A review is given of the analogous dependence on reciprocal square root of grain size or crack size of fracture strength measurements reported for steel and other potentially brittle materials. The two dependencies have much in common. For onset of cleavage in steel, attention is focused on relationship of the essentially athermal fracture stress compared with a quite different viscoplastic yield stress behaviour. Both grain-size-dependent stresses are accounted for in terms of dislocation pile-up mechanics. Lowering of the cleavage stress occurs in steel because of carbide cracking. For crack size dependence, there is complication of localized crack tip plasticity in fracture mechanics measurements. Crack-size-dependent conventional and indentation fracture mechanics measurements are described also for results obtained on the diverse materials: polymethylmethacrylate, silicon crystals, alumina polycrystals and WC-Co (cermet) composites.

1. Introduction

Much of the analysis presented here is attributed to mentoring received from Clarence Zener, Norman Petch and George Irwin, as will be made clear in the referenced material. To begin with, Zener, as Director of the Westinghouse Research Laboratories, appointed me to a summer research position in East Pittsburgh in 1955, just prior to attending graduate school at Carnegie Institute of Technology. A post-doctoral year was spent during 1958–1959 under the direction of Norman Petch, then at Leeds University, and research association continued for many years. Research interaction with George Irwin began in 1982 when he became research professor at
the University of Maryland after distinguished careers both at the US Naval Research Laboratory and at Lehigh University.

There is a much longer history of science and engineering concern with fracturing of materials, particularly of brittle fracturing that can occur without warning. A historical example is provided by the late-nineteenth-century editorial comment from Wahl, who stated: ‘from the experiments thus made with perfect iron the strength of materials was formulated; but practical experience showed that large forgings and castings were subjected to blow-holes and honeycombs, which the most vigilant inspection would fail to discover—and these defects would lead to breakage at points of strain where theory said the metal should bear the given pressure. The matter became of so much importance in view of the large number of iron girders used in modern buildings, that in New York a law was passed requiring an actual test of beams, girders, etc.’ [1]. Wahl’s commentary was published only just more than one decade before Sorby [2] introduced a seminal paper on the microscopic examination of steel microstructures.

Just after turn of the twentieth century, Inglis [3] took up investigation of the stress state associated with a hole or crack and Griffith used the result to establish an energy balance for critical crack growth that follows the reciprocal square root of crack size dependence mentioned above and provides the basis for modern fracture mechanics [4]. Establishment of an explicit influence of grain size on the fracture strength of crack-free steel and related metals was slower in development. Taylor [5], co-inventor of the crystal dislocation, had suggested that the yield stress was independent of grain size, at least for aluminium. Kelly [6] has provided a valuable commentary on W.L. Bragg, of X-ray diffraction fame, suggesting a reciprocal grain size dependence of the plastic flow stress. Zener [7] was first to establish a connection between a model of a slip band and a shear crack, leading to prediction of a reciprocal square root of grain diameter dependence for plastic yielding, and later pointed to crack initiation occurring at the tip of a dislocation pile-up [8]. Eshelby et al. [9] produced the key model description of pile-up parameters that Hall [10] and Petch [11] employed to establish a reciprocal square root of grain diameter, \( \ell^{-1/2} \), dependence for measurements of the tensile yield, \( \sigma_y \), and cleavage, \( \sigma_C \), stresses, respectively, of steel in the same-type relationship

\[
\sigma = \sigma_0 + k\ell^{-1/2}.
\] (1.1)

In the Hall–Petch (H–P) equation (1.1), the experimental constants are \( \sigma_0 \) and \( k_y \) for the lower yield point stress and are \( \sigma_0C \) and \( kC \) for the cleavage fracture stress.

Thus begins the present story. The review starts with close examination of Petch’s experimental measurements for the yield and cleavage fracture stress, followed by generalization of equation (1.1) to describe the complete stress–strain behaviour of mild steel at ambient temperature, with true flow stress, \( \sigma_\varepsilon \), at true strain, \( \varepsilon \). Additional extension was to other body-centred (bcc), face-centred cubic (fcc) and hexagonal close-packed (hcp) metals and alloys. An important early application of the H–P relationship came from Cottrell [12] and Petch [13] in their specification of the ductile-to-brittle transition temperature (DBTT) exhibited by steel and related metals. The temperature and strain rate dependence of \( \sigma_0y \) is of paramount importance in determining the DBTT. The period of development occurred nearly simultaneously with the time of Irwin and co-workers extending Griffith’s work to establish the subject of fracture mechanics [14]. And further development by Irwin [15] and by Bilby et al. [16] dealt with modelling of crack tip plasticity that is shown to provide interpretation of modern fracture mechanics results obtained on additional steel, polymethylmethacrylate (PMMA), silicon, alumina and composite WC-Co (cermet) alloy materials.

