Magnetic reconnection is a process of magnetic field topology change, which is one of the most fundamental processes happening in magnetized plasmas. In most astrophysical environments, the Reynolds numbers corresponding to plasma flows are large and therefore the transition to turbulence is inevitable. This turbulence, which can be pre-existing or driven by magnetic reconnection itself, must be taken into account for any theory of magnetic reconnection that attempts to describe the process in the aforementioned environments. This necessity is obvious as three-dimensional high-resolution numerical simulations show the transition to the turbulence state of initially laminar reconnecting magnetic fields. We discuss ideas of how turbulence can modify reconnection with the focus on the Lazarian & Vishniac (Lazarian & Vishniac 1999 Astrophys. J. 517, 700–718 (doi:10.1086/307233)) reconnection model. We present numerical evidence supporting the model and demonstrate that it is closely connected to the experimentally proven concept of Richardson dispersion/diffusion as well as to more recent advances in understanding of the Lagrangian dynamics of magnetized fluids. We point out that the generalized Ohm’s law that accounts for turbulent motion predicts the subdominance of the microphysical plasma effects for reconnection for realistically turbulent media. We show that one
of the most dramatic consequences of turbulence is the violation of the generally accepted notion of magnetic flux freezing. This notion is a cornerstone of most theories dealing with magnetized plasmas, and therefore its change induces fundamental shifts in accepted paradigms, for instance, turbulent reconnection entails reconnection diffusion process that is essential for understanding star formation. We argue that at sufficiently high Reynolds numbers the process of tearing reconnection should transfer to turbulent reconnection. We discuss flare events that are predicted by turbulent reconnection and relate this process to solar flares and γ-ray bursts. With reference to experiments, we analyse solar observations in situ as measurements in the solar wind or heliospheric current sheet and show the correspondence of data with turbulent reconnection predictions. Finally, we discuss first-order Fermi acceleration of particles that is a natural consequence of the turbulent reconnection.

1. Problem of magnetic reconnection in realistically turbulent plasmas

Magnetic fields are known to critically modify the dynamics and properties of magnetized plasmas. It is generally accepted that magnetic fields embedded in a highly conductive fluid retain their topology for all time due to the magnetic fields being frozen-in [1,2]. This concept of frozen-in magnetic fields is a basis of many theories, e.g. the theory of star formation in magnetized interstellar medium (ISM).

In spite of this, there is ample evidence that magnetic fields in highly conducting ionized astrophysical objects, like stars and galactic discs, show evidence of changes in topology, i.e. ‘magnetic reconnection’, on dynamical time scales [3–5]. Historically, magnetic reconnection research was motivated by observations of the solar corona [6–8] and this influenced attempts to find peculiar conditions conducive for flux conservation violation, e.g. special magnetic field configurations or special plasma conditions. For instance, Priest & Forbes [5] showed examples of magnetic configurations that produce fast reconnection and much work has been done showing how reconnection can be accelerated in plasmas with very small collision rates [9–13] (see also reviews [14–16] and references therein). However, it is clear that reconnection is a ubiquitous process taking place in various astrophysical environments. For instance, magnetic reconnection can be inferred from the existence of large-scale dynamo activity inside stellar interiors [17,18]. We would argue that it is also required to enable the eddy-type motions in magnetohydrodynamic (MHD) turbulence, e.g. in the Goldreich & Sridhar turbulence [19]. In fact, it is easy to show that without fast magnetic reconnection, magnetized fluids would behave like Jello or felt, rather than as a fluid.

It is clear that solar flares [20] are just one vivid example of reconnection activity. Other dramatic reconnection events attributed to reconnection include γ-ray bursts (see [21] for a review), while reconnection routinely takes place essentially everywhere both in collisional and collisionless magnetized plasmas. Incidentally, magnetic reconnection occurs rapidly in computer simulations due to the high values of resistivity (or numerical resistivity) that are employed at the resolutions currently achievable. Therefore, if there are situations where magnetic fields reconnect slowly, numerical simulations do not adequately reproduce the realities of astrophysical plasmas. This means that if collisionless reconnection is the only way to make reconnection rapid, then numerical simulations of many astrophysical processes including those of the ISM, which is collisional, are in error. Fortunately, observations of collisional solar photosphere indicate that the reconnection is fast in these environments [22], which contradicts to the idea that being collisionless is the prerequisite for plasma to reconnect fast.

What makes reconnection challenging to explain is that it is not possible to claim that reconnection must always be rapid empirically, as solar flares require periods of flux accumulation time, which corresponds to slow reconnection. Thus, magnetic reconnection should have some sort of trigger, which should not depend on the parameters of the local plasma. In this review, we argue that the trigger is turbulence. This opens a wide avenue for the application of
turbulent reconnection theory to explain astrophysical explosions, e.g. solar and stellar flares and superflares, as well as γ-ray bursts.

A lot of support to models of reconnection based on plasma physics comes from the in situ measurements of magnetospheric reconnection. While important for some practical purposes, e.g. for some aspects of the Space Weather Program, this reconnection happens on scales comparable to the ion inertial length and therefore is atypical for large-scale reconnection that happens in most astrophysical systems. We argue that the large-scale magnetic reconnection is based on MHD turbulence physics making small-scale plasma reconnection processes irrelevant for the reconnection rates that are attained.

With the advent of numerical simulations, it becomes clear that the regular schemes of reconnection, like classical Sweet–Parker or Petschek reconnections, do not work. Instead, the reconnecting systems transfer to a more chaotic state which is characterized by the hierarchy of magnetic islands in two dimensions [23–28] or a more complex chaotic state in three dimensions [29]. We argue that as the scale of reconnection layers increases, the turbulent reconnection will inevitably take over, modifying and suppressing the plasmoid instability that gives rise to the currently observed picture.

Turbulence generation has long been associated with magnetic reconnection processes [30]. This review, however, is mostly dealing with how turbulence changes the rates of magnetic reconnection, although we also consider turbulence generation by reconnection.

Magnetic reconnection is a ubiquitous process in turbulent media, but it is not easy to observe as reconnection transfers most of the energy into kinetic motion related to smaller scale eddies, thus supporting the energy cascade. Apart from solar flares, the dynamics of which can be compared with the predictions of turbulent reconnection, in situ measurements of reconnection available for the solar wind provide ways of testing theoretical predictions. We show that both sets of data are consistent with turbulent MHD-based magnetic reconnection.

It is worth noting that our discussion addresses three-dimensional magnetic reconnection. The change of dimensionality of physical problems changes frequently the physics involved. For the theory based on MHD turbulence, it is very important to note that the properties of MHD turbulence are very different in two and three dimensions.

The theory of turbulent reconnection that we describe is based on the Lazarian & Vishniac work [31] (henceforth LV99) and the extensions of the original model in subsequent publications, for instance in Eyink et al. [32] (henceforth ELV11). The original LV99 model was supported by numerical simulations, some results of which have been published [32–34], as well as different pieces of observational evidences that we describe in the review. Additional theoretical support for the model comes from very recent work by Eyink [35]. While our review reflects our optimism based on the successes of the LV99 model in explaining different astrophysical phenomena, e.g. γ-ray bursts ([36] and references therein) and removal of magnetic fields from molecular clouds (e.g. [37] and references therein), we feel that the challenges presented by the variety of astrophysical conditions are very stimulating for further studies of magnetic reconnection. We also accept the limitations of our model that is intended for describing the astrophysical phenomena at large scales and therefore adopting MHD approximation. Therefore, magnetic reconnection happening at the scale of ion Larmor radius, as is the case of the Earth magnetosphere, cannot be described by the model. Important cases of magnetic reconnection in the presence of plasma effects as well as plasmoid instabilities are described in an extensive review by Yamada et al. [16]. Other cases when magnetic reconnection can be fast in MHD regime without turbulence are discussed at length, for example, in an excellent book by Priest & Forbes [38]. Thus, this review should be viewed as a personalized outlook on the reconnection problem by the authors who are exploring the connection of the two ubiquitous processes, namely, magnetic reconnection and astrophysical turbulence, while many issues of the problem are far from being finally settled and different ideas and alternative models are being tested and explored by different research groups. We accept that magnetic reconnection, similar to magnetic turbulence, is a very deep subject where the synergy of different approaches and techniques may prove to be beneficial eventually. We also note that the claim that turbulence can accelerate
magnetic reconnection predates the LV99 model [39–43]. Some new approaches to turbulent reconnection were formulated more recently [44]. In the review, we provide a comparison of LV99 with these approaches.

