This paper presents a novel approach to achieving high spatial resolution in the demodulation of images produced by a two-dimensional X-ray Talbot interferometry (XTI) system. Currently, demodulation of XTI images is mainly performed by either phase-stepping (PS) or Fourier transform (FT) methods. However, the PS method for two-dimensional XTI demodulation requires a larger number of exposures and a more complex grating control process than that of one-dimensional XTI. On the other hand, although the FT method uses only a single-fringe image, it gives lower spatial resolution than the PS method. For practical application of two-dimensional XTI, a simpler exposure process with high spatial resolution is required. In this paper, we introduce a hybrid method combining the PS and FT methods. This method simplifies the exposure process in comparison with the PS method required in two-dimensional XTI while achieving higher spatial resolution than the FT method in the demodulation of images. The method works by using additional exposures to eliminate unnecessary spectral components that appear in the FT method. Furthermore, the proposed method is demonstrated by using actual two-dimensional XTI data and shown to achieve high spatial resolution in the demodulation of images for both the $x$- and $y$-differential phase components.

1. Introduction

X-ray Talbot interferometry (XTI) has been widely regarded as a promising X-ray phase imaging method since the early 2000s [1,2]. An XTI system contains a set of two gratings, namely phase and amplitude
Coherent X-rays from the source penetrate the object (set on the optical path) and the first phase grating, creating an interference pattern. The interference pattern is partially absorbed by the amplitude gating and forms a detectable moiré fringe pattern on the detector. It is well known that the detected fringe pattern includes three physical parameters regarding the scanned object. The first parameter is the object’s absorption index, which is the same as measured by conventional X-ray imaging. We can determine this conventional parameter by measuring the decrease in intensity of the detected X-rays. The second parameter is differential phase. The phase of incident X-rays is shifted as they pass through an object and the fringe pattern is distorted by the differential change in the shifted phase. This parameter is attractive because it can visualize soft matter, which is not observable in absorption images. The third parameter is internal X-ray scattering. The coherence of incident X-rays is reduced by the object’s microstructure. Thus, we can detect microstructural features from fringe pattern degradation. This parameter has recently been investigated because it can visualize microstructural features smaller than the detector pixel.

These parameters are extracted from the detected fringe pattern through a demodulation process. Currently, phase demodulation methods used in XTI are roughly divided into the following two types: phase-stepping (PS) and Fourier transform (FT) methods.

In previous studies, the PS method has been most widely used owing to the high spatial resolution of the demodulated results. The parameters are demodulated from several fringe patterns by changing the relative positions of the two gratings by translation. As the contrast of the fringe pattern is adjusted by the translation, each parameter mentioned above is demodulated by using the following equations for individual pixels in the one-dimensional case:

\[
A(x, y) = \sum_{n=1}^{N} I_n(x, y),
\]

\[
P(x, y) = \text{Arg} \left( \sum_{n=1}^{N} I_n(x, y) \exp \left( i \frac{2\pi n}{N} \right) \right)
\]

and

\[
S(x, y) = \text{Abs} \left( \sum_{n=1}^{N} I_n(x, y) \exp \left( i \frac{2\pi n}{N} \right) \right).
\]

Here, \( I_n(x, y) \) is the \( n \)th captured fringe pattern intensity. The demodulated parameters \( A(x, y) \), \( P(x, y) \) and \( S(x, y) \) are the absorption image, the differential phase and the degradation by internal X-ray scattering, respectively. In the case of two-dimensional XTI, a set of PS along \( x \) - and \( y \)-directions is required to obtain the two-dimensional information of \( P(x, y) \) and \( S(x, y) \).