2. Hall–Petch relations for steel

Petch measured at liquid nitrogen temperature the tensile yield and cleavage fracture stresses of mild steel, ingot iron and spectrographic iron materials having different polycrystal grain sizes [11]. Additional measurements were reported of the reduction in area exhibited by the
Figure 1. Combined tensile (lower) yield point and (higher) cleavage fracture stress measurements reported by Petch for several iron and steel materials [11].

failed specimens. The yield and cleavage strength measurements are combined in figure 1. The measurements shown on the left side of the segmented vertical line are solely cleavage fracture stresses. The right-side short line segments connect the clearly separated measurements at smaller grain sizes.

Within experimental accuracy, Petch chose to extrapolate the yield and cleavage stresses in figure 1 to a same value of $\sigma_{0C}$. However, the true fracture stresses were reduced at smaller grain sizes to correct for an increased plastic strain preceding fracture, thus producing a relatively low value of $k_C \{\varepsilon = 0\} \approx 80 \text{ MPa mm}^{1/2}$. The value compares with $k_C \approx 104 \text{ MPa mm}^{1/2}$ (and $\sigma_{0C} \approx 350 \text{ MPa}$) as given by Madhava [17] in a comprehensive compilation of grain-size-dependent $\sigma_C$ measurements, including (lower) intercrystalline failure stresses and (higher) post-yield cleavage measurements (see [18]). The issue relates importantly to the question of whether yielding would always precede fracturing, as will be discussed. Low [19] produced similar results to those of Petch at larger grain size but, in contrast, reported a much steeper grain-size-dependent cleavage stress, uncorrected for strain dependence, and taken to extrapolate to $\sigma_{0C} = 0$, thus mimicking a Griffith-type dependence.

In figure 1, the lower dashed line applies for the H–P dependence of the ambient temperature lower yield point stress of mild steel, $\sigma_{l,y,p}$ [20]. The result is shown more fully in figure 2 to include also a subsequent flow stress, $\sigma_\varepsilon$, dependence on grain size with constants, $\sigma_0$ and $k_\varepsilon$, such that $k_\varepsilon < k_{l,y,p}$, and leading to the earlier reported ductile fracture stress dependence on grain size [21]. For this material, the value of $\sigma_{0,18}$ was very nearly equal to the true stress at maximum load, and so the increase in stress until reaching a strain and ‘necking’-corrected ductile fracture stress, $\sigma_{d.f.}$, was largely accounted for by strain hardening during the necking strain. Comparison of results in figures 1 and 2 shows that $k_{d.f.} < k_C$ even for Petch’s lowered $\sigma_C$ dependence. The strain hardening during uniform straining past the lower yield point is seen to be mainly contained in $\sigma_{0\varepsilon}$ that is taken to measure plastic flow within the grain volumes.

The level of the lower yield point stress in figure 2 is linked through its temperature, $T$, and strain rate, $(\varepsilon/\varepsilon)$, dependence to the generally higher value of the athermal cleavage stress (figure 1). Reference is often made to Zener & Hollomon [22] for describing the coupled temperature–strain rate dependencies of $\sigma_\varepsilon$ on the basis of a chemical rate theory type of thermal activation description. The summer appointment mentioned earlier was to investigate the coupled relationship for molybdenum material tested in compression so as to avoid brittle fracture in tension [23,24]. Later development with Zerilli led to the so-called Z-A dislocation-mechanics-based constitutive relations for material dynamics calculations and are expressed for bcc metals in the form [25]

$$\sigma_\varepsilon = \sigma_G + B_0 \exp[-\beta T] + A\varepsilon^n + k_\varepsilon \varepsilon^{-1/2}. \tag{2.1}$$

In equation (1.1), $\sigma_G$ is an athermal stress component determined by the presence of solute, precipitates and an initial dislocation density; $B_0$ is a reference stress at temperature, $T = 0 \text{ K}$; $\beta$ is an exponential temperature coefficient, and $A$ and $n$ are coefficients for power
law strain hardening. The coupled $T$ and strain rate dependence are in $\beta$ that follows the relationship