This review on reconnection is closely related to other reviews dealing with processes in turbulent solar wind, for example those of Matthaeus et al. [45] and Cranmer et al. [46] dealing with turbulent heating, as reconnection provides one of the channels of dissipation. The plasma effects that are subdominant for turbulent reconnection in our settings, but that may be important for other applications, are covered, for example, in the reviews of Howes [47] and Gary [48].

In what follows, we argue that turbulent reconnection is the generic process taking place in astrophysical environments which are turbulent due to the huge Reynolds numbers of the flows involved. The turbulence can be pre-existing and also self-generated by the reconnection process. We provide the MHD description of astrophysical turbulence in §2, describe the LV99 model of turbulent reconnection in §3, provide its elaboration and extension in §4, demonstrate examples of the numerical testing of the model in §5, discuss the observational testing of the model with solar data and solar wind data in §6, outline the implications of the model in §7 and provide a comparison of the model with other ideas of fast stochastic reconnection in §8. We conclude by discussing the general tendency of models of reconnection to get more stochastic in §9.

2. Astrophysical turbulence and its magnetohydrodynamic description

Observations of the ISM reveal a Kolmogorov spectrum of electron density fluctuations [49,50] as well as steeper spectral slopes of supersonic velocity fluctuations (see [51] for a review). Measurements of the solar wind fluctuations also reveal turbulence power spectrum [52]. Ubiquitous non-thermal broadening of spectral lines as well as measures obtained by other techniques [53] confirm that turbulence is present everywhere in astrophysical environments where we test for its existence. This is not surprising as magnetized astrophysical plasmas generally have very large Reynolds numbers due to the large length scales involved and the fact that the motions of charged particles in the direction perpendicular to magnetic fields are constrained. Laminar plasma flows at these high Reynolds numbers are prey to numerous linear and finite-amplitude instabilities, from which turbulent motions readily develop.¹

Indeed, observations show that turbulence is ubiquitous in all astrophysical plasmas. The spectrum of electron density fluctuations in the Milky Way is presented in figure 1, but similar examples are discussed in [52,57] for solar wind, [58] for molecular clouds and [59] for the intracluster medium. The plasma turbulence is sometimes driven by an external energy source, such as supernovae in the ISM [60,61], merger events and active galactic nuclei outflows in the intercluster medium [62–64], and baroclinic forcing behind shock waves in interstellar clouds. In other cases, the turbulence is spontaneous, with available energy released by a rich array of instabilities, such as magneto-rotational instability in accretion discs [65], kink instability of twisted flux tubes in the solar corona [66,67], etc. In all these cases, turbulence is not driven by reconnection. Nevertheless, we mention that an additional driving of turbulence through the energy release in the reconnection zone can sometimes be important, especially in magnetically dominated low β plasmas. We discuss the case of turbulence driven by reconnection in §4c. All in all, whatever its origin, the signatures of plasma turbulence are seen throughout astrophysical media.

As turbulence is known to change dramatically many processes, in particular diffusion and transport processes, it is natural to pose the question to what extent the theory of astrophysical reconnection must take into account the pre-existing turbulent environment. We note that even if the plasma flow is initially laminar, kinetic energy release by reconnection due to some plasma process, e.g. tearing and related plasmoid generation, is expected to generate vigorous turbulent motion in high Reynolds number fluids.

¹In addition, the mean free path of particles can also be constrained by the instabilities developed on the collisionless scales of plasma [54–56]. In this situation not only Alfvénic but also compressible turbulent modes can survive.
Turbulence in plasma happens at many scales, from the largest to those below the proton Larmor radius. To understand at what scales the MHD description is adequate, one needs to reiterate a few known facts \cite{32,68}. Indeed, to describe magnetized plasma dynamics one should deal with three characteristic length scales: the ion gyroradius $\rho_i$, the ion mean free path length $\ell_{\text{mfp},i}$ arising from Coulomb collisions, and the scale $L$ of large-scale variation of magnetic and velocity fields.

The MHD approximation is definitely applicable to ‘strongly collisional’ plasma with $\ell_{\text{mfp},i} \ll \rho_i$. This is the case, for example, of star interiors and most accretion disc systems. For such ‘strongly collisional’ plasmas a standard Chapman–Enskog expansion provides a fluid description of the plasma \cite{69}, with a two-fluid model for scales between $\ell_{\text{mfp},i}$ and the ion skin-depth $\delta_i = \rho_i/\sqrt{\beta_i}$ and an MHD description at scales much larger than $\delta_i$.

Hot and rarefied astrophysical plasmas are often ‘weakly collisional’ with $\ell_{\text{mfp},i} \gg \rho_i$. Indeed, the relation that follows from the standard formula for the Coulomb collision frequency (e.g. \cite{70}) is

$$\frac{\ell_{\text{mfp},i}}{\rho_i} \propto \frac{\Lambda}{\ln \Lambda} \frac{V_A}{c},$$

where $\Lambda = 4\pi n\lambda_D^2$ is the plasma parameter, or the number of particles within the Debye screening sphere. For some media that $\Lambda$ can be large.

\textbf{Figure 1.} Big power law in the sky from [49] extended to scale of parsecs using WHAM data (adapted from [50]).
For the ‘weakly collisional’ but well magnetized plasmas one can invoke the expansion over the small ion gyroradius. This results in the ‘kinetic MHD equations’ for lengths much larger than $\rho_i$. The difference between these equations and the MHD ones is that the pressure tensor in the momentum equation is anisotropic, with the two components $p_\parallel$ and $p_\perp$ of the pressure parallel and perpendicular to the local magnetic field direction [68]. In ‘weakly collisional’, i.e. $L \gg \ell_{mfp,i}$, and collisionless, i.e. $\ell_{mfp,i} \gg L$, systems turbulence is bound to induce instabilities that limit the effective mean free path $[\ell_{mfp, i}]_{\text{eff}}$ by magnetically mediated scattering of particles [55,71]. This effective mean free path is a game changer and it is not surprising that numerical simulations in [72] that accounted for this effect demonstrated that turbulence in ‘collisionless plasmas’ of galaxy clusters is very similar to MHD turbulence on the scales larger than $[\ell_{mfp, i}]_{\text{eff}}$.

We can also note that additional simplifications that justify the MHD approach occur if the turbulent fluctuations are small compared with the mean magnetic field, and having length scales parallel to the mean field much larger than perpendicular length scales. Treating wave frequencies that are low compared with the ion cyclotron frequency, we enter the domain of ‘gyrokinetic approximation’ which is commonly used in fusion plasmas, e.g. [54,73], for which at length scales larger than the ion gyroradius $\rho_i$ the incompressible shear-Alfvén wave modes get decoupled from the compressive modes and can be described by the simple ‘reduced MHD’ equations. These Alfvén modes are most important for fast magnetic reconnection, which we discuss later in the review.

In short, our considerations above confirm the generally accepted notion that the MHD approximation is adequate for most astrophysical turbulent plasmas at sufficiently large scales. In particular, the Goldreich–Snirndhar [19] (henceforth GS95) theory of Alfvénic turbulence should be true for describing Alfvénic part of the MHD turbulent cascade. For Alfvénic turbulence the eddies are elongated along magnetic field with the relationship between the parallel and perpendicular dimensions due to the critical balance condition, namely,

$$\ell^{-1}_\parallel V_A \sim \ell^{-1}_\perp \delta u_\ell,$$

where $\delta u_\ell$ is the eddy velocity, while $\ell_\parallel$ and $\ell_\perp$ are eddy scales parallel and perpendicular to the local direction of magnetic field, respectively. The notion of local magnetic field is the essential part of the modern understanding of Alfvénic turbulence and it was added to the GS95 picture by the later studies (LV99, [78,79]). The use of local magnetic field is expected as at small scale eddies can be influenced only by the magnetic field around them and not by the global mean field.

