Wave-induced response of a floating two-dimensional body with a moonpool

Arnt G. Fredriksen¹, Trygve Kristiansen¹ and Odd M. Faltinsen¹,²

¹Centre for Ships and Ocean Structures (CeSOS), and ²Centre for Autonomous Marine Operations and Systems (AMOS), Department of Marine Technology, NTNU, Trondheim 7491, Norway

Regular wave-induced behaviour of a floating stationary two-dimensional body with a moonpool is studied. The focus is on resonant piston-mode motion in the moonpool and rigid-body motions. Dedicated two-dimensional experiments have been performed. Two numerical hybrid methods, which have previously been applied to related problems, are further developed. Both numerical methods couple potential and viscous flow. The semi-nonlinear hybrid method uses linear free-surface and body-boundary conditions. The other one uses fully nonlinear free-surface and body-boundary conditions. The harmonic polynomial cell method solves the Laplace equation in the potential flow domain, while the finite volume method solves the Navier–Stokes equations in the viscous flow domain near the body. Results from the two codes are compared with the experimental data. The nonlinear hybrid method compares well with the data, while certain discrepancies are observed for the semi-nonlinear method. In particular, the roll motion is over-predicted by the semi-nonlinear hybrid method. Error sources in the semi-nonlinear hybrid method are discussed. The moonpool strongly affects heave motions in a frequency range around the piston-mode resonance frequency of the moonpool. No resonant water motions occur in the moonpool at the piston-mode resonance frequency. Instead large moonpool motions occur at a heave natural frequency associated with small damping near the piston-mode resonance frequency.
1. Introduction

The use of moonpools to perform marine operations is expected to increase significantly. One reason is the rapid increase in development of subsea factories. Operators have defined goals such as near all-year availability for maintenance and repair, requiring operability in, for example, significant wave height $H_s = 4.5\, \text{m}$ in the North Sea. Specialized offshore vessels with moonpools are regarded as one of the key elements for achieving this. However, this requires careful design of the moonpool in order to avoid excessive resonant piston-mode motion. Here, the piston-mode resonance is defined as the resonant liquid motion in the moonpool causing a net liquid flux through the lower entrance of the moonpool. The resonant flow is nearly vertical and one-dimensional in most of the moonpool. The word piston is associated with that the liquid motion appears like the motion of a piston.

This study is aimed towards improved understanding of the moonpool resonance problem. The setting is that of a freely floating vessel subjected to waves. A two-dimensional approach is taken in order to simplify the study. Dedicated experiments are performed in a two-dimensional wave flume. Numerical methods are further developed in order to also study the problem numerically. The results are compared.

For the two-dimensional moonpool problem, there exist several studies concerning the resonant piston-mode elevation in the moonpool under forced oscillations. The problem has previously been looked at experimentally and from both a potential flow and viscous flow point of view by using analytical and numerical methods. Few studies have been reported using incident waves on a floating two-dimensional body with a moonpool. Therefore, this study adds new experimental data to the literature and validation data for our two hybrid methods combining viscous and potential flow solvers.

A two-dimensional freely floating vessel close to a terminal was investigated by [1] using both numerical and experimental methods. Their numerical work was motivated by the following question. What was the main cause of the discrepancies between linear potential flow theory and what was measured? Was it (i) flow separation or (ii) nonlinear free-surface conditions? They found that flow separation was the main physical effect explaining the difference.

By using two-dimensional potential and viscous flow methods, Lu et al. [2] studied the wave amplitude inside narrow gaps between three adjacent boxes subjected to incoming waves. Model tests and numerical calculations in three dimensions of the gap resonance between two rigidly linked side-by-side barges were performed by Molin et al. [3]. The rigid-body motions of two freely floating adjacent barges in three dimensions were considered in [4] by using a first- and second-order potential flow analysis. Recently, Kristiansen et al. [5] validated a hybrid method against a three-dimensional moonpool set-up with sharp corners at the lower moonpool entrance. Their results showed that the resonant free-surface amplitude in the moonpool decreases from around 70 times the forced heave motion to between 10 and 20 times when flow separation is included.

We will start by presenting the experiments. Next, we will describe the two numerical hybrid methods that have been developed. Last, we will present and compare results from the experimental and numerical studies.