$$\beta = \beta_0 - \beta_1 \ln \left( \frac{d\varepsilon}{dt} \right).$$

In equation (2.2), $\beta_0$ and $\beta_1$ provide linkage in equation (2.1) of the coupled $T$ and $(d\varepsilon/dt)$ dependencies. The first three terms on the right-hand side of equation (2.1) are normally combined in the single ‘friction stress’, $\sigma_{0\varepsilon}$, in equation (1.1). Different steel materials were shown to give reasonably constant values of $B_0$, $\beta_0$ and $\beta_1$ [26]. The dislocation mechanics model of $k_{l,y,p}$, though containing a local concentrated shear stress, $\tau_C$, has been found to be essentially athermal for steel. The situation is quite different for an important thermal dependence in $\tau_C$ for fcc and hcp metals [27]. The higher $k_{l,y,p}$ shown in figure 1 for the yield stress at liquid nitrogen temperature is possibly attributed to deformation twinning that follows its own (athermal) H–P dependence with constants $\sigma_{0T}$ and $k_T$, such that $k_T < k_C$ [28]. The relative $k$-values establish that deformation twinning is not responsible for cleavage.

Equations (1.1) and (2.1) were employed in figure 3 to illustrate the occurrence of a tensile DBTT at $\sigma_y = \sigma_C$ for mild steel [29], similar to the reported compression and tensile stress measurements made for molybdenum [24]. The yield stress parameters for steels A and B in equation (2.1) were taken as $B_0 = 1780$ MPa, $\beta = 0.0143$ K$^{-1}$, $\sigma_G = 30$ MPa and $k_y = 23$ MPa mm$^{1/2}$; the cleavage stress parameters were $\sigma_{0C} = 330$ MPa and $k_C = 104$ MPa mm$^{1/2}$. Steel A has a grain size of 100 $\mu$m and steel B, of 5 $\mu$m. The yield stress of A was raised to that of A* by addition of an athermal $\Delta \sigma_{0y}$, component of 260 MPa as might occur from neutron irradiation damage.

Figure 3 shows that the smaller grain size steel B has a lower DBTT because of the H–P inequality: $k_C > k_y$, whereas steel A*, which has an appreciably raised $\sigma_y$ because of the athermal component $\Delta \sigma_{0y}$ that leaves $\sigma_C$ unchanged, has a much higher DBTT owing to the raised friction stress. The situation relates to important measurements made on neutron irradiation damage in steel by Hull & Mogford [30]. The danger indicated in figure 3 for this case, which was deemed important at the time, was that steels B and A* could have the same ambient temperature yield strengths and temperature dependencies of them while exhibiting appreciably different values.

![Figure 2](http://rsta.royalsocietypublishing.org/)

Figure 2. Hall–Petch relations for mild steel stress–strain behaviour [20,21].
of the DBTT. Wechsler et al. [31] have reported quantitative measurements made on irradiation embrittlement of A212B reactor pressure vessel steel material.

3. Grain-size-dependent ductile–brittle transition

Cottrell and Petch produced a theoretical model description of the DBTT for steel and related bcc metals involving employment of the H–P dependencies and introducing a surface energy, \( \gamma \), for cleavage [12,13]. Cottrell’s version of the DBTT was expressed in terms of the H–P parameters \( \ell \), \( \sigma_0 \) and \( k_y \) as [12]

\[
k_y (\sigma_0 \ell^{1/2} + k_y) = C \gamma .
\] (3.1)

On the right-hand side of equation (1.1), \( G \) is the shear modulus and \( C \) is a numerical constant. By increasing \( k_y \), \( \sigma_0 \) or \( \ell \), the condition for cleavage cracking would be met more quickly and failure would occur. Equation (3.1) was shown to be in agreement with the Hull & Mogford [30] results giving an increase in the DBTT produced by an increase in \( \sigma_0 \). Al Mundheri et al. [32] have demonstrated that neutron irradiation clearly raises the yield stress by a constant \( \Delta \sigma_{0y} \) without changing the cleavage fracture stress.

Petch [13] obtained an explicit temperature dependence of the DBTT. The temperature-dependent component of \( \sigma_y \) was employed with the stress coefficient, \( B_0 \), and exponential temperature coefficient, \( \beta \), as in equation (2.1). Petch also employed in place of \( \sigma_y \) the true ductile fracture stress, \( \sigma_t \), with constants, \( \sigma_{0f} \) and \( k_f \) [21]. On such basis, an equation was obtained for \( T_C \) as

\[
T_C = \left( \frac{1}{\beta} \right) \ln B_0 - \ln \left\{ \left( \frac{C \gamma}{k_f} \right) - k_f \right\} - \ln \ell^{-1/2} .
\] (3.2)

Equation (3.2) provides indication in the \( \beta \) factor of the importance of the exponential temperature coefficient that is reduced at higher strain rate in accordance with equation (2.2). Experimental support was reported by Heslop & Petch [33]. It may be noted that the figure 3 DBTT description removes the need for the relatively uncertain parameter, \( \gamma \) [34].
Figure 4. The thermal component of stress dependence obtained from data in [36].

