those defined by the probabilistic reading of unitary quantum mechanics.

Secondly, there are the attempts to **dissolve** the problem, by finding a consistent interpretation of unitary quantum theory. There appear to be\(^10\) only two real options here.

Definition 2.4 (The quantum state is always probabilistic). This approach tries to interpret quantum state space as systematically like the space of classical probability distributions. This has proved extremely difficult, though, for essentially the reasons given above: interference phenomena do not seem to be understandable as probabilistic phenomena. More rigorously, the theoretical results of Bell [22], Kochen and Specker [23], and Gleason [24] make it clear that any such strategy will have the apparently pathological feature of **contextuality**.\(^11\) For this reason, most\(^12\) adherents of this strategy have given up on the idea of interpreting the ‘probabilities’ as probabilities of any microphysical even, and just read them directly as probabilities of measurement outcomes, treated as primitive. That is, they fall back on an instrumentalist reading of quantum theory. (For a clearly articulated example, see [30].)

Definition 2.5 (The quantum state is always physical). This approach, most famously associated with the name of Everett [31], tries to interpret quantum state space as systematically like classical phase space. This deals straightforwardly with the problem of microscopic interference, but struggles with the problem of macroscopic superpositions. If Schrödinger-cat states like 2.1 are to be understood as physical states, what physical situation are they representing? The only coherent answer seems to be: a situation with two independently

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\(^9\)Examples include the GRW theory ([12]; see [13] for discussion) and proposals due to Penrose [20]; cf. also Philip Stamp’s discussion [21]).

\(^10\)It is of course always possible that some third option exists but has escaped my attention (and third parties should always treat an author’s classification schemas with healthy suspicion). However, it is important that any such move genuinely resolves our conceptual confusion (or at least explains why that confusion is misguided). Attempting to say that the quantum state is somehow both physical and probabilistic, for instance, just redescribes the problem; saying that it is probabilistic but does not have the properties of classical probability is in danger of breaking our grip on what ‘probability distribution’ means and making it simply a synonym for ‘quantum state’.

\(^11\)For a detailed discussion, see [25].

\(^12\)Not all: [26] is a counter-example; the ‘quantum logic’ strategy (cf. [27] and references therein for discussion), and certain variants of the consistent histories approach [28,29] might also count.
evolving cats, one alive and one dead. But what justifies this interpretation, and
where do the probabilities enter the theory, if they are not to be added directly
through either a probabilistic reading of the state, or through additional hidden
variables, or through stochastic dynamics?13

However, once again it will be useful not to consider the conceptual question
of how to make sense of a state space that seems sometimes to be a space
of physical states and sometimes a space of probability distributions, but to
ask, more practically, how it is that we sometimes get away with treating the
state space as a space of probability distributions, given that fundamentally that
interpretation does not seem consistent.

3. The role of decoherence

Let us consider more carefully just when the probabilistic interpretation can be
consistently applied. Recall that it is interference that blocks a probabilistic
interpretation at the microscopic scale; if a probabilistic interpretation can
be applied at any scale, then, it must be because interference phenomena
can be neglected.

This, of course, is precisely the question that decoherence theory is designed
to answer. In the following, I apply the environment-induced decoherence
framework14 used by Zeh [34] and Zurek [35]; essentially, the same conclusions,
though, can be derived in the decoherent-histories framework (see [36–38]; see
also my discussion in ch. 3 of [16]).