The FT method first presented by Takeda et al. [3] provides the fringe pattern phase from a single-fringe pattern. By using the concept of that method, parameters are demodulated from the following equations for two-dimensional XTI:

\[
A(x, y) = \mathcal{F}^{-1}\{\tilde{I}(k_x, k_y)G(k_x, k_y)\},
\]

\[
P(x, y) = \text{Arg}(\mathcal{F}^{-1}\{\tilde{I}(k_x - q_x, k_y - q_y)G(k_x, k_y)\})
\]

and

\[
S(x, y) = \text{Abs}(\mathcal{F}^{-1}\{\tilde{I}(k_x - q_x, k_y - q_y)G(k_x, k_y)\}).
\]

Here, \( \tilde{I}(k_x, k_y) \) is the FT of fringe pattern \( I(x, y) \) and \( \mathcal{F}^{-1}\{ \cdots \} \) indicates the inverse FT operation. \( G(k_x, k_y) \) is an even filter function for cropping the spectrum in Fourier space and \( (q_x, q_y) \) is the peak position of the Fourier spectrum. The parameters are demodulated from the Fourier spectrum corresponding to the carrier frequency. Note that the filter function size determining the spatial resolution is limited to avoid overlap of neighbouring spectra. Therefore, in principle, the spatial resolution is worse than that in the PS method.

Most existing XTI experimental set-ups use a set of one-dimensional gratings with a stripe pattern (one-dimensional XTI). In this case, the parameters of differential phase and internal X-ray scattering demodulated by the XTI system are obtained in one direction (perpendicular to the stripe pattern), and it is impossible to obtain the parameters in the direction parallel to the stripe.
pattern. Therefore, if the integrated phase parameters of the object are required, especially in the case of projection imaging, the integral constant must be set along the axis parallel to the grating stripe pattern in advance of the integration process. In general, the shape of the target object on the exposure is limited because a reference region without a target object should be chosen to set the integral constant.

On the other hand, studies on two-dimensional phase imaging that aim to resolve the above-mentioned problem have been conducted [4–9]. Zanette et al. [8,9] demonstrated the two-dimensional PS method and showed the two-dimensional information of the object was successfully obtained. However, if we adopt the PS method for two-dimensional XTI, the control process becomes more complicated than that for one-dimensional XTI because precise alignment of the gratings and multiple exposures would be required in two directions. As an example of a study on the FT method, Wen et al. [10] applied the FT method with a one-dimensional grating to demonstrate the use of the scattering images for the analysis of the microstructure of objects. Fundamentally, the FT method requires only one shot for demodulating the parameters, and unlike the PS method, precise control of grating alignment is not required. For these reasons, Itoh et al. [11] and Sato et al. [12] have also studied XTI with sets of two-dimensional gratings. In this case, the differential phases in the x- and y-directions are obtained simultaneously, and integrated phase images with a high signal-to-noise ratio can be obtained by using a relevant integration method excluding the reference area. However, the spatial resolution in demodulation by the FT method is known to be lower than that by the PS method.

From the viewpoint of practical XTI application, high spatial resolution is a key feature required for revealing objects that are undetectable by existing imaging techniques. The differential phase image, which is the primary information obtained from XTI, has particularly high intensity at the edges of the object. Thus, higher spatial resolution is advantageous for expanding the application scope. Two-dimensional XTI has the complexity of the PS method and the low-resolution achievable with the FT method. Therefore, demodulation methods employing a simple procedure while providing high spatial resolution for two-dimensional XTI should be investigated.

This paper introduces a demodulation approach that combines the PS and FT methods in order to improve the spatial resolution of the FT method. This approach provides a simple exposure procedure with high spatial resolution for two-dimensional XTI.

2. Hybrid demodulation approach for two-dimensional X-ray Talbot interferometry

(a) Fourier transform method

In contrast to one-dimensional XTI, several combinations of grating patterns for two-dimensional XTI are allowed, as shown by Zanette et al. [6]. Here, we adopt a grating set combining a $\pi$-shift checker-patterned phase grating and grid-patterned absorption grating [6,8,9,11,12] because this set yields the highest contrast for the fringe pattern. The moiré fringe pattern is approximated as

$$I_1(x, y) = a(x, y)(1 + b_x(x, y) \cos(P_x(x, y) - \omega_xx)) \times (1 + b_y(x, y) \cos(P_y(x, y) - \omega_yy)),$$