(a) Charpy v-notch transition results

In a later quantitative assessment of the DBTT measured in Charpy v-notch (CVN) tests, Petch [35] dealt with the influence on $\beta$ and $B_0$ of the substitutional alloying elements: manganese, nickel and silicon. The assessment connected with valuable experimental results that had been reported much earlier by Leslie et al. [36] for titanium-gettered interstitial-free steel and with other results reported later by Sandström & Bergström [37], including in the latter case a description of DBTT measurements reported for CVN tests. For relation to results reported by Petch, figure 4 leads for a 1.5% manganese steel to values of $B_0 = 1570$ MPa, $\beta_0 = 0.0092$ K$^{-1}$ and $\beta_1 = 0.00045$ K$^{-1}$, as determined from the measurements reported by Leslie et al. [36]. For the titanium-gettered low-carbon steel, $k_y$ was reduced to $5.5$ MPa mm$^{1/2}$, $\sigma_C = 70$ MPa and $\ell^{-1/2} = 5.6$ mm$^{-1/2}$, thus leading to an athermal stress of $100$ MPa being subtracted from the measured $\sigma_y$ values. Leslie et al. [36] had remarked that the lowered carbon concentration (of 0.004 wgt. %) had largely eliminated most of the grain size dependence of the DBTT, presumably because of the reduction in $k_y$, otherwise, Petch had noted as well the important influence of $k_y$ lowering [35].

A valuable historical note is that Orowan [38] had drawn attention to the condition of the yield stress being raised to the level of the cleavage fracture stress as a criterion for the onset of brittleness in steel. With a plastic constraint factor, $\alpha$, taken for the notch effect in a CVN test, the DBTT is specified as

$$\sigma_C = \alpha \sigma_y. \quad (3.3)$$

With $\alpha = 1.0$, the condition is the same as described for the tensile result in figure 3. Krafft et al. [39] had commented previously on the importance of the strain rate in accounting for CVN impact test results. Chlup et al. [40] have described application of higher strain rate testing to more brittle cast-basalt and soda-lime glass materials. Otherwise, equation (3.3) provides a quantitative description of the DBTT for a notched impact test [41,42] analogous to that given by Cottrell [12] and Petch [13].

Figure 5 illustrates the use of equation (3.3) in evaluating very complete $\sigma_y$, $\sigma_C$ and CVN measurements reported by Sandström and Bergström [37,41,42]. The 10 and 65 $\mu$m grain-size-dependent CVN results shown in figure 5 are seen to provide a counterpart description of the tensile transition shown in figure 3. A value of $\sigma_{0C} = 330$ MPa and $k_C = 107$ MPa mm$^{1/2}$

---

Figure 4. The thermal component of stress dependence obtained from data in [36].
had been measured by Sandström and Bergström. The constants $\sigma_{\text{C}} = 130 \text{ MPa}$, $\beta_0 = 0.0075 \text{ K}^{-1}$, $\beta_1 = 0.00040 \text{ K}^{-1}$ and $B_0 = 1000 \text{ MPa}$, and $k_y = 14.9 \text{ MPa mm}^{1/2}$ were determined from tabulated $T$, $(d\varepsilon / dt)$ and grain size measurements. The $\beta_0$ and $B_0$ parameters, though different from those determined from figure 4, were shown to be in excellent agreement with corresponding values reported for other results by Curry [43] and by Zerilli & Armstrong [25]. The value of $\beta_1$ is in good agreement with the value determined from the Leslie et al. [36] measurements. An effective strain rate of 400 s$^{-1}$ was determined to apply in the Charpy test and a value of $\alpha = 1.94$ was reported for the $v$-notch plastic constraint. The value of $T_C$ was taken to be in good agreement with the value determined from the Leslie et al. [36] measurements. A relatively low value of $\gamma = 3 \text{ J m}^{-2}$ was estimated. The higher than predicted measurement of $T_C$ for the smaller grain size microalloyed steel has been tracked to a role for carbide cracking. The importance of large grains in initiating cleavage in bimodal grain size distributions was established by Chakrabarti et al. [44]. Other Charpy impact energy results have been reported for (Brazilian) ASTM A508 class 3A dual phase steel for which the true strain at fracture was shown to follow an H–P dependence on effective grain size [45], in line with previous prediction [46].