A description of MHD turbulence that incorporates both weak and strong regimes was presented in LV99. In the range of length scales where turbulence is strong, this theory implies that

$$\ell_\parallel \approx L_i \left( \frac{\ell_\perp}{L_i} \right)^{2/3} M_A^{-4/3}$$

and

$$\delta u_\ell \approx u_L \left( \frac{\ell_\perp}{L_i} \right)^{1/3} M_A^{1/3},$$

when the turbulence is driven isotropically on a scale $L_i$ with an amplitude $u_L$. As we see further, driving of turbulence by reconnection may be different from the isotropic driving assumed for the derivation of the expressions above.

We do not discuss theories of Alfvénic turbulence that were developed to obtain the spectral index of $-3/2$ which was suggested by limited-resolution numerical simulations (e.g. [79]).

---

2We will concentrate on Alfvénic modes, while disregarding the slow and fast magnetosonic modes of MHD turbulence [74–76], which is possible as the backreaction of fast and slow modes on Alfvénic cascade is insignificant [19,74,77].

3Low-resolution numerical simulations are notorious in being ambiguous in terms of spectral slope. For instance, the initial compressible simulations suggested the spectral index of high Mach number hydrodynamic turbulence to be $-5/3$, which prompted theoretical attempts to explain this (e.g. [80]). However, further high-resolution research [81] revealed that the flattening of the spectrum observed was the result of a bottleneck effect, which is more extended in compressible than...
The additional physics that was considered included, e.g. dynamical alignment [82], polarization intermittency [83], turbulence non-locality [84]. In particular, Boldyrev’s [82] study predicts the Kraichnan index of \(-\frac{3}{2}\) [85,86] rather than Kolmogorov index \(-\frac{5}{3}\) that follows from GS95. We feel that more recent high-resolution numerical simulations [87,88] provide results in agreement with the GS95 expectations, while the more shallow spectra are likely to be due to the bottleneck effect arising from MHD turbulence being less local compared with hydrodynamic one [89].

The measurements in the solar wind show evidence for the \(-\frac{3}{2}\) spectrum at 1 AU from the Sun and \(-\frac{5}{3}\) spectrum at distances larger than 1 AU [90]. We believe that more relevant to MHD turbulence is the spectrum measured at larger distances where there is less influence from the imbalance as well as the transient processes of spectrum evolution. While the discussion of the exact scaling of MHD turbulence is ongoing (see papers and comments by Perez et al. [91,92] and answers to them in [87,88]), we stress that our results on reconnection marginally depend on the exact spectral index of turbulence. In LV99, which was developed when GS95 theory was far from being accepted, in the appendix the reconnection rates were provided for arbitrary spectral indexes of turbulence and scale-dependent anisotropies.

More discussions of astrophysical turbulence can be found in recent reviews (e.g. [93–95]. In particular, there are many additional effects discussed, e.g. compressibility, effect of partial ionization as well as the effect of imbalance of turbulence. The latter may be a consequence of having sources and sinks of turbulent energy that are not coincident in space. All these effects are not of principal importance for our discussion of turbulent reconnection and therefore we do not discuss them here.

Finally, we point out that we concentrate our attention on sub-Alfvénic turbulence as the reconnection of weakly perturbed magnetic fields is the natural generalization of the classical formulation of the reconnection problem. The opposite extreme is the turbulence in the dynamically unimportant magnetic field, where the magnetic fields are reversing at the resistive dissipative scale. This is a degenerate example employed in the model of kinetic dynamo and it is of no interest for the reconnection research. If turbulence is super-Alfvénic, magnetic field becomes dynamically important and stiff at the scale of \(L_i M_A^{-\frac{3}{2}}\) [96] and the reconnection ideas below can be applied to such fields.

The most important points of this section are

- astrophysical fluids are generically turbulent,
- MHD description of Alfvénic turbulence is valid at sufficiently large scales, and
- we have an adequate theory of Alfvénic turbulence.

In what follows we refer to these points dealing with the problem of turbulent reconnection.

### 3. Turbulent reconnection model

The model of turbulent reconnection in LV99 generalizes the classical Sweet–Parker model [97,98].\(^4\) In the latter model, two regions with uniform laminar magnetic fields are separated by a thin current sheet. The speed of reconnection is given roughly by the resistivity divided by the sheet thickness, i.e.

\[
V_{rec} \approx \frac{\eta}{\Delta}.
\]  

in incompressible fluids. In the MHD simulations that are indicative of \(-\frac{3}{2}\) spectrum, similar to the aforementioned low-resolution hydrodynamic simulations showing \(-\frac{5}{3}\), no bottleneck effect is seen. As the bottleneck is a physical effect, the fact that it is not seen in simulations to our mind means that it is just extended and higher resolution simulations are necessary. Therefore, choosing between theories on the basis of just spectral slope of low-resolution simulations may be tricky.

\(^4\)The basic idea of the model was first discussed by Sweet and the corresponding paper by Parker refers to the model as ‘Sweet model’.
Figure 2. (a) Sweet–Parker model of reconnection. The outflow is limited to a thin width $\delta$, which is determined by Ohmic diffusivity. The other scale is an astrophysical scale $L_x \gg \delta$. Magnetic field lines are laminar. (b) Turbulent reconnection model that accounts for the stochasticity of magnetic field lines. The stochasticity introduced by turbulence is weak and the direction of the mean field is clearly defined. The outflow is limited by macroscopic field line wandering. (c) An individual small-scale reconnection region [99].

For steady-state reconnection, the plasma in the current sheet must be ejected from the edge of the current sheet at the Alfvén speed, $V_A$. Thus, the reconnection speed is

$$V_{\text{rec}2} \approx \frac{V_A \Delta}{L_x},$$

(3.2)

where $L_x$ is the length of the current sheet, which requires $\Delta$ to be large for a large reconnection speed. As a result, the overall reconnection speed is reduced from the Alfvén speed by the square root of the Lundquist number, $S \equiv L_x V_A / \eta$, i.e.

$$V_{\text{rec,SP}} = V_A S^{-1/2}.$$  

(3.3)

The corresponding Sweet–Parker reconnection speed is negligible in astrophysical conditions as $S$ may be $10^{16}$ or larger.

The corresponding model of magnetic reconnection is illustrated in figure 2.

Similar to the Sweet–Parker model, the LV99 model deals with a generic configuration, which should arise naturally as magnetic flux tubes try to make their way one through another. However, in the LV99 model the large-scale magnetic field wandering determines the thickness of outflow. Thus, the LV99 model does not depend on resistivity and can provide both fast and slow reconnection rates depending on the level of turbulence.

To obtain the reconnection rate in the LV99 model, one should use the scaling relations for Alfvénic turbulence from §2. A bundle of field lines confined within a region of width $y$ at some particular point spreads out perpendicular to the mean magnetic field direction as one moves in either direction following the local magnetic field lines. The rate of field line diffusion
is given by
\[
\frac{d\langle y^2 \rangle}{dx} \sim \frac{\langle y^2 \rangle}{\lambda},
\] (3.4)
where $\lambda \approx \ell_\parallel^{-1}$, $\ell_\parallel$ is the parallel scale and the corresponding transversal scale, $\ell_\perp$, is $\sim \langle y^2 \rangle^{1/2}$, and $x$ is the distance along an axis parallel to the magnetic field. Therefore, using equation (2.3) one gets
\[
\frac{d\langle y^2 \rangle}{dx} \sim L_i \left( \frac{\langle y^2 \rangle}{L_i^2} \right)^{2/3} \left( \frac{u_L}{V_A} \right)^{4/3},
\] (3.5)
where we have substituted $\langle y^2 \rangle^{1/2}$ for $\ell_\perp$. This expression for the diffusion coefficient will only apply when $y$ is small enough for us to use the strong turbulence scaling relations, or in other words when $\langle y^2 \rangle < L_i^2 (u_L/V_A)^4$.

