Using the split Hopkinson pressure bar to validate material models

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This paper gives a discussion of the use of the split-Hopkinson bar with particular reference to the requirements of materials modelling at QinetiQ. This is to deploy validated material models for numerical simulations that are physically based and have as little characterization overhead as possible. In order to have confidence that the models have a wide range of applicability, this means, at most, characterizing the models at low rate and then validating them at high rate. The split Hopkinson pressure bar (SHPB) is ideal for this purpose. It is also a very useful tool for analysing material behaviour under non-shock wave loading. This means understanding the output of the test and developing techniques for reliable comparison of simulations with SHPB data. For materials other than metals comparison with an output stress \(\sigma\) strain curve is not sufficient as the assumptions built into the classical analysis are generally violated. The method described in this paper compares the simulations with as much validation data as can be derived from deployed instrumentation including the raw strain gauge data on the input and output bars, which avoids any assumptions about stress equilibrium. One has to take into account Pochhammer–Chree oscillations and their effect on the specimen and recognize that this is itself also a valuable validation test of the material model.

1. Introduction

The material models in QinetiQ have evolved with, and as a result of, a close collaboration between experiments, analysis and numerical simulation. The constitutive and fracture models are a good example of this [1–3]. The reason for their development is an inability to predict all...
of our problems of interest using models widely available in the external literature and deployed in commercially available numerical simulation codes. In developing our models, we desired those models to be as physically based as possible. To have as few constants as possible: for those constants to have physical meaning so as to facilitate characterization and implementation; to require only a small overhead for characterization; and for the model constants for a particular material to be valid over as wide a range of strain, strain rate and temperature as possible. We also wished to avoid iteration in the numerical scheme and the use of optimizers in determination of constants. Ideally, each material would have a unique set of constants for its model.

The split Hopkinson pressure bar (SHPB) has been in use for many years to measure the compression response, principally of metals, but also more recently for much softer materials such as polymers. The detailed experimental technique and analyses are well documented and there have been several review papers [4–6]. The SHPB test is regarded as the key validation test for material models at high strain rate. The main reason for this is that the specimen is considered to be in a state of uniaxial stress after attaining stress equilibrium and high strains can be achieved through purely elastic loading in the bars where there are no shock waves present. Thus, the stress state applied to the material is well defined. In addition, for metals, equilibrium is seen to be obtained after three stress wave transits in the specimen based on either a one- or three-wave analysis and the onset of plasticity ensures no volume change in the specimen. Thus, one simply needs to monitor the stress \( v \) strain response in the specimen to compare with the experimental output. Our view is that for general metals (e.g. copper and iron), this method and analysis is perfectly adequate for the purpose of defining the dynamic deformation behaviour of the material and validating constitutive models.

However, this classical analysis starts to break down when investigating the behaviour of softer non-metallic materials such as polymers, polymer composites, foams, etc., and also for porous and granular materials. For these materials, the deformation mechanisms are driven by elastic behaviour and thus it is not always clear when stress equilibrium is obtained—if indeed the sample is ever in equilibrium during the test. The assumption of no volume change is also not always appropriate as there can be significant damage in the specimen, equating to voids or cracks and is certainly violated for foams. Using simulations of the SHPB for validating the material or constitutive model can also be misleading because the nature of the material model will actually determine whether the sample is in equilibrium in the simulation. This approach can become a circular argument and lead to erroneous conclusions on the validity of the model.

Different approaches, therefore, are required to overcome these issues both experimentally and in numerical simulations. This discussion paper describes these methods and how they can be used to provide much more effective validation of the material models as well as shedding useful insights on the nature of the SHPB test itself.

2. Improvements to experiments

The standard SHPB apparatus is very straightforward and consists of a striker bar hitting an input bar and an output bar with a specimen between the two bars, as shown in figure 1.

The general technique employs strain gauges on the input and output bars to monitor the input, reflected and transmitted pulses. However, this basic experimental arrangement can be greatly improved so that data on frequency-dependent attenuation can be derived directly, timing of force measurements obtained and comparisons of force and strain development in the sample
performed. This is a good test of predictive models due to the cyclic nature of the imposed load. If the test is viewed simply as a way of imposing a stress wave on a sample in a less extreme manner then comparisons can be made with the plate impact test.