(b) Carbidicracking

An indication of progress past Wahl’s historical editorial comment on the need for testing to avoid the use of defective material [1] is indicated by the finer scale focus over the intervening years on microstructural elements involved in determining the fracture strength of engineering materials. McMahon & Cohen [47] were pioneers in investigating the initiation of cleavage caused by cracking of carbide plates at grain boundaries. In a later publication dedicated to Orowan, Cohen and Vukcevik reported a model analysis for such carbide-initiated cleavage of ferrite in which the H–P yield stress was equated to the Griffith stress for carbide cracking, expressed in
the latter case as [48]

\[ \sigma_C^* = \left( \frac{2E \gamma_C}{\pi(1-\nu^2)t_{\text{max}}} \right)^{1/2}, \] (3.4)

where \( E \) is Young’s modulus, \( \gamma_C \) is the surface energy of the cracked carbide, \( \nu \) is Poisson’s ratio and \( t_{\text{max}} \) is the maximum carbide plate thickness. The model gave a similar grain size dependence of \( T_C \) to that given by Petch [13] except for \( T_C \) increasing with increase in carbide thickness. More recently, Roberts, Noronha, Wilkinson and Hirsch have employed an analogous relation to equation (3.4), with addition of a plastic work term, \( w_p \), to \( \gamma_C \) and specified \( \sigma_C^* \) to be determined by emission of dislocations from the cracked carbide [49]. Consideration of any grain size effect was excluded.

Petch [50] too had earlier considered an influence of carbide cracking on the predicted grain size dependence of \( T_C \). Emphasis was given to the dynamics of slip-induced cracking of grain boundary carbide that then required critical growth of cleavage through the ferrite even for relatively thick carbide plates. Substitution of \( 2\gamma_p \) for \( \gamma_C \) was made in equation (3.4). Calculations were presented for carbide modification of \( \sigma_C \) and reversal of the decreasing \( T_C \) dependence on \( \ell^{-1/2} \) for different carbide plate thicknesses in the range of 0.25 to 5 \( \mu \)m. Interesting association of larger carbide thickness and reduced sharpness of the temperature range for the transition was pointed out by Petch, thus adding credence to the modified model of predicting \( T_C \). In fact, Sandström & Bergström [37] had reported a relatively thicker 0.8 \( \mu \)m carbide thickness for the 10 \( \mu \)m grain size microalloyed steel and this has very probably contributed to the higher than predicted value of \( T_C \) shown in figure 5.

The calculations reported by Petch for carbide modification of the grain size dependence of \( \sigma_C^* \) were re-cast by Armstrong et al. [26] in terms of the reverse consideration of dependence on \( t \). The model description compared favourably with notched test results reported by Bowen & Knott [51]. At the smallest \( t \)-values, a comparison was made with Petch’s DBTT predictions for the \( \sigma_C^* \) dependence on \( \ell^{-1/2} \). The indication was that carbide control for conventional grain size material is normally restricted to carbide thicknesses of approximately 1 \( \mu \)m or larger. A statistical analysis of main crack linkage with larger carbide particles in a spheroidized steel material was presented by Strnad et al. [52]. Relatively larger \( \gamma_p \) values were required to describe the measurements, presumably because of neglect of any slip-induced stress concentration effect. Armstrong [53] has suggested that \( \sigma_C^* \) should enter into the concentrated stress term in the H–P dislocation pile-up model relation for the cleavage stress, \( \sigma_C^* \), that is expressed as

\[ \sigma_C' = \sigma_C'^0 + m \left( \frac{\pi G b}{\alpha^* \ell} \right)^{1/2} \sigma_C^*^{1/2}, \] (3.5)

where \( m \) is an orientation factor, \( b \) is dislocation Burgers vector and \( \alpha^* \approx 0.8 \) is for an average dislocation character. Equation (3.5) provides for possibility of part of the scatter observed in \( \sigma_C^* \) measurements to be accounted for by influence of local grain size and crystallographic grain orientation.

Zhang, Armstrong and Irwin investigated carbide or inclusion cracking influence on the brittleness transition for an opposite condition of the first vestiges of isolated cleavage regions (ICRs) occurring in A533B pressure vessel and related steels either at the upper shelf level of CVN specimens or from spring-loading of side-grooved compact tensile specimens [54]. In the project, large-scale structural inhomogeneity influences associated with dendritic solidification effects had been tracked on the cleavage surface of cast thick-plate material through mismatched surface elevation measurements revealed on mated fracture surfaces [55]. The ICRs were found to be initiated often at localized regions of ductile hole-joining failures [56].