Firstly: suppose that the physical system we are considering has a Hilbert space
\( \mathcal{H} \), which can be decomposed as

\[ \mathcal{H} = \mathcal{H}_M \otimes \mathcal{H}_E, \]  

where \( \mathcal{H}_M \) is the Hilbert space of the degrees of freedom that interest us—say, the
centre-of-mass degrees of freedom of a macroscopic object, or the low-momentum
vibrational modes of an extended solid—and \( \mathcal{H}_E \) is the Hilbert space of all the
other degrees of freedom that interact with those that interest us. \( \mathcal{H}_M \) is normally
said to represent the ‘system’, and \( \mathcal{H}_E \) the ‘environmental’ degrees of freedom,
but it is important to recognize that the ‘environment’ need not be spatially
external: for the extended solid, for instance, \( \mathcal{H}_E \) might be the Hilbert space of
vibrational modes above some momentum cutoff.15

Suppose also that there is some basis \( \{ |z\rangle \} \), of \( \mathcal{H}_M \), labelled by some (discrete
or continuous) variable \( z \), with the following property: the state

\[ \left( \int dz \alpha(z)|z\rangle \right) \otimes |\psi\rangle \]  

13For detailed discussions of the Everett interpretation, see [16,32].
14See [33] and references therein for a review of this framework.
15It is worth noting that this is still not completely general; in some situations of physical interest,
the division between system and environment does not have a simple tensor-product structure.
In this context, the framework of decoherent histories can provide an appropriate generalization;
see [39] for further discussion.
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(where the integral sign schematically denotes summation or integration over \( z \), as appropriate) evolves rapidly, for fairly generic \( |\psi\rangle \in \mathcal{H}_E \), to

\[
\int dz \alpha(z) |z\rangle \otimes |\psi(z)\rangle,
\]

(3.3)

where (i) ‘rapidly’ means ‘rapidly as compared to the dynamical time scales of the system’s own dynamics’ and (ii) \( \langle \psi(z)|\psi(z')\rangle \simeq 0 \) unless \( z \simeq z' \). Effectively, dynamics like this consists of the environment ‘measuring’ the system state. The classic example has \( \{|z\rangle\} \) as a wave-packet basis, reasonably localized in both centre-of-mass position and momentum. Since the system–environment interactions are local, two wave packets with significantly distinct locations will cause significantly different evolution of the environment. And two wave packets with significantly different momentum will swiftly evolve into two wave packets with significantly different position. (Handling this case carefully requires us to deal with the fact that wavepackets form an overcomplete basis; I will ignore this issue for simplicity.)

If this occurs, we will say that the system is \textit{decohered} by the environment, with respect to the \( \{|z\rangle\} \) basis.

Now suppose we want to apply the probability interpretation with respect to the \( \{|z\rangle\} \) basis: that is, we want to treat a state like (3.3) as having probability\(^{16}\) \( |\alpha(z)|^2 \) of having whatever physical property (call it \( Z \)) that \( z \) is supposed to represent (some given centre-of-mass position and/or momentum, say). If the density operator of the system at time \( t \) is \( \rho(t) \), then we have

\[
\Pr(Z = z, t) = \langle z|\rho(t)|z\rangle,
\]

(3.4)

where \( \Pr(Z = z, t) \) is the probability at time \( t \) of the system having \( Z = z \).

For this really to be interpretable as a \textit{probability}, though, its evolution needs to be free from interference effects: that is, it needs to evolve \textit{like a probability function}. So: let \( \Pr(Z = z', t'; Z = z, t) \) be the probability\(^{17}\) that the system has \( Z = z' \) at time \( t' \), \textit{given} that it has \( Z = z \) at time \( t \). Then, the standard laws of probability tell us that

\[
\Pr(Z = z', t') = \int dz \Pr(Z = z', t'|Z = z, t) \times \Pr(Z = z, t).
\]

(3.5)

The dynamics will have this form only if \( \Pr(Z = z', t') \) is a linear functional of \( \Pr(Z = z, t) \); that is, if \( \langle z|\rho(t')|z\rangle \) (considered as a function of \( z \)) is a linear functional of \( \langle z|\rho(t)|z\rangle \).

In general, this will not be the case. Assuming we can write down autonomous dynamics for \( \rho \) in the first place, then of course the overall linearity of unitary dynamics implies that \( \rho(t') \) is a linear functional of \( \rho(t) \). But in general, \( \langle z|\rho(t')|z\rangle \) depends not only on diagonal terms of \( \rho(t) \) like \( \langle z|\rho(t)|z\rangle \), but also on off-diagonal terms like \( \langle z|\rho(t)|w\rangle \).