There have also been extensive attempts at improving the signal transmitted through the specimen. One method to achieve this is to use polymer bars so that the impedance of low-density samples is much better matched to the loading bar. The required analysis becomes more complicated as the elastic wave propagation is affected by more external and environmental effects than a metal bar. However, this can be overcome by using additional strain gauges on the bars so that dispersive- and frequency-dependent attenuation can be measured directly. Such attenuation provides a direct description of the physical behaviour and should be able to be predicted by polymer models. This method has been pioneered by several research groups and significant advances have been made [7,8].

Although the method was designed for polymer bars, it can also be used with metal bars and here it shows clear advantages for viscoelastic samples. The analysis also gives more information than just force and strain, modulus is of prime interest, and this can be used in validation. An example is given in figure 2 where hydroxyl-terminated poly(butadiene) (HTPB) has been tested in the SHPB using metal bars and stress versus strain determined by two methods: the classical metals analysis including all the inherent assumptions and the analysis due to Aleyaasin & Harrigan [8] which has no inherent assumptions. It is immediately notable that the output of Harrigan’s analysis reproduces the expected strain stiffening of the material and contains useful modulus information, in contrast to the classical analysis.

There are also novel methods of measuring the strains directly on the specimen surface. An example of this is digital image cross-correlation (DICC) methods [9]. These data have been used by a number of researchers to obtain useful additional information such as displacement and accelerations of the sample and also to measure the wave propagation within the sample [9]. This technique has the potential to directly measure the localization within the specimen as a function of time and space in response to a well-defined stress state. The full utility of DICC to capture velocity information for both characterization and validation is only now being realized particularly when combined with velocity measurement perpendicular to the specimen surface using VISAR or PDV. The Hopkinson bar set-up is entirely suited to this because the entire outer surface is available for viewing.

### 3. Comparison of simulation with experiments

As stated previously, one of the key uses for the SHPB is as a validation method for material models at high strain rate. This is very difficult to achieve when comparing with the usual
analysis as there are so many in-built assumptions. This is illustrated for the three-wave analysis, which uses the relation between the input, reflected and transmitted pulse to calculate the stress–strain response and is therefore a more complete analysis, as opposed to the one-wave analysis, which simply uses the transmitted pulse. However, even the three-wave analysis assumes stress equilibrium and volume conservation in the specimen. To illustrate how complex these analyses are a comparison of a one-wave and a three-wave analysis for a polymer composite is shown in figure 3.

The assumption is made that the specimen is in equilibrium when the one- and three-wave analyses are similar. From the figure, this is only true at relatively high strains above 0.2 and even then the judgement is subjective. The stress/strain trace is further complicated as there are significant Pochhammer–Chree (P-C) oscillations present. The oscillations arise from the dispersion of the different wavelengths as they propagate along the input bar. Therefore, it is very difficult to ascertain the true stress/strain behaviour in the sample.

The way to overcome this is to simply use the raw gauge data from the input and output bars for the incident, reflected and transmitted pulses directly. Thus, there are no implicit assumptions about stress equilibrium or volume conservation in the specimen as the gauges only measure the waves in elastic bars. For soft materials, the transmitted pulse is very small and thus the gauges have to be sufficiently sensitive to resolve these small stresses. This requires the use of semiconductor gauges to ensure that the signal to noise ratio is sufficiently large. An example of the input, reflected and transmitted pulses for a polymer composite is shown in figure 4, where it is noted that the ‘noise’ level on the traces is sufficiently low to enable the stress to be monitored.

4. Validation of material models

The conclusion of §3 is that to effectively compare the simulations with the experiments, then it is necessary for the comparison to be made with the raw gauge output. This uses the full power of an explicit simulation code. While this is a step forward in that it removes restrictions on the geometry of the specimen, it is still somewhat superficial and hides the latent potential of the SHPB for validation of material models. It also masks some serious issues concerning an understanding of the test.