Figure 6 was developed in a model description of the observations [57]. Variations in microstructure were taken to produce irregularities in the local stress–strain behaviour typified, for example, by a dislocation pile-up stress concentration at a relatively stiffer particle clump that would undergo abrupt plastic instability thus transferring rapid load increase onto an adjacent grain that was favourably oriented to produce a cleavage crack in the manner described by
Cottrell [12]. A large initiating grain size and rapid load transfer of a plastically unstable particle clump were important features of the model. The several requirements are in line with expectation of an ICR being a relatively rare occurrence in a sea of ductile fracturing. Less favourability of the same conditions in the encompassing microstructure halts the spread of any ICR. The model relates to another analysis of the Cottrell mechanism using a dislocation/crack interaction solution [58]. And the figure 6 model description may be compared also with a model description given by Ritchie et al. [59] for cleavage initiation at the DBTT.

4. Crack size in fracture mechanics

Griffith [4] tested his theory against experimental results obtained on soda lime silicate glass material. Early difficulty in directly applying the theory to metal fracturing is well-known. Recently, Wiederhorn et al. [60] reviewed the case for cracking in silicate glass materials. Emphasis was given to new nanoscale measurement capabilities that are being applied to characterizing such cracks, especially with regard to determining the nonlinear size and structure of the distorted region at the crack tip. The experimental evidence for crack tip plasticity is overwhelming and detailed mechanisms have been reported, for example, proceeding from a simple calculation of dislocation nucleation in the plane of the crack [61] to a more general description of crack-assisted dislocation multiplication linked with the pre-existing dislocation density [62]. Antolovich & Armstrong [63] have reviewed the issue.

(a) Modification by plastic zone size

A continuum mechanics basis for critical crack growth with a leading plastic zone at the crack tip was reported by Dugdale [64], Irwin [15] and Bilby et al. [16]. Dugdale provided connection with results on mild steel. Modification of the inverse square root of crack size, \( c \), dependence was
obtained from the Bilby et al. analysis in terms of a leading plastic zone size, $s$, in the form [65,66]:

$$\sigma_F = \left( \frac{8^{1/2}}{\pi} \right) \sigma_{F0} \left[ \frac{s}{c + s} \right]^{1/2}. \tag{4.1}$$

Figure 7 shows application of equation (4.1) to results reported for PMMA material. Crack-free yield and tensile fracture stresses are shown on the ordinate scale axis. Heald et al. [69] reported similar application of the same-type analysis to measurements obtained on other polymeric materials as well as on the mild steel sheet material investigated by Dugdale, and to tungsten fibre-reinforced copper composite materials.

For $(s/c) \ll 1.0$, equation (4.1) follows the Griffith inverse square root of crack size dependence as shown in figure 7. By comparison with the fracture mechanics relation, $\sigma_F = K/(\pi c)^{1/2}$, the result is obtained that $K = \sigma_{F0}(8s/\pi)^{1/2}$. A value of $K = 32.3$ MPa mm$^{1/2}$ applies in figure 7 for the measurements of Berry [67]. A close value of $K = 36.4$ MPa mm$^{1/2}$ is obtained for the Williams & Ewing [68] measurements. The equation (4.1) dependence has been employed to describe a DBTT for material with different crack sizes in comparison with a constitutive equation description of the material strain rate dependence [70]. At crack velocities greater than approximately 330 m s$^{-1}$ imposed on PMMA materials, the relatively constant fracturing energy increases because of initiation of multiple small crack branchings [71]. Recently, Carpenteri & Taplin [72] have reported a larger range in $K$-values for polypropylene material relating to test specimen width effects in determining the plane strain fracture mechanics stress intensity, $K_{1c}$.

In equation (4.1), the value of $\sigma_{F0}$ was specified as the crack-free fracture stress thus providing for incorporation of H–P grain size dependence. The same relationship was shown to be obtained for critical crack tip opening displacement [66]. For comparison with Irwin’s result [15], his plastic zone size radius, $r = (s/2)$. Conversion of equation (4.1) to plane strain for a von Mises yield stress condition leads to the relationship [53]

$$K_{1c} = \left( \frac{8}{3\pi} \right)^{1/2} \left[ \sigma_{0C} + k_C \epsilon^{-1/2} \right] s^{1/2}. \tag{4.2}$$

Experimental verification of the H–P-type prediction in equation (4.2) was demonstrated for a compilation of iron and steel measurements [65,73–77]. The large plastic zone sizes associated with the measurements were found to be relatively independent of the material grain size. The condition may be altered according to the acuity of the pre-crack size as will be discussed here, and later in connection with indentation fracture mechanics measurements. Yokobori & Konosu
[78] had investigated the effect of notch size and acuity for steel material tested at 77 K. For relatively blunt notches, a positive H–P dependence was obtained for $K$, whereas for fatigue pre-cracked material with a very large ratio of crack size to grain size, a negative H–P dependence was obtained. Armstrong explained the reversed effect as being caused by a reduction in $s$, then becoming proportional to the material grain size [66]. More recently, Zeng & Hartmaier [79] have reported a (dislocation dynamics/cohesive zone) model for evaluating the fracture toughness of tungsten material in which the fracture toughness decreased with increase in grain size. The result was attributed to obtainment of relatively small plastic zone sizes being limited by grain boundaries.