However, in the case we are considering, the interaction with the environment guarantees that off-diagonal terms in \( \rho \) are suppressed, and suppressed on time

\(^{16}\)Or probability density, if \( z \) is a continuous variable.

\(^{17}\)Again, this should be interpreted as a probability density if \( z \) is continuous.
scales much quicker than those that characterize the dynamics of the system itself. In this situation, then, the effective evolution equation for \( \rho \) reduces to an evolution equation specifically for the diagonal elements of \( \rho \) in the \( \{ |z \rangle \} \) basis.

Let us sum up. If the total system we are studying can be decomposed into ‘system’ and ‘environment’ degrees of freedom, such that for some basis \( \{ |z \rangle \} \) of the system, the system is decohered by the basis with respect to that basis, then we can consistently treat the total system as a probabilistic mixture of states like \( |z \rangle \otimes |\psi(z) \rangle \).

Furthermore, we have excellent reason—from general physical arguments, from specific models, and increasingly from experiment—to think that the macroscopic degrees of freedom of a system are decohered by the residual degrees of freedom with respect to a wave-packet basis for those macroscopic degrees of freedom. So macroscopic systems can consistently be treated as probabilistic mixtures of states with different—but definite—values of those macroscopic degrees of freedom.

Note that nothing in this analysis relies on the environment being in any way discarded, except in the pragmatic sense that we are not terribly interested in what it is doing. The total system continues to evolve unitarily, but by virtue of the particular form of that unitary dynamics, it can consistently be given a probabilistic interpretation with respect to its macroscopic degrees of freedom.

Note also that it is the dynamical aspects of decoherence that are important here (a point also stressed by Zurek [40,41]). The rapid suppression of the off-diagonal elements of the density operator, which is usually taken as the signature of decoherence, is significant not in itself, but because it implies that the dynamics of the diagonal elements must be probabilistically interpretable. (The decoherent-histories framework makes this more explicit: in that framework, the possibility of interpreting the system’s evolution probabilistically is definitional of decoherence.)

4. Decoherence and the measurement problem

Decoherence, then, explains why the measurement problem is a philosophical rather than a practical problem. Given the ubiquity of decoherence, the strategy of applying the probability interpretation of the state to decohered systems (and to the basis with respect to which the decoherence occurs) will not actually give us contradictory predictions—at least, not to the levels of accuracy that we have any hope of probing empirically. Those readers inclined towards the so-called ‘shut up and calculate interpretation’, therefore, can stop reading now. For the rest, we should now return to the previous taxonomy of strategies for solving, or dissolving, the problem, and see how they fare in the light of decoherence.

We begin with the strategies for solving the problem: the dynamical-collapse and hidden-variable strategies. Recall that dynamical-collapse strategies effectively turn the shift from a physical to a probability reading of the quantum state into a dynamical process, while hidden-variable strategies effectively hold on to a physical reading of the quantum state and add additional dynamical variables over which the probabilities are defined.

At first sight, both strategies are made straightforward by decoherence. We can fairly straightforwardly specify a dynamical collapse theory by stipulating that superpositions of states in the decoherence-preferred basis spontaneously...
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collapse, with collapse probabilities given by the probability rule; we can fairly straightforwardly specify a hidden-variable theory by treating the variable that labels the decoherence-preferred basis as a hidden variable and stipulating that the probability distribution over that variable is given by the probability rule. Neither of these recipes completely specifies the theory in question (how quickly does collapse occur? What dynamics govern the hidden variables, and does it ensure that the probabilities remain in line with the quantum predictions after the initial stipulation?) but—it might seem—the hard work has been done.

Unfortunately, things are not so simple, for a straightforward reason: 

— Decoherence suppresses interference only approximately. Interference between alternative ‘possibilities’ is far too low to be experimentally detectable (which is another way to say that to within the limits of experimental accuracy, the probability interpretation gives consistent predictions), but it is not zero.