2. Experiments

The experiments were performed in a wave flume at the Marine Technology Centre at NTNU in Trondheim, Norway, in the same location as the experiments in [1,6,7]. The wave flume is 12 m long, 0.6 m wide and has a 1.0 m water depth. A flap-type wavemaker hinged 5 cm above the tank bottom is mounted at one end, and at the other end a parabolic beach is used to absorb the waves (figure 1). The present model was similar to the model used in the forced oscillation experiments in [6]. The draft-to-beam and moonpool gap-to-beam ratios were the same as in [6]. The model consisted of two rectangular-shaped hulls of $20 \times 20 \times 58.6\, \text{cm}$ mounted 10 cm apart to form a moonpool. They were rigidly connected by two aluminium L-profiles. The model was ballasted
with weights strapped to the inside of the model, such that the model floated with a 10 cm draft. Table 1 describes parameters in the model test set-up. We tuned the natural period in roll to be higher than the moonpool piston-mode natural period. Nearly horizontal mooring lines with an angle of $3^\circ$ were connected to each side of the hull to prevent the model from drifting. This angle gave a small, but negligible, coupling between heave and the other degrees of freedom. At the ends of these lines, springs were connected, such that the model was free to move in sway, heave and roll. Between the hull and the springs each line went through a pulley, such that the springs were connected vertically to the roof by a force gauge (figure 1). Owing to the spring stiffness, the uncoupled natural period in sway was 3.2 s. The position of the centre of gravity (COG) and the moment of inertia of the model were determined by Norwegian Marine Technology Research Institute (MARINTEK) personnel by following standard procedures in commercial testing, called cradle tests.

The experimental programme was performed for 35 different wave periods and three different wave height-to-wavelength ratios (wave steepnesses): $H/\lambda = \frac{1}{60}$, $\frac{1}{45}$ and $\frac{1}{30}$, where $H$ is the wave height and $\lambda$ is the wavelength from the linear dispersion relation. The experiments with the highest wave steepness were repeated once and documented with a video camera. Prior to the experiments, a wave calibration series was performed without the model.

We performed system tests to ensure that the frictional coefficient in each individual pulley was sufficiently low so as not to influence the motion of the model. There was no hysteresis effect in the mooring system.

Three acceleration sensors were used to measure rigid-body motions in 3 d.f. Two of them measured acceleration in the body-fixed heave direction. They were placed on opposite sides of the hull (figure 1a). The third measured acceleration in the body-fixed sway direction. To find roll acceleration, the heave accelerometers were subtracted from each other and divided by the distance between them. The heave acceleration at the COG was found by taking the average from the two heave accelerometers. The position was found by integration of the acceleration signals. These were first band-pass filtered between 0.5 and 4 times the incoming wave frequency. Finally, the sway acceleration at the COG was found by removing the parasitic g-component and by use of the roll angle and simple geometric relations.
Four wave gauges were used to measure the surrounding wave field (figure 1). To measure the wave elevation inside the moonpool, copper tape was glued to the inside of each side hull. It was found that using conventional wave gauges inside the moonpool gap and fixed to the model would influence the inertia properties of the model too much.

(a) Discussion on error sources

In addition to the inherent uncertainties in the calibration factors and the measurement equipment, a few additional error sources are discussed. The properties of the waves generated by a hinged-type wavemaker are not fully reflected in the numerical simulations. Higher order effects in the generated waves are not captured in the numerical simulations. The incoming waves are specified by a known linear velocity potential on the free surface and the incoming wall boundary. The difference this represents is thought to be negligible, as only first-order motions and how they are affected by the wave steepness are considered herein.

Friction from the pullies should preferably be modelled as a Coulomb friction, i.e. a friction coefficient multiplied by a normal force. This has not been implemented as the friction was found to be negligible from individual tests on each pulley.

The model covers 98% of the width of the tank. There was a 7 mm gap between the model and the glass wall on each side. There are therefore three-dimensional flow effects at the model ends, which influence the pressure at the ends. In addition, there are viscous shear forces at the gap between the glass walls of the tank and the model to be accounted for. In accordance with Faltinsen [8] and Jonsson [9], laminar flow can be assumed on a smooth plane surface with oscillating flow if the Reynolds number defined as $Re = \frac{U_{lm}^2}{\nu \omega}$ is less than $10^5$. Here, $U_{lm}$ is the maximum tangential relative velocity just outside the boundary layer, and $\nu$ and $\omega$ are the kinematic viscosity coefficient and circular frequency of oscillation, respectively. It is found that laminar flow can be assumed for all variations of wave period and amplitude that are experimentally tested. The well-known solution of the Stokes second problem for laminar flow can be used to assess the influence of the viscous stress between the model and the water inside the narrow gap between the model and the glass. Solutions to this problem can be found