When the turbulence injection scale is less than the extent of the reconnection layer, i.e. $L_x \gg L_i$, magnetic field wandering obeys the usual random walk scaling with $L_x/L_i$ steps and the mean-squared displacement per step equal to $L_i^2 (u_L/V_A)^4$. Therefore,
\[
\langle y^2 \rangle^{1/2} \approx (L_i x)^{1/2} \left( \frac{u_L}{V_A} \right)^2 x > L_i.
\] (3.6)

Combining equations (3.5) and (3.6), one can derive the thickness of the outflow $\Delta$ and obtain (LV99)
\[
V_{\text{rec}} \approx V_A \min \left[ \left( \frac{L_x}{L_i} \right)^{1/2}, \left( \frac{L_i}{L_x} \right)^{1/2} \right] M_A^2,
\] (3.7)
where $V_A M_A^2$ is proportional to the turbulent eddy speed. This reconnection rate represents a large fraction of the Alfvén speed when $L_i$ and $L_x$ are not too different and $M_A$ is not too small.

Owing to the importance of the turbulent reconnection, it is advantageous to consider re-deriving the reconnection rates in another way. This was is based on the Lagrangian properties of magnetized plasma, in particular on the Richardson dispersion (see [100] and references therein).

Richardson diffusion/ dispersion can be illustrated with a simple hydrodynamic model. Consider the growth of the separation between two particles $d(l(t))/dt \sim u(l)$, which for Kolmogorov turbulence is $\sim \alpha t^{1/3}$, where $\alpha$ is proportional to the energy cascading rate, i.e. $\alpha \approx V_L^3/L$ for turbulence injected with superAlfvénic velocity $V_L$ at the scale $L$. The solution of this equation is
\[
l(t) = \left[ t_0^{1/3} + \alpha(t - t_0) \right]^{3/2},
\] (3.8)
which at late times leads to Richardson diffusion/ dispersion or $l^2 \sim t^3$ compared with $l^2 \sim t$ for ordinary diffusion. Both terms ‘diffusion’ and ‘dispersion’ can be used interchangeably, but keeping in mind that the Richardson process results in superdiffusion (see [101] and references therein) we feel that it is advantageous to use the term ‘dispersion’. Although the Richardson dispersion process was introduced for hydrodynamic turbulence, a similar process is valid for magnetized fluids. We will not distinguish the magnetized and not magnetized case by name and instead of magnetic Richardson dispersion will use just Richardson dispersion. In magnetized turbulence, Richardson dispersion is important in terms of spreading magnetic fields which provides a way to re-derive the LV99 relations.

The fact that time dependence of the magnetic field diffusion induces magnetic reconnection can be illustrated with the Sweet–Parker reconnection, where magnetic field lines are subject to Ohmic diffusion. The latter induces the mean-squared distance across the reconnection layer that a magnetic field line can diffuse by resistivity in a time $t$ given by
\[
\langle y^2(t) \rangle \sim \lambda t,
\] (3.9)
where $\lambda = c^2/4\pi \sigma$ is the magnetic diffusivity. The field lines are advected out of the sides of the reconnection layer of length $L_x$ at a velocity of order $V_A$. Therefore, the time that the lines can
spend in the resistive layer is the Alfvén crossing time $t_A = L_x/V_A$. Thus, field lines that can reconnect are separated by a distance

$$
\Delta = \sqrt{\langle y^2(t_A) \rangle} \sim \sqrt{\lambda t_A} = \frac{L_x}{\sqrt{S}},
$$

(3.10)

where $S$ is Lundquist number. Combining equations (3.2) and (3.10), one gets again the well-known Sweet–Parker result, $v_{\text{rec}} = V_A/\sqrt{S}$.

The difference with the turbulent case is that instead of Ohmic diffusion one should use the Richardson one [32]. In this case, the mean-squared separation of particles $\langle |x_1(t) - x_2(t)|^2 \rangle \approx \epsilon t^3$, where $t$ is time, $\epsilon$ is the energy cascading rate and $\langle \ldots \rangle$ denote an ensemble averaging [102]. For sub-Alfvénic turbulence $\epsilon \approx u_\perp^4/\left(V_A L_i\right)$ (see LV99) and therefore analogously to equation (3.10), one can write

$$
\Delta \approx \sqrt{\epsilon t^3} \approx L \left(\frac{L}{L_i}\right)^{1/2} M_A^2,
$$

(3.11)

where it is assumed that $L < L_i$. Combining equations (3.2) and (3.11), one obtains

$$
v_{\text{rec,LV99}} \approx V_A \left(\frac{L}{L_i}\right)^{1/2} M_A^2
$$

(3.12)

in the limit of $L < L_i$. Similar considerations allow one to recover the LV99 expression for $L > L_i$, which differs from equation (3.12) by the change of the power $\frac{1}{2}$ to $-\frac{1}{2}$ and recover equation (3.7).

4. Extending LV99 reconnection theory

(a) Recent theoretical developments: rigorous mathematical approach

Recently, the LV99 notion of magnetic line wandering has played a central role in the extension of ‘general magnetic reconnection’ (GMR) theory to turbulent plasmas. Recall that GMR theory [103,104] attempts to quantify the changes of magnetic connections between plasma elements. It is assumed in the standard approach to GMR that such changes occur only in narrow, sparsely distributed current layers or ‘diffusion regions’ of small total volume. This assumption is invalid for turbulent plasmas. By tracking along field lines wandering in space, Eyink [35] has developed an extended version of GMR theory valid for both laminar and turbulent plasmas.

The study by Eyink [35] provides a rigorous mathematical treatment of the motion of magnetic field lines in turbulent plasmas. The slip source vector which is defined as the ratio of the curl of the non-ideal electric field in the generalized Ohm’s law and the magnetic field strength was introduced, and it was demonstrated that this vector gives the rate of development of slip velocity per unit arc length of field line. It diverges at magnetic nulls, unifying GMR with magnetic null-point reconnection. In a turbulent inertial range, the curl becomes extremely large while the parallel component is tiny, so that line slippage occurs even while ideal MHD is accurate. This means that ideal MHD is valid for a turbulent inertial range only in a weak sense which does not imply magnetic line freezing (see also §7). By rigorous estimates of the terms in the generalized Ohm’s law for an electron–ion plasma the paper shows that all of the non-ideal terms (from collisional resistivity, Hall field, electron pressure anisotropy and electron inertia) are irrelevant compared with the effects of turbulence and large-scale reconnection is thus governed solely by ideal dynamics. It is encouraging that in terms of magnetic reconnection the results of this study correspond to the LV99 model and thus provide more rigorous theoretical foundations for turbulent reconnection. The results for the slippage velocity by Eyink [35] are identical to the expression of the reconnection velocities in LV99. Together with the earlier discussed results on Richardson dispersion in magnetic turbulence, these provide new outlook on the nature of magnetic reconnection in turbulent fluids.
(b) Effect of energy dissipation in the reconnection layer

In LV99, expressions were derived assuming that only a small fraction of the energy stored in the magnetic field is lost during large-scale reconnection and the magnetic energy is instead converted nearly losslessly to kinetic energy of the outflow. This can only be true, however, when the Alfvénic Mach number $\mathcal{M}_A = u_L/V_A$ is small enough. If $\mathcal{M}_A$ becomes large, then energy dissipation in the reconnection layer becomes non-negligible and there is a reduction of the outflow velocity (see ELV11). Note that even if $\mathcal{M}_A$ is initially small, reconnection may drive stronger turbulence (see §4c) and increase the fluctuation velocities $u_i$ in the reconnection layer. This scenario may be relevant to post-coronal mass ejection (CME) reconnection, for example, where there is empirical evidence that the energy required to heat the plasma in the reconnection layer (‘current sheet’) to the observed high temperatures is from energy cascade due to turbulence generated by the reconnection itself [105]. In addition, $V_A$ within the reconnection layer will be smaller than the upstream values, because of annihilation of the anti-parallel components, which will further increase the Alfvénic Mach number.