The most effective way of illustrating some of these issues, our approach and lessons, we have learned is through a case study [10]. An SHPB comprising aluminium alloy bars was used to test several polymer-bonded explosive (PBX) specimens, comprising RDX explosive mixed with an HTPB binder to various fill levels. The PBX test samples were cylinders 10 mm diameter, 5 mm thick; the average strain rate in the test was about $2000\,\text{s}^{-1}$. These tests were then simulated in our-modified version of the Lagrange hydrocode DYNA using the Porter–Gould constitutive model [11,12] to describe the response of the PBX specimens. An overlay of the experimental and predicted stress–strain curves is shown in figure 5. The predicted amplitude, and average trend

![Figure 3. Comparison of one- (solid line) and three-wave (dashed line) analyses of SHPB data for a PBX.](http://rsta.royalsocietypublishing.org/Downloaded from http://rsta.royalsocietypublishing.org/ on June 20, 2017)
of the stress–strain response, is in reasonable agreement with experiment; the difference at the beginning is due to the time required for stress equilibrium to occur within the sample which is, in itself, indicative of the sorts of issues that need to be addressed in the constitutive model. It also indicates the difficulty of measuring normal material parameters such as Young’s modulus and damage evolution from the test.

The most striking feature of the comparison is the wildly oscillatory nature of the DYNA prediction which is not observed in the experiment. Similar comparisons were observed in most of the early comparisons of prediction and experiment. These results raised several important

Figure 4. Experimental input (a) and output SHPB (b) traces for a PBX.

Figure 5. Two predictions of PBX sample response on SHPB compared with experiment. Solid line, experimental data; dashed line, analytic P-G model; dotted line, DYNA simulation.
questions: ‘Were the oscillations real?’, ‘Why were they not observed in the experiment?’ and ‘Was there a flaw in the constitutive model?’

To eliminate the Porter–Gould constitutive model itself as the cause of these oscillations, a simple elastic-plastic model was used to approximate the PBX sample. As before, the predictions exhibit large oscillations (figure 6), demonstrating that the choice of constitutive model is not their cause. It was subsequently suspected that the source of these oscillations may be P-C dispersion in the aluminium bars.

It was further considered that polymeric materials may be particularly susceptible to such oscillations due their compliant nature and Poisson’s ratio close to \( \nu = 0.5 \). To investigate the latter, the elastic-plastic approximation was re-run with the Poisson’s ratio reduced from the original value of \( \nu = 0.496 \)–0.2 but maintaining the same equation of state. The stress curve in this instance, figure 7, was found to be much smoother with the oscillations being dramatically reduced in relation to the stress level achieved in the test. This dependence of apparent size of oscillation on the Poisson’s ratio can be understood by examining the relation between the elastic moduli

\[
G = \frac{3}{2} \frac{1 - 2\nu}{1 + \nu} K, 
\]

where \( K \) and \( G \) are the bulk and shear modulus, respectively. As \( \nu \) approaches 0.5, the ratio \( K/G \) becomes large and, for a constant bulk modulus, the stress is dominated by the shear component of the response (\( G \)). Thus, for \( \nu \sim 0.5 \), the oscillations appear to be large despite being of equivalent absolute amplitude in both cases.
To test the P-C oscillation hypothesis, a dispersion analysis was performed on the input and output bars in the simulation based on Bancroft [13]. Corrections were made to the wave speeds of each Fourier component according to Bancroft’s formula

$$\phi_n = 2\pi \omega n z \left( \frac{1}{C_0} - \frac{1}{C_n} \right),$$

where \( \phi \) is the applied phase shift, \( \omega \) is the angular frequency, \( z \) is the position, \( C_n \) is the wave speed of the \( n \)th Fourier component and \( C_0 \) is the wave speed at long wavelengths. The raw and dispersion corrected stress-time pulses for the simulated input and output bars are presented in figure 8. It was found that applying Bancroft’s dispersion corrections dramatically reduced the oscillations in the incident, reflected and transmitted pulses, thus confirming the P-C hypothesis.

The apparent absence of P-C oscillations in the experiments is explained by the need for precise alignment between the striker and input bar for high Fourier frequencies to be generated. It is these high frequencies that produce the observed P-C oscillations. In most experimental cases, there is an angle, albeit often very small, between these bars resulting in no P-C oscillations or reduced levels being observed. Simulations generally are carried out with perfect alignment.