(2.1)

where $a(x, y)$ is the absorption of X-rays by the object; $b_x(x, y)$ and $b_y(x, y)$ are fringe contrast factors in the x- and y-directions, respectively, and indicate the degradation of the fringe pattern owing to internal X-ray scattering in the object; $P_x(x, y)$ and $P_y(x, y)$ are the differential phases of the object along the x- and y-directions; and $\omega_x$ and $\omega_y$ are the fundamental angular frequencies of the carrier fringe pattern in the x- and y-directions, respectively. The fringe pattern distortion owing to the object is described by five parameters: $a(x, y), b_x(x, y), b_y(x, y), P_x(x, y)$ and $P_y(x, y)$. 
The FT of equation (2.1) is obtained as follows:

$$\tilde{I}(k_x, k_y) = \mathcal{F}(I_1(x, y))(k_x, k_y) = A(k_x, k_y) + B_x(k_x - \omega_x, k_y) + B'_x(k_x + \omega_x, k_y) + B_y(k_x, k_y - \omega_y) + B'_y(k_x, k_y + \omega_y) + B_{xy}(k_x - \omega_x, k_y - \omega_y) + B'_{xy}(k_x + \omega_x, k_y + \omega_y) + B_{yx}(k_x - \omega_x, k_y + \omega_y) + B'_{yx}(k_x + \omega_x, k_y - \omega_y),$$

(2.2)

where $\mathcal{F}\{\cdots\}$ is the FT operator and $*$ denotes the complex conjugate. $A(k_x, k_y), B_x(k_x, k_y), B_y(k_x, k_y), B_{xy}(k_x, k_y)$ and $B_{yx}(k_x, k_y)$ are expressed as follows:

$$A(k_x, k_y) = \mathcal{F}(a(x, y))(k_x, k_y),$$

$$B_x(k_x, k_y) = \mathcal{F} \left\{ \frac{a(x, y)b_x(x, y)}{2} \exp(-iP_x(x, y)) \right\} (k_x, k_y),$$

$$B_y(k_x, k_y) = \mathcal{F} \left\{ \frac{a(x, y)b_y(x, y)}{2} \exp(-iP_y(x, y)) \right\} (k_x, k_y),$$

$$B_{xy}(k_x, k_y) = \mathcal{F} \left\{ \frac{a(x, y)b_x(x, y)b_y(x, y)}{4} \exp(-iP_x(x, y) - iP_y(x, y)) \right\} (k_x, k_y),$$

and

$$B_{yx}(k_x, k_y) = \mathcal{F} \left\{ \frac{a(x, y)b_x(x, y)b_y(x, y)}{4} \exp(-iP_x(x, y) + iP_y(x, y)) \right\} (k_x, k_y).$$

Equation (2.2) indicates that there are nine spectrum peaks in Fourier space, and each spectrum includes information about the five parameters. The first term in equation (2.2) corresponds to the absorption of X-rays by the objects. The second to fifth terms in equation (2.2) denote the differential phase and scattering parameters, where the second and third terms denote the parameters in the $x$-direction and the fourth and fifth terms denote the parameters in the $y$-direction.

In accordance with equations (1.4)–(1.6), all the parameters can be extracted by demodulating the spectra. Each spectrum is extracted by the filter function $G(k_x, k_y)$ and centred at the origin and a map image of the parameters included in the spectrum is obtained by the inverse FT. The sixth to ninth terms are cross-terms including cross-information on the $x$- and $y$-directional parameters; they are not used for demodulation because the spectral strength of these terms is low compared with that of the second to fifth terms.

The spatial resolution of the map image in Fourier space is defined by the radius of the filter function $G(k_x, k_y)$. The radius is limited by the distance between the spectrum peaks in order to avoid spectral overlap. In other words, the carrier frequency of the fringe pattern limits the spatial resolution because the spectrum location in Fourier space is determined by the carrier frequency as shown in equation (2.2). Higher frequencies of the fringe pattern therefore yield higher spatial resolution. However, the spatial resolution is also limited by the Nyquist frequency. Furthermore, a high frequency (corresponding to a short period of the fringe pattern) reduces the modulation amplitude of the fringe because it is affected by the modulation transfer function of the detector. Therefore, 4 px/fringe was used in the case of the highest frequency in experimental studies by Sato et al. [12]. In this case, the angular frequency is $\pi/2$ rad/px, and a von Hann window with a radius of $\pi/2$ is used as the filter function.