The effect of turbulent dissipation can be estimated from steady-state energy balance in the reconnection layer

$$\frac{1}{2} v_{\text{out}}^3 = \frac{1}{2} V_A^2 v_{\text{rec}} L_x - \varepsilon L_x \Delta, \quad (4.1)$$

where kinetic energy carried away in the outflow is balanced against magnetic energy transported into the layer minus the energy dissipated by turbulence. Here we estimate the turbulent dissipation using the formula $\varepsilon = u_i^4 / V_AL_i$ for sub-Alfvénic turbulence [86]. Dividing (4.1) by $\Delta = L_x v_{\text{rec}} / v_{\text{out}}$, we get

$$v_{\text{out}}^3 = V_A^2 v_{\text{out}} - 2 \frac{u_i^4}{V_A L_i} L_x, \quad (4.2)$$

which is a cubic polynomial for $v_{\text{out}}$. The solutions are easiest to obtain by introducing the ratios $f = v_{\text{out}} / V_A$ and $r = 2\mathcal{M}_A^4 (L_x / L_i)$ which measure, respectively, the outflow speed as a fraction of $V_A$ and the energy dissipated by turbulence in units of the available magnetic energy, giving

$$r = f - f^3. \quad (4.3)$$

When $r = 0$, the only solution of (4.3) with $f > 0$ is $f = 1$, recovering the LV99 estimate $v_{\text{out}} = V_A$ for $\mathcal{M}_A \ll 1$. For somewhat larger values of $r$, $f \approx 1 - (r/2)$, in agreement with the formula $f = (1 - r)^{1/2}$ that follows from eqn (65) in ELV11, implying a slight decrease in $v_{\text{out}}$ compared with $V_A$. Note that formula (4.3) cannot be used to determine $f$ for too large $r$, because it has then no positive, real solutions! This is easiest to see by considering the graph of $r$ versus $f$. The largest value of $r$ for which a positive, real $f$ exists is $r_{\text{max}} = 2 / \sqrt{27} \approx 0.385$ and then $f$ takes on its minimum value $f_{\text{min}} = 1 / \sqrt{3} \approx 0.577$. This implies that the LV99 approach is limited to $\mathcal{M}_A$ sufficiently small, because of the energy dissipation inside the reconnection layer and the consequent reduction of the outflow velocity. This is not a very stringent limitation, however, because $r$ is proportional to $\mathcal{M}_A^4$. If one assumes $L_x \simeq L_i$, one may consider values of $\mathcal{M}_A$ up to 0.662. Given the neglect of constants of order unity in the above estimate, we may say only that the LV99 approach is limited to $\mathcal{M}_A \lesssim 1$. At the extreme limit of applicability of LV99, $v_{\text{out}}$ is still a sizable fraction of $V_A$, i.e. 0.577, not a drastically smaller value.

The effect of the reduced outflow velocity may be, somewhat paradoxically, to *increase* the reconnection rate. The reason is that field lines now spend a time $L_x / v_{\text{out}}$ exiting from the reconnection layer, greater than assumed in LV99 by a factor of $1 / f$. This implies a thicker reconnection layer $\Delta$ due to the longer time interval of Richardson dispersion in the layer, greater than LV99 by a factor of $(1 / f)^{3/2}$. The net reconnection speed $v_{\text{rec}} = v_{\text{out}} \Delta / L_x$ is thus larger by a factor of $(1 / f)^{1/2}$. The increased width $\Delta$ more than offsets the reduced outflow velocity $v_{\text{out}}$. However, this effect can give only a very slight increase, at most by a factor of $3^{1/4} \approx 1.31$ for $f_{\text{min}} = 1 / \sqrt{3}$. We see that for the entire regime $\mathcal{M}_A \lesssim 1$, where LV99 theory is applicable, energy dissipation in the reconnection layer implies only very modest corrections. It is worth emphasizing that ‘large-scale reconnection’ in super-Alfvénic turbulence with $\mathcal{M}_A > 1$ is a very
different phenomenon, because magnetic fields are then so weak that they are easily bent and twisted by the turbulence. Any large-scale flux tubes initially present will be diffused by the turbulence through a process much different from that considered by LV99. For a discussion of this regime, see [106].

(c) Reconnection in partially ionized gas

On sufficiently small scales, Alfvénic turbulence in partially ionized gas differs from our description provided in §2. Owing to viscosity caused by neutral atoms, the fluid viscosity becomes substantially larger than the fluid resistivity, which means that the Prandtl number of the fluid is high. Turbulence in high Prandtl number fluids has been studied numerically [71,107,108] and theoretically [99]. However, for our present discussion, it is important that for scales larger than the viscous damping scale the turbulence follows the usual GS95 scaling and the considerations about Richardson dispersion and magnetic reconnection that accompany are valid at these scales.

In high Prandtl number media, the GS95-type turbulent motions decay at the scale $l_{\perp, \text{crit}}$, which is much larger than the scale of Ohmic dissipation. Thus, over a range of scales less than $l_{\perp, \text{crit}}$ to some much smaller scale, magnetic field lines preserve their identity. To establish the range of scales at which magnetic fields perform Richardson diffusion, one can observe that the transition to the Richardson dispersion is expected to happen when the field line separation reaches the perpendicular scale of the critically damped eddies $l_{\perp, \text{crit}}$. The separation in the perpendicular direction starts with the scale $r_{\text{init}}$ following the Lyapunov exponential growth with the distance $l$ measured along the magnetic field lines, i.e. $r_{\text{init}} \exp(l/l_{\parallel, \text{crit}})$, where $l_{\parallel, \text{crit}}$ corresponds to critically damped eddies with $l_{\perp, \text{crit}}$. It seems natural to associate $r_{\text{init}}$ with the separation of the field lines arising from the action of Ohmic resistivity on the scale of the critically damped eddies

$$ r_{\text{init}}^2 = \frac{\eta l_{\perp, \text{crit}}}{V_A}, \quad (4.4) $$

where $\eta$ is the Ohmic resistivity coefficient.

At scales smaller than $l_{\perp, \text{crit}}$, the magnetic line separation obeys the laws established by Rechester & Rosenbluth [109]. The distance along the local magnetic field over which anisotropic turbulence separates the magnetic field lines by $l_{\perp, \text{crit}}$ is the Rechester–Rosenbluth length [96]

$$ L_{RR} \approx l_{\parallel, \text{crit}} \ln \left( \frac{l_{\perp, \text{crit}}}{r_{\text{init}}} \right). \quad (4.5) $$

Taking into account equation (4.4) and that

$$ l_{\perp, \text{crit}}^2 = \frac{\nu l_{\parallel, \text{crit}}}{V_A}, \quad (4.6) $$

where $\nu$ is the viscosity coefficient, one can rewrite equation (4.5) as

$$ L_{RR} \approx l_{\parallel, \text{crit}} \ln Pt, \quad (4.7) $$

where $Pt = \nu/\eta$ is the Prandtl number. This means that when the current sheets are much longer than $L_{RR}$, then magnetic field lines undergo Richardson dispersion and according to Eyink et al. [32] the reconnection follows the laws established in LV99. At the same time, on scales less than $L_{RR}$ magnetic reconnection may be slow.5

(d) Self-sustained turbulent magnetic reconnection

Reconnection releases energy and induces outflows. Even if the initial magnetic field configuration is laminar, magnetic reconnection ought to induce turbulence due to the outflow

5Incidentally, this can explain the formation of density fluctuations on scales of thousands of AU, that are observed in the ISM.
In terms of MHD simulations, Beresnyak [112] was the first to study turbulent reconnection with turbulence arising from the reconnection itself. However, the periodic boundary conditions adopted in [112] limited the time span over which magnetic reconnection can be studied and therefore the simulations focus on the process of establishing reconnection.

Analytical description of the results in the framework of the LV99 model was adopted by Beresnyak [112] (A. Beresnyak 2013, personal communication). Below we provide our theoretical account of the results in [112] using our understanding of LV99 turbulent reconnection. We obtain expressions which are different from those of Beresnyak [112].