| Table 1. Dimensions and properties of the model test set-up used in the experiments. |
|-------------------------------------------------|-----|
| model draft (m) | 0.097 |
| width of each side hull (m) | 0.201 |
| moonpool gap width (m) | 0.100 |
| model length in transverse tank direction (m) | 0.586 |
| mass (kg) | 22.885 |
| radius of gyration $r_g$ (m) in roll about the COG | 0.18 |
| COG from the hull bottom (m) | 0.091 |
| spring constants (N m$^{-1}$) | no. 1 | no. 2 | no. 3 |
| | 43.7 | 42.5 | 88.2 |
| pre-tensions (N) | no. 1 | no. 2 | no. 3 |
| | 7.5 | 7.5 | 15 |
| line connection height above the calm water surface on the hull (m) | 0.05 |
| line angle (°) | ~ 3 |
| length between hull and left pulley (m) | 2.1 |
| length between hull and right pulley (m) | 2.0 |
in [10] and other textbooks. The oscillating boundary layer thickness is estimated to be at most \( \delta = 6.5 \sqrt{\nu/\omega} \leq 3 \text{ mm} \), which is less than the narrow gap in the model tests. The wet surface area \( A_g \) towards the glass is 0.08 m\(^2\), and the tangential viscous force is at most \( F_t/U_M = \sqrt{\rho \omega \mu A_g} \leq 0.20 \text{ Ns m}^{-1} \). Here, \( U_M \) is the maximum absolute velocity of the model and \( \mu \) is the dynamic viscosity of the water. This is negligible when compared with the forces measured by the force rings in the end of the mooring lines.

The parabolic beach was adjusted before the start of the experimental programme such that the top of the beach was 2–3 mm below the free surface. This has previously been found effective in damping of small-amplitude waves [7]. However, the wave flume had a small leak such that the water level slightly decreased by about 1–2 mm during the experimental programme. Repetition tests performed at the end of the experimental programme showed notable reflections of the longest waves for periods above 1.0 s, and a variation in the body motions of up to 5% for these periods.

There was significantly more drift in the recorded wave elevation from the copper tape than from the other conventional wave gauges. It is also possible that the meniscus effect is different on conventional wave gauges and the copper tape. Furthermore, there may be a very thin ‘run-up’ on the hull which is detected by the copper tape. This type of run-up is not numerically modelled.

### 3. Numerical methods

A semi-nonlinear and a nonlinear hybrid method are used. In both methods, the Navier–Stokes equations of an incompressible liquid are solved for in the vicinity of the body and the Laplace equation in the rest of the liquid domain. The main difference is that, in the semi-nonlinear method, both the body-boundary and free-surface boundary conditions are linearized, while in the nonlinear method they are satisfied at the exact free surface and body position. Furthermore, the governing equations in the nonlinear method are solved for in a body-fixed coordinate system. The nonlinearity in the semi-nonlinear method is due to the advective acceleration in the Navier–Stokes equations.

In the potential flow domain, the Laplace equation is solved using the recently developed harmonic polynomial cell (HPC) method from [11]. In the viscous flow domain, the flow is solved using the finite volume method (FVM) on a staggered rectangular mesh using Chorin’s projection method [12] involving a Poisson equation for the pressure as the most CPU-demanding step in our procedure. The main reason for choosing the HPC method as the potential flow solver is the high spatial accuracy. Low dispersion errors and low numerical damping in propagating waves are then achieved, compared with a lower order FVM or finite difference method. Waves can therefore be generated and propagated towards the structure with high accuracy in a numerical wave tank (NWT). Furthermore, the far-field waves generated by the structure can be described with high accuracy.

The following limitations apply in the developed code: (i) the body geometry must fit on a rectangular mesh, (ii) vorticity should not reach the free surface, and (iii) turbulence is not accounted for. The importance of turbulence depends on the problem being considered. For instance, it is crucial in determining the flow separation points for bodies without sharp corners in super- and transcritical flow conditions. Even though free vorticity flow becomes turbulent at much lower Reynolds numbers than for boundary layer flow, turbulence modelling is believed to be secondary when flow separation from a sharp corner occurs. However, the fact that turbulent diffusion of free vorticity flow is much larger than viscous diffusion does have an effect, in particular in ambient oscillatory flow. We are not aware of quantitative investigations of the latter effect with relevance to our problem. Turbulent flow modelling would require a three-dimensional model.

Using the present hybrid method instead of using the Navier–Stokes equation with a state-of-the-art numerical scheme in the whole liquid domain is significantly more time efficient.
Figure 2. Example of a mesh used in simulations by the semi-nonlinear hybrid method. The wave period is $T = 0.73\,\text{s}$. Grey cells belong to the potential flow domain, and black cells to the viscous flow domain. (a) The entire NWT. Note the difference in scales on the horizontal and vertical axes. (b) A close-up of the mesh close to the hull. Here, the scale ratio is correct. The free surface is included in both views. (Online version in colour.)

(a) Semi-nonlinear hybrid method

Our semi-nonlinear hybrid numerical method has similarities to the work by Kristiansen & Faltinsen [7], except that the low-order FVM used in their method is replaced by the HPC method in the potential flow domain and that the motions of the body are not forced, but follow from rigid-body dynamics. All boundary conditions are linearized around the initial position. The governing equations are solved in an Earth-fixed coordinate system in both flow domains. The computational benefit is that the mesh is constant in time. Figure 2 shows an example of a mesh used in a semi-nonlinear simulation. In the semi-nonlinear hybrid method, we are only able to match the pressure and the normal liquid velocity at the intersection. Therefore, we cannot guarantee that the tangential liquid velocity is continuous across the intersection. The staircase pattern at the intersection as shown in figure 2 is introduced as an attempt to also force continuous tangential velocity. This is not a perfect approach, but it increases the stability of the solution.