— The basis picked out by decoherence is itself only approximately given. In the standard examples, for instance, we might specify it as ‘a wave packet, not too localized in position or momentum’. But ‘not too localized’ is scarcely a precise criterion; nor is there anything particularly privileged about the Gaussians usually used for wave packets, save their mathematical convenience. Criteria like Zurek’s ‘predictability sieve’ [40] can be used to pick a particular choice of basis, but these have more a pragmatic than a fundamental character.

— The analysis of decoherence that I gave above relies on a certain decomposition of the total Hilbert space into ‘system’ and ‘environment’ degrees of freedom. Varying that decomposition will vary the probability rules defined by the decoherence process. Granted, any remotely sensible decomposition can be expected to give essentially similar results, but ‘remotely sensible’ and ‘essentially similar’ are not really the kind of terms we would like to see in the specification of a fundamental theory. But Dowker & Kent [42] have provided strong arguments (in the rather different mathematical framework of decoherent histories) that the mere criterion that the probability calculus applies to a given decoherence basis, divorced from other considerations, is overwhelmingly too weak to pick out a unique basis, even approximately.

These observations all stem from the same feature of decoherence: that it is a high-level, dynamical process, dependent on details of the dynamics of the world and even on contingent features of a given region of the world. It is not the kind of thing that can be captured in the mathematical language of microscopic physics.

Note that this is not to say that decoherence is not a real, objective process. Science in general, and physics in particular, is absolutely replete with real, objective processes that occur because of high-level dynamical effects. Pretty much any regularity in chemistry, biology, psychology, etc. has to have this character, in fact. But 

emergent processes like this do not have a place in the

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axioms of fundamental physics, precisely because they emerge from those axioms themselves.18

An analogy: it has been well understood for many years that particles—whether in relativistic quantum field theory, or in condensed matter physics—are themselves not fundamental. The particle spectrum of a quantum field theory is determined by the dynamical features of the theory, and in general it is determined only approximately, and in a way that varies according to the contingent features of the regime that interests us. (In some situations, we analyse quantum chromodynamics in terms of quarks; in others, in terms of protons and neutrons; not only the masses and charges of the particles, but which particles we use in the first place, vary according to the energy levels at which the theory is analysed.)

Adrian Kent puts it rather well (in a slightly different context):

> It’s certainly true that phase information loss is a dynamical process which needs no axiomatic formulation. However, this is irrelevant to our very simple point: no preferred basis can arise, from the dynamics or from anything else, unless some basis selection rule is given. Of course, [one] can attempt to frame such a rule in terms of a dynamical quantity— for example, some measure of phase information loss. But an explicit, precise rule is needed. [46, p.11]

But if decoherence cannot be used to define a set of hidden variables, or a collapse law, nonetheless it serves to constrain both concepts. For if the collapse rule does not cause collapse onto a basis that is approximately decohered, it will fail to reproduce the experimental predictions of quantum mechanics; likewise, if a probability function over the values of the hidden variables does not determine a probability function over the decoherence-selected basis, the hidden variables will not allow us to recover the empirical predictions of quantum theory.

In the case of non-relativistic quantum theory, this is unproblematic. The decoherence-preferred basis is basically a coarse-graining of the position basis, so a collapse rule that collapses the system onto wavepackets fairly concentrated around a particular centre-of-mass position, or a choice of position as the hidden variable, will do nicely. And indeed, we find that the main examples of non-relativistic collapse and hidden-variable theories—the GRW theory and Bohm’s theory—do indeed select position in this way.