The logic of the derivation below is straightforward. As the opposite magnetic fluxes enter in contact, the width of the reconnection layer $\Delta$ grows. The rate at which this happens is limited by the mixing rate induced by the eddies at the scale $k_1$, i.e.

$$
\frac{1}{\Delta} \frac{d\Delta}{dt} \approx g \frac{V_\Delta}{\Delta} \tag{4.8}
$$

with a factor $g$ which takes into account possible inefficiency in the diffusion process. $V_\Delta$ is a part of the turbulent cascade, i.e. the mean value of $V_\Delta^2 \approx \int \Phi(k_1) dk_1$, where

$$
\Phi = C_K \epsilon^{2/3} k_1^{-5/3} \tag{4.9}
$$

and $C_K$ is a Kolmogorov constant, which for ordinary MHD turbulence is calculated in [114], but in our special case may be different. If the energy dissipation rate $\epsilon$ were time-independent, then the layer width would be implied by equations (4.8) and (4.9) to grow according to Richardson’s law $\Delta^2 \sim \epsilon t^3$. However, in the transient regime considered, energy dissipation rate is evolving. If the $y$-component of the magnetic field is reconnecting and the cascade is strong, then the mean value of the dissipation rate $\epsilon$ is

$$
\epsilon \approx \frac{\beta V_A^2}{(\Delta/V_\Delta)}, \tag{4.10}
$$

where $\beta$ is another coefficient measuring the efficiency of conversion of mean magnetic energy into turbulent fluctuations. This coefficient can be obtained from numerical simulations.

The ability of the cascade to be strong from the very beginning follows from the large perturbations of the magnetic fields by magnetic reconnection, while magnetic energy can still dominate the kinetic energy. The latter factor that can be experimentally measured is given by a parameter $r_A$. With this factor and making use of equations (4.9) and (4.10), the expression for $V_\Delta$ can be rewritten in the following way:

$$
V_\Delta \approx C_K r_A (V_A^2 V_A^2 \beta)^{2/3}, \tag{4.11}
$$

where the dependencies on $k_1 \sim 1/\Delta$ cancel out.

This provides the expression for the turbulent velocity at the injection scale $V_\Delta$

$$
V_\Delta \approx (C_K r_A)^{3/4} V_A \beta^{1/2} \tag{4.12}
$$

as a function of the experimentally measurable parameters of the system. Thus, the growth of the turbulent reconnection zone is according to equation (4.8)

$$
\frac{d\Delta}{dt} \approx g \beta^{1/2} (C_K r_A)^{3/4} V_A, \tag{4.13}
$$

which predicts the nearly constant growth of the outflow region as seen in fig. 3 of [112].

For the steady-state regime, one expects the outflow to play an important role. The equations for the reconnection rate were obtained in LV99 for the isotropic injection of energy. For the case
of anisotropic energy injection of turbulence, we should apply the following approach. Using equation (6.2) and identifying $V_\Delta$ with the total velocity dispersion, which is similar to the use of $U_{\text{obs,turb}}$ in equation (6.1), one can get

$$V_{\text{rec}} \approx V_\Delta \left( \frac{\Delta}{L_x} \right)^{1/2}, \quad (4.14)$$

where the mass conservation condition provides the relation $V_{\text{rec}} L_x \approx V_A \Delta$. Using the latter condition, one gets

$$V_{\text{rec}} \approx V_A \left( C K r_A \right)^{3/2} \beta, \quad (4.15)$$

which is somewhat slower than the rate at which the reconnection layer was growing initially.

### (e) Flares of turbulent reconnection

On the basis of LV99 theory, a simple quantitative model of flares was presented in [110]. There it is assumed that since stochastic reconnection is expected to proceed unevenly, with large variations in the thickness of the current sheet, one can expect that some unknown fraction of this energy will be deposited inhomogeneously, generating waves and adding energy to the local turbulent cascade.

For the sake of simplicity, the plasma density is assumed to be uniform so that the Alfvén speed and the magnetic field strength are interchangeable. The nonlinear dissipation rate for waves is

$$\tau_{\text{nonlinear}}^{-1} \sim \max \left[ \frac{k^2 v^2_{\text{wave}}}{k_i V_A}, k^2_\perp V L \right], \quad (4.16)$$

where the first rate is the self-interaction rate for the waves and the second is the dissipation rate induced by the ambient turbulence [115]. The important point here is that $k_\perp$ for the waves falls somewhere in the inertial range of the strong turbulence. Eddies at that wavenumber will disrupt the waves in one eddy turnover time, which is necessarily less than $L/V_A$. Therefore, the bulk of the wave energy will go into the turbulent cascade before escaping from the reconnection zone.

An additional simplification is achieved by assuming that some fraction $\epsilon$ of the energy liberated by stochastic reconnection is fed into the local turbulent cascade. The evolution of the turbulent energy density per area is

$$\frac{d}{d\tau} \left( \frac{\Delta V^2}{V^2} \right) = \epsilon V_A^2 V_{\text{rec}} - V^2 \Delta \frac{V_A}{L_x}, \quad (4.17)$$

where the loss term covers both the local dissipation of turbulent energy, and its advection out of the reconnection zone. Since $V_{\text{rec}} \sim v_{\text{turb}}$ and $\Delta \sim L_x(V/V_A)$, it is possible to rewrite this by defining $\tau \equiv tV_A/L_x$ so that

$$\frac{d}{d\tau} M_A^3 \approx \epsilon M_A - M_A^3. \quad (4.18)$$

If $\epsilon$ is a constant then

$$V \approx V_A \epsilon^{1/2} \left( 1 - e^{-2\tau/3} \right)^{1/2}. \quad (4.19)$$

This implies that the time during which reconnection rate rises to $\epsilon^{1/2} V_A$ is comparable to the ejection time from the reconnection region ($\sim L_x/V_A$).

Within this toy model $\epsilon$ is not defined. Its value can be constrained through observations. Given that reconnection events in the solar corona seem to be episodic, with longer periods of quiescence, this is suggestive that $\epsilon$ is very small, for example, depending strongly on the ratio of the thickness of the current sheet to $L_x$. In particular, if it scales as $M_A$ to some power greater than two then initial conditions dominate the early time evolution.

Another route by which magnetic reconnection might be self-sustaining via turbulence injection would be in the context of a series of topological knots in the magnetic field, each of which is undergoing reconnection. For simplicity, one can assume that as each knot undergoes
reconnection, it releases a characteristic energy into a volume which has the same linear
dimension as the distance to the next knot. The density of the energy input into this volume
is roughly \( \epsilon V^2 A V / L_x \), where here \( \epsilon \) is defined as the efficiency with which the magnetic energy
is transformed into turbulent energy. Thus, one gets

\[
\epsilon V^2 A \approx \frac{v^3}{L_k},
\]

where \( L_k \) is the distance between knots and \( v' \) is the turbulent velocity created by the reconnection
of the first knot. This process will proceed explosively if \( v' > V \) or

\[
V^2 L_k \epsilon > V^2 L_x.
\]

The condition above is easy to fulfil. The bulk motions created by reconnection can generate
turbulence as they interact with their surroundings, so \( \epsilon \) should be of order unity. Moreover,
the length of any current sheet should be at most comparable to the distance to the nearest
distinct magnetic knot. The implication is that each magnetic reconnection event will set
off its neighbours, boosting their reconnection rates from \( V_L \), set by the environment, to
\( \epsilon^{1/2} V A (L_k / L_x)^{1/2} \) (as long as this is less than \( V_A \)). The process will take a time comparable to the
crossing time \( L_x / V_L \) to begin, but once initiated will propagate through the medium with a
speed comparable to speed of reconnection in the individual knots. The net effect can be a kind
of modified sandpile model for magnetic reconnection in the solar corona and chromosphere. As
the density of knots increases, and the energy available through magnetic reconnection increases,
the chance of a successfully propagating reconnection front will increase.

(f) Relativistic reconnection

Magnetic turbulence in a number of astrophysical highly magnetized objects, accretion discs near
black holes, pulsars, \( \gamma \)-ray bursts may be in the relativistic regime when the Alfvén velocity
approaches that of light. The equations that govern magnetized fluid in this case look very
different from the ordinary MHD equations. However, studies by Cho and co-workers [116,117]
show that for both balanced and imbalanced turbulence, the turbulence spectrum and turbulence
anisotropies are quite similar in this regime and the non-relativistic one. This suggests that the
Richardson dispersion and related processes of LV99-type magnetic reconnection should carry on
to the relativistic case [118]. This prediction was confirmed by the recent numerical simulations
of M. Takomoto (2014, personal communication), who with his relativistic code adopted the
approach in [33] and showed that the rate of three-dimensional relativistic magnetic reconnection
gets independent of resistivity.