(b) Nonlinear hybrid method

The basics of the nonlinear hybrid numerical method follows the description given in [6]. There, a fully nonlinear hybrid method in a body-fixed, non-rotating coordinate system was presented. In this work, we have developed the hybrid method further to include freely floating body motions in a body-fixed rotating coordinate system.

The Earth-fixed $(y_e, z_e)$-coordinate system has the origin at the mean free surface at the initial centre position of the body. The vertical $z_e$-axis is positive upwards. The body-fixed $(y, z)$-coordinate system follows the COG and rotates with the roll motion of the body. The governing equations for the liquid flow are modified according to Faltinsen & Timokha [13, pp. 43–49]. For
Figure 3. Snapshot of body-fixed mesh at a specific time step used in the nonlinear simulation in the body-fixed coordinate system. The wave period is $T = 0.95 \text{s}$ and wave steepness $\frac{1}{16}$. Grey cells belong to the potential flow domain, and black cells to the viscous flow domain ($\Omega_{\text{CFD}}$). (a) The entire NWT. Note the difference in scales on the $y$- and $z$-axes. (b) A close-up of the mesh close to the hull. Here, the scale ratio is correct.

instance, the Navier–Stokes equations in the liquid domain $\Omega_{\text{CFD}}$ are transformed in a body-fixed rotating coordinate system to

$$
\frac{d^b \mathbf{u}_r}{dt} + \mathbf{u}_r \cdot \nabla \mathbf{u}_r = -\frac{1}{\rho} \nabla p + \mathbf{g} + \nu \nabla^2 \mathbf{u}_r - \mathbf{a}_0
$$

$$
- (\omega_0 \times \mathbf{u}_0) - \dot{\omega}_0 \times \mathbf{r}
$$

$$
- 2(\omega_0 \times \mathbf{u}_r) - \omega_0 \times (\omega_0 \times \mathbf{r}) \quad \text{in} \quad \Omega_{\text{CFD}}.
$$

Furthermore, incompressible liquid requires

$$
\nabla \cdot \mathbf{u}_r = 0 \quad \text{in} \quad \Omega_{\text{CFD}}.
$$

Here, $d^b \mathbf{u}_r/dt = (d\mathbf{v}_r/dt)\mathbf{j} + (d\omega_r/dt)\mathbf{k}$ means the time derivative of the liquid velocity vector relative to the body-fixed coordinate system for a fixed point in the body-fixed coordinate system, without time-differentiating the unit vectors $\mathbf{j}$ and $\mathbf{k}$ defined along the $y$- and $z$-axes, respectively. $\nabla$ is the nabla operator in body-fixed $(y, z)$-coordinates. $\mathbf{a}_0$ ($\mathbf{u}_0$) is the translatory acceleration (velocity) vector of the COG of the body. And $\dot{\omega}_0$ ($\omega_0$) is the angular acceleration (velocity) vector of the body. Further, $\mathbf{r}$ is the radial vector from the COG to the liquid particle, $\mathbf{g} = g[0, \sin(\eta_4), \cos(\eta_4)]$ is the gravitational vector and $\eta_4$ is the time-dependent roll angle.

The free-surface boundary conditions are formulated in a semi-Lagrangian manner, to follow a point on the free surface in the $z$-direction in the body-fixed, rotating coordinate system. The equations of motion are posed such as to reflect that all forces and moments are given in a body-fixed coordinate system (e.g. [14, pp. 421–424]).

To avoid a time-changing mesh in the viscous flow domain, the intersection between the potential and viscous flow domains needs to be below the expected minimum $z$-coordinate value of the free surface in the simulation. At every time step, the free-surface nodes are moved in the $z$-direction to the new position on the free surface. Parts of the HPC mesh are then stretched/compressed, as exemplified in figure 3. There are three challenges related to this remeshing strategy. One is related to large roll angles, the second is related to large moonpool
piston-mode motions and the third is related to the viscous flow domain around the hull edges where flow separation occurs. To satisfy these challenges, we are required to create a mesh with as large as possible viscous flow domain around the body, while still allowing the potential flow domain enough area to follow the free surface in time in the body-fixed coordinate system.