Petch & Armstrong [80] investigated the influences both of $\sigma_y$ and $k_y$ on $K_C$ measurements reported for cleavage of a number of steel materials. A role for work-hardening was established in a model description of plastic yielding being initiated at the root of a notch in a fracture mechanics test and essentially athermal work hardening ensuing until the cleavage fracture stress was reached and the test specimen failed. Such bcc-type athermal work hardening characteristic has been confirmed, for example, for tantalum material in line with application of equation (2.1) [81]. For grain size dependence, the yield stress is increased, but the fracture stress is increased more because of $k_C > k_y$, and a greater amount of work hardening is required to reach a higher stress intensity, $K_C$. An H–P dependence for $K_C$ was shown by Petch and Armstrong to apply for 0.08 carbon steel results reported by Lin et al. [82] in tests at $-80$ and $-120^\circ$C.

An opposite effect on $K_C$ occurs for increase in $\sigma_y$ at fixed grain size, because such increase does not change the cleavage fracture stress, as described for figure 3. Consideration of the relationship between work-hardening and crack tip opening displacement led to prediction of a logarithmic decrease in $K_C$ with increase in $\sigma_y$ according to the expression [79]

$$\ln K_C = \ln K'_C - \frac{\sigma_y}{2n' A'}$$

In equation (4.3), $n'$ and $A'$ are power law work-hardening coefficients analogous to $n$ and $A$ in equation (2.1) except being taken to represent the total stress. Petch and Armstrong obtained reasonable agreement with equation (4.3) over a wide range of $\sigma_y$ measurements made on various steels. Based on a ‘weakest link’ model, Tyson [83] also accounted for a dependence of $K_C$ on $\sigma_y$ with involvement of an important role for $n'$. A log/log dependence of $K_C$ on $\sigma_y$ was obtained for cleavage-initiated failure measurements reported for AISI 1020 plain carbon steel material by Couque et al. [84]. Figure 8 shows agreement of equation (4.3) with the same results. The T1, T7 and T9 designations apply for $\sigma_y$ values obtained in different thermal treatments. As noted in figure 8, Couque et al. had reported test results for both quasi-static and dynamic loading.

There is additional complication to be expected for both grain size and crack size effects in ductile fracture toughness measurements. Srinivas et al. [85] reported an H–P dependence of ductile fracture toughness measurements made for Armco iron material, beginning from a lowest $K$ determined for cleavage initiation at a largest millimetre-scale grain size and extending downwards to fully ductile hole-joining failure occurring at a grain size, $\ell = 38 \mu m$. A value of $k_f = 276 \text{MPa mm}^{1/2}$ was determined for a necking-corrected $\sigma_y$ that corresponded to reductions in area increasing from approximately 80% to approximately 90% with decreasing grain size, far greater than Petch’s approximately 65% for mild steel. An H–P dependence was determined for $K_{JC}$ on the basis of J-integral methods involving critical stretch width zone measurements and load line displacement plots. Estimation of plastic zone sizes were compared with measurements obtained in microhardness tests. Chen et al. [86] have reviewed the J-integral procedure for determination of crack growth resistance (J–R) curves.

(b) Indentation fracture mechanics

Beyond the multiscale dimensional aspects of $\ell$, $c$ and $s$, there is diversity both in the available methods of fracture mechanics testing and in the different materials tested. Frank & Lawn [87] produced a Griffith-based breakthrough description of long-standing observations made
of ‘cone-cracking’ configurations produced at hardness indentations in otherwise brittle glass materials. A dependence of load on crack length to the (3/2) power was predicted. The theory was soon extended to radial and median crack geometries associated with indentations made in essentially brittle glass and crystalline ceramic-type materials. Atkinson et al. [88] have reviewed the theory. Figure 9 shows the (3/2) dependence in relation to elastic and plastic load measurements determined for indentation tests made on silicon single crystals [89].