The suggestion that LV99 is applicable to relativistic reconnection motivated the use of the
model for explaining \( \gamma \)-ray bursts [36,119] and in accretion discs around black holes and pulsars
[120,121]. Now, as the extension of the model to relativistic case has been confirmed, these and
other cases where the relativistic analogue of LV99 process was discussed to be applicable [21] are
given numerical support.

Naturally, more detailed studies of both relativistic MHD turbulence and relativistic magnetic
reconnection are required. It is evident that in magnetically dominated, low-viscous plasmas
turbulence is a generic ingredient and thus it must be taken into account for relativistic magnetic
reconnection. As we discuss elsewhere in the review, the driving of turbulence may be by external
forcing or it can be driven by reconnection itself.

5. Numerical testing of turbulent reconnection theory

Figure 3 illustrates results of numerical simulations of turbulent reconnection with turbulence
driven using wavelets in [33] and in real space in [34].
Figure 3. Visualization of reconnection simulations in [33,34]. (a) Magnetic field in the reconnection region. (b) Current intensity and magnetic field configuration during stochastic reconnection. The guide field is perpendicular to the page. The intensity and direction of the magnetic field are represented by the length and direction of the arrows. The colour bar gives the intensity of the current. (c) Representation of the magnetic field in the reconnection zone with textures.

Figure 4. (a) The dependence of the reconnection velocity on the injection power for different simulations with different driving. The predicted LV99 dependence is also shown. \( P_{\text{inj}} \) and \( k_{\text{inj}} \) are the injection power and scale, respectively, \( B_z \) is the guide field strength and \( \eta_u \) the value of uniform resistivity coefficient. (b) The dependence of the reconnection velocity on the injection scale (adapted from [34]).

As we show later, simulations in [33,34] provided very good correspondence to the LV99 analytical predictions for the dependence on resistivity, i.e. no dependence on resistivity for sufficiently strong turbulence driving, and the injection power, i.e. \( V_{\text{rec}} \sim P_{\text{inj}}^{1/2} \). The corresponding dependence is shown in figure 4

The simulations did not reveal any dependence on the strength of the guide field \( B_z \) (figure 4). To address this dependence, in the limit where the parallel wavelength of the strong turbulent eddies is less than the length of the current sheet, we can rewrite the reconnection speed as

\[
V_{\text{rec}} \approx \left( \frac{P L_x}{V_{Ax}} \right)^{1/2} \frac{1}{k_{\parallel} V_{A}}.
\]

Here \( P \) is the power in the strong turbulent cascade, \( L_x \) and \( V_{Ax} \) are the length scale and Alfvén velocity in the direction of the reconnecting field, respectively, and \( V_A \) is the total Alfvén velocity, including the guide field.
In a physically realistic situation, the dynamics that drive the turbulence, whatever they are, provide a characteristic frequency and input power. Since the guide field enters only in the combination $k_{\perp} V_A$, i.e. through the eddy turnover rate, this implies that varying the guide field will not change the reconnection speed. In the simulations, the periodicity of the box in the direction of the guide field complicates the analysis (see more discussion in [122]).

The injection of energy in LV99 is assumed to happen at a given scale and the inverse cascade is not considered in the theory. Therefore, it is not unexpected that the measured dependence on the turbulence scale differs from the predictions. In fact, it is slightly more shallow compared with the LV99 predictions (figure 4b).

Figure 5a shows the dependence of the reconnection rate on explicit uniform viscosity obtained from the isothermal simulations of the magnetic reconnection in the presence of turbulence [34]. The open symbols show the reconnection rate for the laminar case when there was no turbulence driving, whereas filled symbols correspond to the mean values of reconnection rate in the presence of saturated turbulence. All parameters in those models were kept the same, except the uniform viscosity, which varied from $10^{-4}$ to $10^{-2}$ in the code units. We notice the lack of any scaling for the laminar case, which is somewhat in contradiction to the scaling $V_{rec} \propto \nu^{-1/4}$ derived in [123]. We should note that the authors introduced the viscosity dependency using the energy equation balance, which cannot be applied in the isothermal case. They also stress that the proper scaling might be sensitive to the chosen boundaries, which in their numerical tests were closed. In the models presented in figure 5, we use outflow boundaries. The viscosity scaling for the case when turbulence is present is shown by filled symbols. This scaling is also $V_{rec} \propto \nu^{-1/4}$, but can be explained rather as the effect of the finite inertial range of turbulence than the effect of energy balance affected by viscosity or boundary conditions. For an extended range of motions, LV99 does not predict any viscosity dependence, if the dissipation scale lies much below the scale of current sheet. However, for numerical simulations, the range of turbulent motions is very limited and any additional viscosity decreases the resulting velocity dispersion and therefore the field wandering thus affecting the reconnection rate.

LV99 predicted that in the presence of sufficiently strong turbulence, plasma effects should not play a role. The accepted way to simulate plasma effects within MHD code is to use anomalous...
The results of the corresponding simulations are shown in figure 5b, and they confirm that the change of the anomalous resistivity does not change the reconnection rate.

Within the derivation adopted in LV99, current sheet is broad with individual currents distributed widely within a three-dimensional volume, and the turbulence within the reconnection region is similar to the turbulence within a statistically homogeneous volume. Numerically, the structure of the reconnection region was analysed by Vishniac et al. [124] based on the numerical work by Kowal et al. [33]. The results support LV99 assumptions with reconnection region being broad, the magnetic shear is more or less coincident with the outflow zone, and the turbulence within it is broadly similar to turbulence in a homogeneous system. In particular, this analysis showed that peaks in the current were distributed throughout the reconnection zone, and that the widths of these peaks were not a strong function of their strength. The illustration of the results is shown in figure 6 which shows histograms of magnetic field gradients in the simulations with strong and moderate driving power, with no magnetic field reversal but with driven turbulence, and with no driven turbulence at all, but a passive magnetic field reversal (i.e. Sweet–Parker reconnection). A few features stand out in this figure. First, all the simulations with driven turbulence have a roughly Gaussian distribution of magnetic field gradients. In the case with no field reversal (figure 6c), the peak is narrow and symmetric around zero. In the presence of a large-scale field reversal, the peak is slightly broadened and skewed. It is turbulent reconnection that does not produce any strong feature corresponding to a preferred value of the magnetic field gradient. Instead one sees a systematic
bias towards large positive values. We conclude from the lack of coherent features within the outflow zone, and the broad distribution of values of the gradient of the magnetic field, that the current sheet and the outflow zone are roughly coincident and this volume is filled with turbulent structures.

As we discussed, the LV99 model is intrinsically related to the concept of Richardson dispersion in magnetized fluids. Thus by testing the Richardson diffusion of magnetic field, one also provides tests for the theory of turbulent reconnection.

The first numerical tests of Richardson dispersion were related to magnetic field wandering predicted in LV99 [99,112,125]. In figure 7, we show the results obtained by Lazarian et al. [99]. There we clearly see different regimes of magnetic field diffusion, including the $y \sim x^{3/2}$ regime. This is a manifestation of the spatial Richardson dispersion.

As we discussed in §3, the LV99 expressions can be obtained by applying the concept of Richardson dispersion to a magnetized layer. Thus by testing the Richardson diffusion of magnetic field, one also provides tests for the theory of turbulent reconnection.

The numerical tests of Richardson dispersion in space correspond to magnetic field wandering predicted in LV99. In figure 7, we show the results obtained by Lazarian et al. [99]. There we clearly see the Richardson regime corresponding to $y \sim x^{3/2}$ regime (see more discussion in ELV11).