It is crucial to note what makes this possible. Position has a dual role in non-relativistic quantum theory: it is at one and the same time (i) one of the fundamental microphysical variables in terms of which the theory is defined and (ii) such that a coarse-grained version of it is preferred by the high-level, dynamical, emergent process of decoherence. As such, it is possible to formulate modifications or supplements to non-relativistic quantum theory that are both precisely defined in terms of the microphysical variables used to formulate quantum mechanics, and appropriately aligned with the macrophysical variables picked out by decoherence.19

18Emergence has a long and tangled history in philosophy of science; see [43,44], and references therein, for details. In physics, perhaps the most influential discussion of recent years has been [45].

19Incidentally, this is what makes testing dynamical collapse theories so difficult: the fact that they must succeed in reproducing the empirical predictions of quantum theory in normal circumstances pretty much guarantees that it will be very difficult to distinguish genuine collapse from mere decoherence. (Difficult, but not impossible; see Stamp [21])
Unhappily for modificatory strategies, there does not appear to be a variable in extant relativistic quantum theory—in quantum electrodynamics, say, or in the Standard Model—that manages to play this dual role. In the fermionic sector, decoherence seems to prefer states of definite particle number—and, as we have already seen, ‘particle number’ is itself a dynamically emergent concept, determined by the complex details of renormalization theory and dependent, to a considerable degree, on the energy levels at which we wish to study the systems of interest to us. In the bosonic sector—at least where electromagnetism is concerned—decoherence seems instead to select out coherent states \[47\], but these are coherent states defined with respect to the effective field operators governing the system at low energies. In neither case is there any remotely straightforward definition of the decoherence-preferred basis in terms of the quantities used to formulate the quantum field theory at the microphysical level. Indeed, it is commonplace of renormalization theory\[^{20}\] that these variables are largely hidden from view at the level of observable phenomena.

For this reason, I suspect—if it is accepted that modifications of quantum theory should not themselves be stated in a way which makes essential reference to dynamically emergent and high-level features of the theory—that the prospects of solving the measurement problem in the relativistic domain by modifying quantum theory are dim. It is notable that, to my knowledge, there is no dynamical-collapse theory even purporting to be applicable to relativistic quantum theory in the presence of interactions\[^{21}\] and those hidden-variable theories that have been proposed in the relativistic domain\[^{22}\] are largely silent about renormalization.

Turning to the strategies for *dissolving* the measurement problem, recall that there are again two: treat the wave function as always probabilistic, or treat it as always physical. The former strategy makes essentially no contact with decoherence: the point of decoherence (as I have presented it here) is to give an account of when a probabilistic reading of the wave function is consistent, but the probabilistic strategy treats it as *always* consistent. It does so, in general, by retreating from any attempt to interpret the probabilities as probabilities of anything except measurement outcomes.

This strategy, in effect, is a retreat to the idea that measurement is a primitive. Insofar as that makes sense, it suffices to resolve the puzzles of interference without any concern about decoherence (as it would have to: after all, it has to explain why the probabilistic reading is *always* possible, even in situations where decoherence is negligible). But, as I have alluded to earlier, it does *not* make sense, so far as I can see, partly on philosophical grounds but largely on the

\[^{20}\] See [48], or any other modern textbook on quantum field theory, for technical details and references.

\[^{21}\] The nearest thing to such a theory is Tumulka’s theory [49], which is explicitly formulated for a multi-particle theory, on the assumption that there are no interactions. It was, of course, precisely the need to incorporate interactions that drove the pioneers of relativistic quantum mechanics to field theory.

\[^{22}\] The main examples are [50,51], which take the hidden variables to be particle positions; [52], which takes them to be bosonic field configurations; and [53], which takes them to be local; fermion-number densities.
straightforward grounds that experimental physicists, and theorists who study experiment, cannot treat measurement as primitive but invariably fall back on the need to analyse it, using quantum theory itself.

To be fair, this objection has received at least some attention from advocates of the strategy (for recent examples, see [54,55]).