To overcome this limitation, we introduce a novel approach to improve the spatial resolution.

(b) Fourier transform method using phase stepping

The approach proposed in this paper combines the FT and PS methods: an additional shot is taken with the phase shifted to eliminate unnecessary spectra in the FT method.
We can obtain another fringe pattern by shifting the phase in equation (2.1) by \((\pi, \pi)\)

\[
I_2(x, y) = a(x, y)(1 + b_x(x, y) \cos(P_x(x, y) - \omega_x x + \pi)) \\
\times (1 + b_y(x, y) \cos(P_y(x, y) - \omega_y y + \pi)). \tag{2.3}
\]

This additional fringe pattern can be obtained by changing the relative position between the two gratings by half of the grating pitch, as in the PS method. Note, however, that the difference from the conventional PS method is that in our method the fringe pattern in the captured image is necessary and the period should be kept the same as that used in the FT method. By subtracting equation (2.3) from equation (2.1), we obtain the following simple form:

\[
I_1 - I_2 = 2a(x, y)(b_x(x, y) \cos(P_x(x, y) - \omega_x x) \\
+ b_y(x, y) \cos(P_y(x, y) - \omega_y y)). \tag{2.4}
\]

In equation (2.4), the first term and the sixth to ninth terms in equation (2.2) are eliminated. Likewise, it is easy to obtain the absorption term, which is eliminated in equation (2.4), by calculating \(I_2 + I_1\)

\[
I_1 + I_2 = 2a(x, y)(1 + b_x(x, y)b_y(x, y) \cos(P_x(x, y) - \omega_x x) \cos(P_y(x, y) - \omega_y y)). \tag{2.5}
\]

Here, in contrast to equation (2.4), the second to fifth terms in equation (2.2) are eliminated. Equations (2.4) and (2.5) describing the neighbouring spectra of the objective spectrum in equations (1.4)–(1.6) are removed from the Fourier space, and the second-nearest spectra become the nearest. As a result, the filter radius used in the FT method can be increased in the Fourier space. This also means that the spatial resolution of demodulated parameters can be improved by using only one additional exposure.

This concept can be extended to four-shot methods. In addition to equations (2.1) and (2.3), we use additional exposures \(I_3(x, y)\) and \(I_4(x, y)\) as follows:

\[
I_3(x, y) = a(x, y)(1 + b_x(x, y) \cos(P_x(x, y) - \omega_x x)) \\
\times (1 + b_y(x, y) \cos(P_y(x, y) - \omega_y y)) \tag{2.6}
\]

and

\[
I_4(x, y) = a(x, y)(1 + b_x(x, y) \cos(P_x(x, y) - \omega_x x)) \\
\times (1 + b_y(x, y) \cos(P_y(x, y) - \omega_y y + \pi)). \tag{2.7}
\]

Expressions that are even simpler than equation (2.4) can be obtained by calculating \(I_1 - I_2 - I_3 + I_4, I_1 - I_2 + I_3 - I_4\) and \(I_1 + I_2 + I_3 + I_4\)

\[
I_1 - I_2 - I_3 + I_4 = 4a(x, y)b_x(x, y) \cos(P_x(x, y) - \omega_x x), \tag{2.8}
\]

\[
I_1 - I_2 + I_3 - I_4 = 4a(x, y)b_y(x, y) \cos(P_y(x, y) - \omega_y y) \tag{2.9}
\]

and

\[
I_1 + I_2 + I_3 + I_4 = 4a(x, y). \tag{2.10}
\]

In contrast to the two-shot method, the second-nearest neighbour spectra remaining in equations (2.4) and (2.5) are eliminated from the Fourier space. We obtain a pair of individual expressions including information about phase and scattering in the \(x\)- and \(y\)-directions individually, and to achieve higher spatial resolution, we can use a filter that is roughly two times larger than that in the one-shot FT method.