For the mesh inside the moonpool gap, an estimate of the expected moonpool piston-mode amplitude should be given and used to create the intersection between the potential flow and viscous flow domains. For the mesh outside the vessel, the limiting factor is the expected maximum roll amplitude $\eta_{4}^{4\text{Max}}$ in the simulation. This is illustrated in figure 3a, from which $\eta_{4}^{4\text{Max}}$ can be interpreted. The expected moonpool wave amplitude and roll amplitude are input to the simulation with a safety margin. For higher wave steepnesses and higher roll angles, the viscous domain close to the hull edges are decreased compared with figure 3, and vorticity will easily reach the intersection between the potential flow and viscous flow domains. The simulations with wave steepness $\frac{1}{45}$ and $\frac{1}{30}$ are stabilized by applying the five-point Chebychev smoothing from [15] in the near field of the structure on the free surface, one longitudinal body length on each side. To achieve stable simulations, the smoothing algorithm is applied once each time step.

To simplify the re-meshing algorithm, we keep the bottom of the NWT fixed in the body-fixed coordinate system. This simplification will, for large roll angles, result in the far-field free surface colliding with the ‘artificial’ numerical tank bottom. The numerical simulations are for larger roll angles performed with a larger water depth than in the experiments. However, the water-depth influence is secondary for the considered wave periods.

In the nonlinear hybrid method, we can guarantee continuous pressure, normal liquid velocity and tangential liquid velocity across the intersection. However, the method will become unstable if vorticity reaches the intersection. The staircase pattern is not needed, but is retained because of the re-meshing strategy.

(c) Equations of motion

The time-stepping procedure for solving the equations of motion is different for the two hybrid methods. For the semi-nonlinear hybrid method subtracting the proper infinite frequency-added mass multiplied by the acceleration on each side of the matrix system, and then integrating the equations forward in time, as done by Kristiansen [16], was found to be adequate for stability and accuracy. It is also beneficial that we in the semi-nonlinear method solve for the acceleration potential $\psi$ in the potential flow domain. For the nonlinear hybrid method, this approach was not stable enough. Owing to the body-fixed coordinate system, the Poisson equation for the pressure in the viscous domain has values related to the acceleration of the body on the right-hand side. In order to achieve a stable solution, an iteration scheme is implemented here. This means that, for every iteration on the solution, the acceleration is checked against the last guess on the acceleration. If the latter is higher than a preset tolerance value, the Poisson equation is solved again with a new guess on the acceleration. This is coupled to the iterative matrix solver, such that only one iteration with the iterative matrix solver is performed between each new guess on the acceleration.

We are not able to keep the integration of the equations of motion in accordance with the explicit fourth-order Runge–Kutta method and are solving the equations of motion in time using a linear forward Euler method. This means that the overall accuracy of the method is order $\Delta t$, not $\Delta t^4$. However, we keep the explicit fourth-order Runge–Kutta method for the propagation of free-surface waves.

Convergence and sensitivity studies were presented in [6] and assumed valid for the present numerical simulations. More convergence and sensitivity studies are presented in [17].

4. Results

First, we describe the numerical set-up, then we compare our numerical simulations with the experiments. The steady-state amplitude response in sway, heave and roll, and the free-surface
amplitudes on both sides of the moonpool gap oscillating with the fundamental frequency $\omega = 2\pi / T$ of the incident waves will be compared with the experiments and discussed. Furthermore, we present the following numerical studies:

- damping from forced oscillations in heave and roll; and
- comparison with linear potential flow theory.

(a) Numerical set-up

In between the wavemaker and the model, the horizontal mesh resolution is set to 30 cells over a wavelength. The mesh size gradually changes and becomes equal to the mesh size across the hull. This applies from either half a wavelength or two times the length of one side hull, depending on which is the longest. In the horizontal direction, the number of FVM cells across one side hull is set to be 30 (mesh size = 0.0067 m). The mesh size is kept constant across the moonpool gap, i.e. 15 cells across the gap. As the details of the boundary layer flow are secondary in our problem, we do not use a fine mesh in the boundary layers.

The mesh is symmetric around the hull, except that in the numerical beach the mesh size is increasing. The numerical beach starts three wavelengths after the hull and is four wavelengths long. The procedure for simulating energy dissipation by the numerical beach is similar to that in [18]. The total length of the NWT in addition to the length of the hull is 14 wavelengths, and therefore different for each wave period.

In the $z$-direction, the mesh resolution is constant from the free surface up to half the hull draft below the hull bottom, using 20 cells across the draft of the body (mesh size 0.005 m), then gradually increasing the size until the bottom of the tank. A total number of 60 cells in the $z$-direction is used.

Because of limitations in the implemented numerical method, only rectangular cells can be used in the viscous flow FVM domain. The HPC potential flow domain has no such limitation. However, to simplify the re-meshing scheme the HPC nodes in the same liquid column will all have the same $y$-coordinate. Figure 2 shows an example of a mesh used in the semi-nonlinear simulation and figure 3 shows an example from the nonlinear simulation for the $1/60$ wave steepness case.

(b) Validation of the numerical methods

Figure 4 presents non-dimensional numerical and experimental rigid-body motion amplitudes in 3 d.f. and moonpool free-surface amplitudes. The translatory motion amplitudes for sway ($\eta_2$) and for heave ($\eta_3$) are for the COG. The moonpool wave amplitudes are given from a fisherman’s (body-fixed) point of view. The rigid-body translatory motions are given in the Earth-fixed coordinate system. Furthermore, the roll amplitudes ($\eta_4$) are presented.