To be fair, this objection has received at least some attention from advocates of the strategy (for recent examples, see [54,55]).23 I leave it to readers to judge for themselves whether these responses really do justice to physical practice. It is worth pointing out, though, that in general its advocates tend to work in quantum information, a field whose raison d'être is to abstract away the messy details of quantum-mechanical processes and look at their abstract structure. This strategy has been remarkably successful, yielding deep insights about quantum theory that would not have come easily if we had kept the messy details in play; for all that, it is possible to worry that some quantum-information approaches to the measurement problem mistake the map for the territory.

The final strategy is Everett’s: treat the quantum state as always giving the physical state of the system. If decoherence can contribute directly to any (dis)solution to the measurement problem, it is here. For we have seen that decoherence is an emergent process; what it tells us, interpreted as Everett suggests, is that even if the Universe is fundamentally a unitarily evolving whole, at the emergent level, it has the structure of a probability distribution over states, each of which describes approximately classical goings on. There is no mechanism by which one of those states is preferred (is ‘actual’, if you like) and the others are mere possibilities: at the fundamental level, all are components in a single, unitarily evolving state, and no one is preferred over another. (Any such mechanism would amount to a dynamical-collapse theory, as discussed previously.) But at the emergent level, each term evolves independently of the others; furthermore, their mod-squared amplitudes behave as if they were probabilities.

Does this mean that decoherence suffices to solve the measurement problem, provided that we understand the quantum state as a physical state as Everett proposed? The answer turns on two problems.

— The ontological problem. For something to be a collection of quasi-classical worlds, does it suffice for it to have the structure of a collection of quasi-classical worlds—or is more needed?

— The probability problem. For something to be a probability measure over a set of quasi-classical worlds, does it suffice for it to have the structure of a probability measure over a collection of quasi-classical worlds—or is more needed?

Space does not permit an extensive engagement with these questions. My own view24 is that neither problem is truly problematic. But it is notable that both problems are essentially philosophical in nature: in the light of decoherence, if an Everettian solution to the measurement problem is to be rejected, then it will have to be for subtle philosophical reasons rather than any structural deficiency in quantum theory.

23 And, as I mentioned earlier, it is not universally accepted that only a primitivist view of measurement can justify the strategy.

24 Developed in full in [16]; for earlier versions, see [56,57] in the first case and [58,59] in the second. For further discussion—on both sides—see [32] and references therein.
5. Conclusion

In twenty-first-century physics, the ‘measurement problem’ is best understood, not as an illegitimate intrusion of a primitive ‘measurement’ postulate into physics, but as a conceptual incoherence in our interpretation of quantum states: it seems impossible to understand the macroscopic predictions of quantum mechanics without interpreting the state probabilistically, yet because of interference, quantum states cannot systematically be thought of as probability distributions over physical states of affairs. We can attempt to resolve that incoherence either by philosophical methods (thinking hard about how to understand quantum states so as to come up with a non-incoherent way) or by modifying the physics (replacing quantum mechanics with some new theory that does not even prima facie lead to the conceptual incoherence).

Decoherence explains why it is that quantum theory nonetheless works in practice: it explains why interference does not, in practice spoil the probabilistic interpretation at the macro-level. But because decoherence is an emergent, high-level, approximately defined, dynamical process, there is no hope of incorporating it into any modification of quantum theory at the fundamental level. Decoherence does, however, act as a significant constraint on such modifications—a constraint that, in the case of relativistic quantum field theory, is likely to be exceedingly hard to satisfy.

Decoherence could, however, play a role in a solution to the measurement problem that leaves the equations of quantum theory alone and treats the objective macroscopic reality we see around us as itself an emergent phenomenon. Such a strategy is committed to the claim that, at the fundamental level, the quantum state continues to describe the physical state of the world: it is, therefore, ultimately Everett’s strategy. Decoherence finds its natural role in the measurement problem as the process that explains why quantum mechanics, interpreted as Everett advocates, can fundamentally be deterministic and non-classical, but emergently classical. It does not, however, in any way blunt the metaphysically shocking aspect of Everett’s proposal: no one quasi-classical branch is singled out as real; all are equally part of the underlying quantum reality.

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