Figure 8. Dispersion correction in the input bar (a) and output bar (b) for PBX sample. Solid line, original pulse; dashed line, corrected pulse.
When developing or validating new material algorithms, the benefits or hindrances of P-C oscillations need to be considered. The presence of P-C oscillations can be exploited to produce a load/unload cycle in the specimen. This feature represents a vigorous test of the material model, particularly its ability to properly reproduce hysteresis, and is a powerful demonstration of the benefits of the integration of numerical simulations, with experimental data and analytic analysis. Conversely, it can be beneficial to reduce P-C dispersion so that the true effect of the material on the input pulse can be seen as clearly as possible. It allows a direct comparison with an analytic form of the material model where there are simpler wave propagation effects. A method commonly used to achieve this reduction in Hopkinson bar tests is to curve the end of the striker bar as shown in figure 9. The effects of alignment and bar curvature on response of polymeric test specimens have since been confirmed both experimentally and with the aid of DYNA simulations using the Porter–Gould constitutive model.

This important result demonstrates that the hydrocode is capable of capturing all the physics in the SHPB in terms of the input, reflected and output pulses.

An example of suppressing P-C oscillations for comparison with the Porter–Gould constitutive model of a polymer composite is shown in figure 10 but it must be remembered that, while this type of analysis can give some confidence that the material model is predicting the correct behaviour, it gives no indication of how this model would behave in a hydrocode where there are more complex wave propagation effects.

![Figure 9. Comparison of numerically simulated input pulses for flat (dashed line) and curved (solid line) impact face on the striker bar for SHPB.](image)

**Figure 9.** Comparison of numerically simulated input pulses for flat (dashed line) and curved (solid line) impact face on the striker bar for SHPB.

![Figure 10. Stress vs strain fit of Porter–Gould model to SHPB test at room temperature for a PBX. Solid line, data; dashed line, analytic P-G model.](image)

**Figure 10.** Stress vs strain fit of Porter–Gould model to SHPB test at room temperature for a PBX. Solid line, data; dashed line, analytic P-G model.
When this comparison is performed with an SHPB output trace which naturally includes the P-C oscillations, the result is shown in figure 11.

The level of agreement for the general stress level and pulse length is good, although it is noted that the model exhibits significantly more oscillations than the experiment. The main reason for this is that the load/unload capability of the model is still limited and requires a fully physically based model to describe the polymer, which is currently being developed. However, without a robust method of comparing hydrocode with experiment, it would be unclear where the deficiencies in the model lay. In addition, the model does not pick up the very high initial modulus in the output pulse, which also requires further theoretical analysis. However, this level of agreement is a significant improvement on the previous models we were using to describe polymer composites.

Another interesting feature of this output trace in figure 11 is the large reduction in the stress after 150 µs which may indicate the onset of failure or major damage in the specimen. The model appears to predict this effect very well although the subsequent reload is not well captured suggesting some missing physics. Even more insight is obtained when the comparison is performed with an SHPB test performed at 223 K on the same material as shown in figure 12.

The level of agreement is very impressive in terms of the stress level and the apparent oscillations seem less severe, although they are of the same magnitude as observed in the room temperature test and only seem reduced as the overall stress level in the sample is increased. The length of the pulse is different because the specimen exhibited radial cracks during the test. Indeed, the time of the pulse can be directly correlated to the timescale of macroscopic fracture
in the specimen giving another parameter for validation. This general level of agreement gives enhanced confidence that the models are suitable for use in real applications such as hazard assessment of munitions.

5. Summary

The SHPB is a very useful apparatus for generating test data for understanding material response and for validating material constitutive models. It can support a wide range of instruments and this allows a wider range of validation data than just stress versus strain. For materials other than metals, this relies upon the use of explicit numerical simulations compared with direct output of the instrumentation. Various features of the method and of the numerical simulation such as varying intensities of P-C oscillations need to be accounted for but also provide further benefit to verification and validation.

Data accessibility. The data underlying this work may be available on request from the Ministry of Defence.

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