The purpose of our approach is to expand the filtering area of the FT method by taking additional shots to eliminate unnecessary spectra. Although the proposed method requires additional shots, fewer shots are required in the hybrid method than in the PS method, which requires at least five shots. The proposed method achieves a shorter exposure sequence than the conventional PS method with high spatial resolution.
Figure 1. (a) Phase image of the cross-piled rods pattern used as the target object, (b) fringe pattern distorted by the object and (c) magnified fringe pattern without the object.

Figure 2. Results of demodulation by the two-shot hybrid method (top) and the four-shot hybrid method (bottom). Columns (a, b) show differential phase images along the x- and y-directions, column (c) shows absorption images and columns (d, e) represent scattering images in the x- and y-directions, respectively. Column (f) shows integrated phase images calculated from differential phase images by using Fourier integration.

3. Comparison of demodulation results and discussion

(a) Results for a synthetic phantom

Firstly, we compare the demodulation results obtained by using the two demodulation methods described above and the conventional FT method. Here, we use the synthetic phantom of cross-piled rods (shown in figure 1a). This is a typical example of the advantage of two-dimensional XTI because one-dimensional XTI cannot visualize the rod parallel to the grating pattern. A simulated fringe pattern obtained by two-dimensional XTI is shown in figure 1b, where the fringe pattern period is set to 4 px/fringe. Figure 1c shows a magnified original fringe pattern without the objects.

The results of demodulation using the proposed methods are shown in figure 2. Each row in figure 2 represents the demodulation results obtained by the two-shot hybrid and four-shot hybrid method, respectively. Columns (a, b) in figure 2 represent differential phase images in the x- and y-directions for each method. Although x-direction differential images show the horizontal rods, vertical rods appear in only the y-direction differential phase images, and two-dimensional imaging reveals rods in both directions. Conventional absorption images are also obtained, as shown in column (c) in figure 2. Columns (d, e) in figure 2 show the degradation of the fringe pattern in two directions. It represents edge scattering and internal scattering of X-rays in the
object. Recently, scattering imaging has attracted considerable interest because of its subpixel resolution. This scattering information has an anisotropic structure, and Jensen et al. [13] showed that the anisotropy of X-ray scattering can be visualized and reflects the object’s microstructure. Two-dimensional XTI is expected to be able to visualize the anisotropic properties of the structure by using scattering information in two directions. In columns (d,e), the two-dimensional edge shapes of rods are visualized. In this case, the alignment of the rods becomes clear, as in the differential phase images. Furthermore, although the rods extend beyond the exposure area, the integrated phase image is successfully calculated by Fourier integration with two-directional differential phases, as shown in column (f).

The improvement in spatial resolution is shown in figure 3, where the edges in the differential phase images are sharper. Figure 3a,b and c shows magnified x-direction differential images taken by the FT method, the two-shot hybrid method and the four-shot hybrid method, respectively.

Figure 3 shows a comparison of the magnified x-direction differential images obtained by (a) the FT method, (b) the two-shot hybrid method and (c) the four-shot hybrid method. Figure 3d presents a cross-sectional plot at the rod edge with the ideal curve, which is calculated from the phase data of figure 1a. The rod edge appears sharper than that of the FT method, indicating that the primary shape of the edge is more accurately retrieved by our proposed method as a result of improving the spatial resolution. Among the tested methods, the four-shot hybrid method achieves the best results and gives the profile closest to the ideal curve, owing to this method having the largest filter radius. The two-shot hybrid method gives a similar profile to the four-shot method, except for the shape of the top edge.
(b) Experimental results

The proposed method is also applied to experimental data collected in our laboratory using a Talbot–Lau system. The experiments are performed using a conventional X-ray source (Rigaku ultraX 18) with a silver target. The effective energy of incident X-rays is assumed to be 22 keV and the gratings used in the system are designed to that effective energy. The phase grating is fabricated with a 4.3 µm checkerboard pattern. The moiré fringe pattern is captured by the detector (Radicon Shad-o-Box; pixel size: 48 µm), and the fringe period is set to 4 px wide along both the x- and y-directions. The fringe pattern is captured at the third self-image distance. The magnification is set to 1.41.