In general, the agreement between experiments and numerical calculations is good, in particular for the nonlinear hybrid method. The moonpool wave amplitude comparisons are reasonable, with some over-prediction on the right-hand side of the gap, and some under-prediction on the left-hand side. One of the reasons could be the quality of the measurements, keeping in mind that they are obtained from the copper tape glued onto the model. The semi-nonlinear hybrid method results suffer some notable discrepancies in sway and roll compared with the experiments around roll resonance. The roll amplitude is, at resonance, over-predicted by up to a factor of 2 depending on the wave steepness. The heave motion is well predicted. A minor discrepancy is that the natural heave period is over-predicted by about 2–3% by the semi-nonlinear hybrid method.

Two principal differences can be found when comparing the rigid-body motion with a single-hull case; see, for instance, two-dimensional experimental studies by Vugts [19]. The first principal difference is the heave resonance introduced by the moonpool, and the second is the cancellation
Figure 4. (a–c) Comparison of numerical and experimental results of rigid-body motion amplitudes ($\eta_2, \eta_3$ and $\eta_4$) and free-surface amplitudes on the left $\zeta_{\text{left}}$ and right $\zeta_{\text{right}}$ of the moonpool for three different wave steepnesses versus wave period. $\zeta_a$ means incident wave amplitude; $k$ means wave number.  

By scaling the resonant period of the piston-mode motion from the forced heave study in [6] to the dimensions used here, we get 0.88 s (the radiation problem). Also when considering incoming waves on a fixed moonpool structure (the diffraction problem), the piston-mode resonant period
Figure 5. Comparison of the space-averaged piston-mode free-surface amplitude $\zeta_{\text{EF}}$ in the Earth-fixed coordinate system due to incoming waves with different steepnesses on a fixed and a floating moonpool section versus wave period for wave steepnesses $1/30$, $1/45$ and $1/60$. The results are obtained using the semi-nonlinear hybrid method. $\zeta_4$ means incident wave amplitude. (Online version in colour.)

is found to be 0.88 s (figure 5). However, for the free-floating problem in waves there is no sign of a resonant piston-mode motion around 0.88 s. This is consistent with theoretical considerations by McIver [20], who found that within linear potential flow theory the main contributions from the diffraction and the radiation potentials are $180^\circ$ out of phase around the piston-mode natural period for a freely floating body. The latter fact was numerically confirmed for this case as well as for a case with twice the moonpool width. Figures 4 and 5 show for the freely floating body in incident waves that there is a pronounced large-amplitude moonpool behaviour only at the natural heave period of $T = 0.75\,\text{s}$.

Many authors have shown before that the potential-flow added mass and damping coefficients in heave for the moonpool problem are highly frequency dependent in the vicinity of the piston-mode resonance frequency. For instance, negative added mass values occur. Furthermore, a clear minimum in the heave damping occurs. The latter implies a clear minimum in the frequency-domain linear vertical wave excitation amplitude according to potential flow [21]. (See [22] for added mass and damping based on potential flow theory for different moonpool sections, where their case 1 corresponds to the case considered in this work.) The heave motion is therefore strongly affected by the moonpool, and resonant heave motion with a clearly peaked response occurs at $T = 0.75\,\text{s}$.

After the first peak in heave at 0.75 s, the moonpool wave motion becomes in phase with the heave response (see the phase angle $\alpha$ in figure 6a), where $\alpha$ is defined as the phase angle between the heave acceleration of the COG and the moonpool wave motion. This causes a decrease in the heave response; the moonpool wave response decreases the heave motion. This is illustrated in figure 7. The heave motion builds up faster than the moonpool wave response. After the initial build-up phase, the moonpool wave response is still increasing while the heave response starts decreasing. The initial heave response is thus larger than the steady-state response. The steady-state heave amplitude has a minimum at $T = 0.80\,\text{s}$ which is associated with a minimum in the amplitude of the steady-state vertical wave excitation force.

The phase angle $\beta$ between heave and roll motion is given in figure 6b. For low wave periods, heave and roll are $180^\circ$ out of phase, causing high local heave motion on the side hull facing the incoming waves. Similarly, the combined heave and roll motion has a cancellation effect on the aft side hull. For higher periods (stiffness dominated), the heave and roll motion are $90^\circ$ out of phase, meaning that the hull motion is in phase with the incident wave motion.

The sway and roll motions have no significant interaction effects with the moonpool motion. However, the sway and roll motion can excite sloshing modes within the moonpool gap, but for the present set-up the highest sloshing mode natural period is 0.36 s.
Figure 6. (a) Phase angle $\alpha$ between heave acceleration and moonpool wave motion (from the copper tape on the right-hand side of the moonpool gap) versus wave period. (b) Phase angle $\beta$ between heave and roll acceleration. Wave steepness $\frac{1}{60}$. $T$ means wave period. (Online version in colour.)