The indentation test method allows determination of $K_{IC}$ measurements from the (3/2) dependence. Related careful measurements were reported for polycrystalline alumina materials over similar ranges of grain size by Franco et al. [90] and by Muchtar & Lim [91]. Figure 10 shows an H–P dependence of $K_{IC}$ for the combined measurements [92]. A range of $K_{IC}$ measurements for single crystals is shown on the ordinate axis [93]. Thermal strain influences occur for larger grain sizes. The three larger filled-circle points in figure 10 were determined for Hertzian-type ring cracks formed under indenter loads of 30–50 kgf applied to sapphire spheres of 5 mm diameter [90]. Ring crack depths of 2–13 µm were measured for surface crack diameters of approximately 300–400 µm. The same trend of measurements was obtained for smaller radial cracks of approximately 10 µm length initiated at a smaller load of 0.5 kgf applied in diamond pyramid indentation tests. The more extensive smaller-filled-circle measurements shown in the same figure were obtained for radial cracking initiated by diamond pyramid indentions made at increasing loads of 0.3, 0.5, 1.0 and 2.0 kgf [91].

Very interestingly, Franco et al. [90] found at larger loads of 1.0–25 kgf in the diamond pyramid hardness tests that the grain size dependence of $K_{IC}$ was gradually reduced and then reversed at the highest applied load. Armstrong and Cazacu accounted for the reversal in terms of the plastic zone size factor, $s$, in equation (4.2) changing over to become proportional to the grain size [92]. Computed values of $s$ were determined from the limiting Griffith form of equation (4.1) using known H–P values for $\sigma_0 F$ and the $K_{IC}$ measurements. Despite the opposite trend for $s$, a positive H–P relation was obtained for $\sigma_F$. In other work, Belenky et al. [94] have reported a relatively low

![Figure 8. Yield stress dependence of cleavage-initiated fracture toughness [80,83,84].](image-url)
Figure 9. Elastic, plastic, cracking measurements: $d$ is Hertzian contact diameter; $D$ is ball diameter; $d_i$ is hardness diagonal length and $d_C$ is tip-to-tip crack length [89].

Figure 10. $K_{IC}$ measurements for single crystal and polycrystalline alumina [90–93].

The value of $K_{IC} = 3.12 \text{ MPa m}^{1/2}$ for static and dynamic nanograined alumina material tested in the form of pre-indentation-cracked bend bars.

Other application of equation (4.2) has involved both indentation fracture mechanics and conventional bend-stress-type fracture mechanics measurements reported for the industrially important WC-Co (cement) composite system [95]. WC particle size, $d$, and Co binder mean-free path length, $\lambda$, influences had been accounted for by individual $H$–$P$ relations being established from microhardness measurements made on each component [96]. The strength and ductility of
the composite material is known to be controlled by the Co phase. Armstrong and Cazacu showed that $\lambda$ could be substituted both for $\ell$ and $s$ in equation (4.2) and so give reasonable fit in figure 11 of combined experimental $K$ measurements reported in bend tests by Sigl & Fischmeister [97] and in indentation fracture mechanics tests by Jia et al. [98]. The single crystal WC measurement was taken from Richter & Ruthendorf [99]. Kotoul [100] had developed a theoretical multi-ligament plastic zone size description for unstable crack control of $K$ that was shown to follow the same $\lambda^{1/2}$ dependence [101].

5. Summary

Grain size and crack size aspects of cleavage fracturing, the ductile–brittle transition and fracture mechanics stress intensity properties of steel and several other potentially brittle materials have been reviewed. In the preceding description:

— a quantitative assessment is given of reported measurements for the H–P $k_C$ value determined for the brittle cleavage strength of a number of steel materials;
— extension of the H–P description to the plastic stress–strain behaviour of mild steel is described, with $k_{1, y, p}$ and $k_C$ being athermal and thermal dependence in $\sigma_0\varepsilon$;
— analysis has been presented for the influence of higher $k_{1, y, p}$ or $k_y$ and $\sigma_0\varepsilon$ on increase in the DBTT in tension and in CVN test results;
— a deleterious effect of cracking produced at grain boundary carbide plates is incorporated within the DBTT analysis;
— a model description is given of transition to first vestiges of cleavage in steel upon lowering of temperature from the higher regime of ductile fracturing;
— a plastic-zone-modified Griffith-type inverse square root of crack size dependence is described for the fracture mechanics stress intensity, $K$;
— grain size and $\sigma_y$ influences are incorporated within the fracture mechanics description of $K$, including an important role for the plastic zone size, $s$; and
— indentation fracture mechanics measurements of $K$ are presented for diverse silicon crystal, alumina polycrystal and WC-Co composite materials.

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