Here, to estimate the RMS error of demodulated images, the radiation dose in each method kept constant. In particular, the exposure time of each shot is derived by dividing the total exposure time by the total shot number in each method (one-shot, two-shot and four-shot methods), and the total exposure time is set to 160 s. As the sample object, three types of meshes are used to show the improvement of spatial resolution.

Figure 4 shows a comparison of x- and y-directional differential phases and integrated phase images obtained using the proposed approach. The fibre diameters for the meshes are 35, 50 and
Table 1. RMS error of demodulated phase along x- and y-directions for each method.

<table>
<thead>
<tr>
<th></th>
<th>one-shot FT</th>
<th>two-shot hybrid</th>
<th>four-shot hybrid</th>
</tr>
</thead>
<tbody>
<tr>
<td>x-direction</td>
<td>0.019</td>
<td>0.027</td>
<td>0.039</td>
</tr>
<tr>
<td>y-direction</td>
<td>0.018</td>
<td>0.028</td>
<td>0.041</td>
</tr>
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110 µm from left to right in each image. The shape of the 110 µm fibre, which is about 3 px wide on the detector, is much clearer as obtained by the hybrid methods than by the one-shot FT method. The 50 µm fibres, which are about 1.5 px wide on the detector, are also barely visible in the hybrid cases. This indicates that the spatial resolution is improved by the proposed method. Although the sharpness of images demodulated by the two-shot hybrid method is less than those demodulated by the four-shot hybrid method, the spatial resolution of the two-shot hybrid method is sufficient for expanding the scope of two-dimensional XTI application.

RMS error of demodulated differential phase along the x- and y-directions is shown in Table 1. This RMS error is calculated from the demodulated differential phase without an object and indicates the noise sensitivity of demodulated phase for each method under the same dosage. The RMS error of the four-shot hybrid method is the worst in spite of the fact that the total dosage is same as in the other methods. However, this is a natural result because the RMS error is affected by the radius of the filter function $G(k_x, k_y)$, which serves as the low-pass filter and apparently reduces RMS error. Therefore, RMS error is proportional to the filter function radius and tends to worsen in the case of hybrid methods using a larger filter function. From this viewpoint, it is understandable that the RMS error of the four-shot hybrid method is twice that of the one-shot FT method.

4. Conclusion

This paper presented a hybrid demodulation two-dimensional XTI approach, which combines the FT and PS methods; this approach achieves higher spatial resolution than the FT method and has a simpler procedure than the PS method. Objects, for example fibres, regardless of orientation, were visible in phase-contrast images because the signal at the object’s edge is stronger than that in the conventional FT method. RMS error is dependent on the total radiation dose and radius of the filter function. Comparing the two-shot and four-shot hybrid methods, the difference is relatively small as shown in figures 3 and 4. Depending on the situation, the two-shot method, which requires only an additional shot while translating the relative position of gratings in one dimension, is capable of improving the spatial resolution.

These methods can be used to augment the advantages of two-dimensional XTI by optimizing the total imaging time. In particular, dynamic XTI imaging with high spatial resolution is a highly attractive area of research, as Momose et al. [14] reported for one-dimensional XTI. In addition to the two-dimensional analysis of the object, the simple exposure process in the proposed methods can also minimize the total imaging time by trading off with respect to the required spatial resolution, which cannot be achieved with the FT method.

To improve the resolution of one-dimensional XTI, our hybrid approach and the underlying concepts can also be applied to one-dimensional systems that use the FT method. In addition to the technical aspects of system design optimization, the improvement in phase demodulation can extend the application scope of XTI systems.

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