Figure 7. Time-series example illustrating how the heave motion ($\eta_3^*$) excites the piston-mode motion, and later how the heave motion is reduced as a consequence of the piston-mode motion inside the moonpool gap by means of a semi-nonlinear simulation, with wave period $T = 0.8$ s and $\frac{1}{60}$ wave steepness. $\zeta_{EF}$, space-averaged wave elevation inside the moonpool in the Earth-fixed coordinate system. (Online version in colour.)

We note the significant difference between semi-nonlinear and nonlinear hybrid methods around roll resonance. The differences between semi-nonlinear and nonlinear hybrid methods are not as large at heave resonance. The latter facts are consistent with the numerically predicted roll damping and heave damping at resonance (figure 8) as functions of roll and heave amplitude, respectively. For example, in the semi-nonlinear simulation with wave steepness $\frac{1}{60}$, the roll amplitude is $7.6^\circ$ at roll resonance and the heave amplitude is 4.5 mm at the heave resonance; in the nonlinear simulation, the roll amplitude is $4.9^\circ$ at roll resonance and the heave amplitude is 3.9 mm at the heave resonance.

The roll damping coefficient in figure 8 is obtained by calculating the damping moment about the COG ($180^\circ$ out of phase with the roll angular velocity) due to forced roll about the COG by both hybrid methods. The corresponding linear roll damping coefficient $B_{44}$ has been calculated. The simulations are performed for forced roll oscillation with different roll amplitudes (figure 8a). Three separate results from the nonlinear hybrid method are given, where the results are for the three different meshes used for the three different wave steepnesses. The roll resonance period $T = 0.96$ s is chosen. The nonlinear method shows nearly linear dependence of the roll amplitude on $B_{44}$, which is not present in the semi-nonlinear method. The wave radiation damping has been separately identified from the energy of the outgoing waves. The predicted wave radiation
damping by the semi-nonlinear and nonlinear methods is quite close. However, the total damping predicted by the two methods differs significantly except when the roll amplitude goes to zero and the damping is mainly due to wave radiation. The difference between the total damping and the wave radiation damping in the nonlinear hybrid method is mainly due to vortex shedding, i.e. eddy-making damping. It is a well-known fact that viscous shear stresses have a small influence on roll damping. The eddy-making damping depends on the instantaneous positions, velocities and strengths of shed vorticity. The latter fact can be qualitatively indicated by using a thin free-shear layer and boundary layer method as presented by Faltinsen & Pettersen [23]. It follows then that the motion of the separation points as it is accounted for in the nonlinear method causes a different expression for the amount of shed vorticity. Figure 8b shows the heave damping coefficient $B_{33}$ predicted by the two hybrid methods as a function of heave amplitude.

In the semi-nonlinear hybrid method, the governing equations in the potential-flow domain are linearized, meaning that contributions from the nonlinear advection terms in the Navier–Stokes equation are not communicated to the potential flow domain. The consequence is that the normal velocity and pressure are continuous across the interface, while the tangential velocity at the interface is discontinuous. The result is artificial local vorticity on the viscous flow side of the intersection. The latter fact was also observed by Greco et al. [24] in their studies with a domain decomposition method involving linear potential flow and the Navier–Stokes equations. Figure 9a illustrates that, during forced roll oscillation, we observed artificial vorticity at the viscous flow side of the horizontal part of the intersection. However, for the nonlinear hybrid method, the advection terms in the Navier–Stokes equations are communicated with the potential flow domain (figure 9c).

From forced heave oscillation tests, the resulting liquid flow and generated vorticity are more equal between the two methods (figure 9b–d). Here, the liquid flow is mostly normal to the horizontal part of the intersection and no artificial vorticity is observed at the intersection.

From figure 8a, we cannot conclude on the relative importance for the roll response of (i) nonlinear body-boundary conditions, (ii) motion of the separation points, and (iii) avoidance of artificial vorticity with associated nonlinear free-surface conditions. As a first step to determining this, studies with a double-body with closed gap in infinite fluid using both linear and exact
Figure 9. Illustration of instantaneous vorticity from the semi-nonlinear hybrid method (S-NL) in (a,b) and from the nonlinear hybrid method (NL) in (c,d). During forced roll oscillations ($\eta_4 = 3.0^\circ$ and $T = 0.96$ s) in (a,c), and during forced heave oscillations ($\eta_3 = 0.008$ m and $T = 0.75$ s) in (b,d). The time instant is the same in (a,c) and the same in (b,d). The colour scale represents the strength of vorticity and is not equal for the heave and roll cases. The colour scale ranges from negative vorticity in blue to positive vorticity in red, and passes through cyan, yellow and orange.

Figure 10. (a) Comparison of the total damping in roll predicted by the semi-nonlinear hybrid method with the sum of half the damping predicted by a double-body in infinite flow with linear body-boundary conditions and the wave radiation damping extracted from the semi-nonlinear case. (b) Same as in (a), but with the nonlinear hybrid method with mesh refinement corresponding to the $1/30$ wave steepness case, and a double-body in infinite flow with exact body-boundary conditions. (Online version in colour.)

body-boundary conditions were performed (figure 10a,b). By adding half the damping predicted by the double-body in infinite fluid to the wave radiation damping from figure 8a and comparing it with the total damping predicted in figure 8, we can examine the importance of nonlinear free-surface conditions versus exact body-boundary conditions. As the two curves in figure 10a are of the same order, it can be concluded that the main reason for the difference between the semi-nonlinear and nonlinear hybrid methods is the difference between linear and exact body-boundary conditions (figure 10a). To check that the result is somewhat general, the additional oscillation periods $T = 0.75$ and 0.83 s have been simulated, and the findings are the same.

The results in figure 8a agree with the results from Braathen [25], who found that vortex shedding had a secondary effect on the wave radiation damping in roll. The latter effect is associated with small far-field flow due to local vorticity at the body edges. The opposite effect, the influence from free-surface waves on vorticity generation and eddy-making damping,
can, according to the results in figure 10b, also be assumed secondary. The result is confirmed with additional numerical simulations at two other oscillation periods, $T = 0.75$ s and $T = 0.83$ s. Similar results for the eddy-making roll damping were obtained by using the rigid free-surface condition.

(c) Comparison with linear potential flow theory

As linear potential flow theory is used in many engineering applications, results are presented in figure 11 together with results from the two hybrid methods and experiments with wave steepness $\frac{1}{60}$. These results are obtained by solving for potential flow in the whole liquid domain.
using the present semi-nonlinear code. All motions are over-predicted around resonance by
the potential flow theory. The large over-prediction of the resonant heave and piston-mode
motions by linear potential flow theory shows the importance of flow separation at the lower
moonpool entrance in the floating two-dimensional moonpool problem. The resonant roll motion
is further over-predicted by a factor larger than 2 relative to experiments. The damping from
flow separation is one important reason for this difference. The damped natural roll period is
predicted to be 4% lower in the pure potential flow case than that predicted by the semi-nonlinear
hybrid method.

5. Conclusion

The rigid-body motions of a two-dimensional ship section with a moonpool and resonant piston-
mode motion of the liquid column inside the moonpool gap has been studied by both numerical
and experimental methods. Dedicated experiments were performed in a wave flume using
incoming waves on a spring-moored two-dimensional ship section.

Two hybrid methods combining potential flow and viscous flow based on [6,7] were applied,
with the viscous flow domain near the body. The first is a hybrid method based on linear
free-surface and body-boundary conditions, here called the semi-nonlinear method. The second
hybrid method is based on a nonlinear free-surface condition and an exact body-boundary
condition in a body-fixed coordinate system, here called the nonlinear method.

In general, the two numerical hybrid methods predict the rigid-body and moonpool responses
quite well. An exception is that the resonant roll motion is clearly over-predicted by the semi-
nonlinear hybrid method (figure 4), while the nonlinear hybrid method captures the roll motion
well. The reason for this is eddy-making damping. The shortcomings of the semi-nonlinear
hybrid method in predicting the roll motion are investigated by a detailed study of the damping
from forced roll oscillations with different forcing amplitudes. It is suggested that the main
reason for the differences is that the body-boundary condition is linearized in the semi-nonlinear
hybrid method.

The moonpool behaviour has a clear effect on the rigid-body motions. The moonpool wave
amplification factor is found to be around 2–2.5 times the incoming wave amplitude around
resonance, depending on the wave amplitude. In comparison, pure linear potential flow theory
predicts a factor of up to 10.

An important observation is that we cannot use the natural piston-mode period to calculate
which period causes maximum piston-mode response of a free-floating body in incident waves.
The linear potential flow response from the radiation and diffraction potentials is 180° out of
phase at the natural piston-mode period, with the consequence that the piston-mode motion at the
piston-mode natural period does not have any resonant behaviour. The maximum piston-mode
response is found at the resonant heave motion in the vicinity of the piston-mode resonance.

Funding statement. This work was supported by the Research Council of Norway through the Centres of
Excellence funding scheme AMOS, project no. 223254.

References

1. Kristiansen T, Faltinsen OM. 2010 A two-dimensional numerical and experimental
(doi:10.1016/j.apor.2009.10.001)
2. Lu L, Cheng L, Teng B, Zhao M. 2010 Numerical investigation of fluid resonance in
(doi:10.1016/j.apor.2009.10.003)
gap resonances in-between two rectangular barges. In Proc. 13th International Congress of the