A direct testing of the temporal Richardson dispersion of magnetic field lines was performed recently [126]. For this experiment, stochastic fluid trajectories had to be tracked backward in time from a fixed point in order to determine which field lines at earlier times would arrive to that point and be resistively ‘glued together’. Hence, many time frames of an MHD simulation were stored so that equations for the trajectories could be integrated backward. The results of this study are illustrated in figure 7. Figure 7a shows the trajectories of the arriving magnetic field lines, which are clearly widely dispersed backward in time, more resembling a spreading plume of smoke than a single ’frozen-in’ line. Quantitative results are presented in figure 7b, which plots the root-mean-square line dispersion in directions both parallel and perpendicular to the local mean magnetic field. Times are in units of the resistive time $1/j_r$, determined by the r.m.s. current value and distances in units of the resistive length $\lambda/j_r$. The dashed line shows the standard diffusive estimate $4\lambda t$, whereas the solid line shows the Richardson-type diffusion, the power law being slightly altered by the numerical effects. We stress that whatever

---

6Owing to the bottleneck effect the measured magnetic energy spectrum is $k^{-3/2}$ [89] and this spectrum corresponds to $r^{8/3}$ Richardson dispersion dependence.
plasma mechanism of line-slippage holds at scales below the ion gyroradius—electron inertia, pressure anisotropy, etc.—will be accelerated and effectively replaced by the ideal MHD effect of Richardson dispersion.

As we discussed in §4c, the self-sustained turbulent reconnection where the turbulence is generated by the reconnection itself can be quantified using the predictions of the LV99 theory. Below we compare the prediction given by equation (4.15) against the results of recent simulations illustrated by figure 8. The figure shows a few slices of the magnetic field strength $|B|$ through the three-dimensional computational domain with dimensions $L_x = 1.0$ and $L_y = L_z = 0.25$. The simulation was done with the resolution $2048 \times 512 \times 512$. Open boundary conditions along the $X$ and $Y$ directions allowed studies of steady-state turbulence. At the presented time $t = 1.0$, the turbulence strength increased by two orders of magnitude from its initial value of $E_{\text{kin}} \approx 10^{-4} E_{\text{mag}}$. Initially, only the seed velocity field at the smallest scales was imposed (a random velocity vector was set for each cell). We expect that most of the injected energy comes from the Kelvin–Helmholtz instability induced by the local interactions between the reconnection events, which dominates in the $Z$-direction, along which a weak guide field is imposed ($B_z = 0.1B_x$). As seen in the planes perpendicular to $B_x$ in figure 8, Kelvin–Helmholtz-like structures are already well developed at time $t = 1.0$. Turbulent structures are also observed within the $XY$-plane, which probably are generated by the strong interactions of the ejected plasma from the neighbouring reconnection events. More detailed analyses of the spectra of turbulence and efficiency of the Kelvin–Helmholtz instability as the turbulent injection mechanism are presented in [113].

The Kelvin–Helmholtz instability due to the interactions of the outflows from neighbouring reconnection events, which takes place in our simulations, is somewhat different from that in the current sheet of Sweet–Parker reconnection, which has been theoretically predicted in [127]. In the laminar reconnection, the profile of the outflow velocity has its maximum in the middle of the current sheet and quickly decays along the direction parallel to the reconnecting magnetic field component. This configuration creates naturally two shear layers in which the Kelvin–Helmholtz instability may develop if the outflow velocity exceeds the Alfvén speed associated with the upstream magnetic field. In order to confirm the predictions obtained in [127], we would need simulations or observations of the thin current sheets with very large resolutions.
6. Observational testing of turbulent reconnection

(a) Solar reconnection

To quantify solar reconnection, one should accept that the energy is injected by reconnection and turbulence is driven by magnetic reconnection. In this situation, one can expect substantial changes of the magnetic field direction corresponding to strong turbulence. Thus, it is natural to identify the velocities measured during the reconnection events with the strong MHD turbulence regime. In other words, one can use

\[ V_{\text{rec}} \approx U_{\text{obs,turb}} \left( \frac{L_{\text{inj}}}{L_x} \right)^{1/2}, \]  

(6.1)

where \( U_{\text{obs,turb}} \) is the spectroscopically measured turbulent velocity dispersion. Similarly, the thickness of the reconnection layer should be defined as

\[ \Delta \approx L_x \left( \frac{U_{\text{obs,turb}}}{V_A} \right) \left( \frac{L_{\text{inj}}}{L_x} \right)^{1/2}. \]  

(6.2)

The expressions given by equations (6.1) and (6.2) can be compared with observations in [128]. There, the widths of the reconnection regions were reported in the range from \( 0.08L_x \) up to \( 0.16L_x \).
while the observed Doppler velocities in units of $V_A$ were of the order of 0.1. It is easy to see that these values are in a good agreement with the predictions given by equation (6.2).7

If we talk about unique predictions that radically differ from LV99 and the present-day plasma reconnection models, then the LV99 prediction of the triggering of reconnection by wave packets coming from the adjacent reconnection sites should be singled out. Thus, a particular series of solar observations is important. In [129], authors explaining quasi-periodic pulsations in observed flaring energy releases at an active region above the sunspot proposed that the wave packets arising from the sunspots can trigger such pulsations. This is exactly what is expected within the LV99 model.

The criterion for the application of LV99 theory is that the outflow region is much larger than the ion Larmor radius $\Delta \gg \rho_i$. This is definitely satisfied for the solar atmosphere where the ratio of $\Delta$ to $\rho_i$ can be larger than $10^6$. Plasma effects can play a role for small-scale reconnection events within the layer, since the dissipation length based on Spitzer resistivity is approximately 1 cm, whereas $\rho_i \sim 10^3$ cm. However, as we discussed earlier, this does not change the overall dynamics of turbulent reconnection.

(b) Solar wind

Reconnection throughout most of the heliosphere appears similar to that in the Sun. For example, there are now extensive observations of reconnection jets (outflows, exhausts) and strong current sheets in the solar wind [130]. The most intense current sheets observed in the solar wind are very often not observed to be associated with strong (Alfvénic) outflows and have widths at most a few tens of the proton inertial length $\delta_i$ or proton gyroradius $\rho_i$ (whichever is larger). Small-scale current sheets of this sort that do exhibit observable reconnection have exhausts with widths at most a few hundreds of ion inertial lengths and frequently have small shear angles (strong guide fields) [131,132]. Such small-scale reconnection in the solar wind requires collisionless physics for its description, but the observations are exactly what would be expected of small-scale reconnection in MHD turbulence of a collisionless plasma [133]. Indeed, LV99 predicted that the small-scale reconnection in MHD turbulence should be similar to large-scale reconnection, but with nearly parallel magnetic field lines and with ‘outflows’ of the same order as the local, shear-Alfvénic turbulent eddy motions. It is worth emphasizing that reconnection in the sense of flux-freezing violation and disconnection of plasma and magnetic fields is required at every point in a turbulent flow, not only near the most intense current sheets. Otherwise, fluid motions would be halted by the turbulent tangling of frozen-in magnetic field lines. However, except at sporadic strong current sheets, this ubiquitous small-scale turbulent reconnection has none of the observable characteristics usually attributed to reconnection, e.g. exhausts stronger than background velocities, and would be overlooked in observational studies which focus on such features alone.

However, there is also a prevalence of very large-scale reconnection events in the solar wind, often associated with interplanetary CMEs and magnetic clouds or occasionally magnetic disconnection events at the heliospheric current sheet (HCS) [130,134]. These events have reconnection outflows with widths up to nearly $10^5$ of the ion inertial length and appear to be in a prolonged, quasi-stationary regime with reconnection lasting for several hours. Such large-scale reconnection is as predicted by the LV99 theory when very large flux-structures with oppositely directed components of magnetic field impinge upon each other in the turbulent environment of the solar wind. The ‘current sheet’ producing such large-scale reconnection in the LV99 theory contains itself many ion-scale, intense current sheets embedded in a diffuse turbulent background of weaker (but still substantial) current. Observational efforts addressed to proving/disproving the LV99 theory should note that it is this broad zone of more diffuse current, not the sporadic

7If we associate the observed velocities with isotropic driving of turbulence, which is unrealistic for the present situation, then a discrepancy with equation (6.2) would appear. Because of that Ciavarella & Raymond [128] did not get quite as good quantitative agreement between observations and theory as we did, but still within observational uncertainties.
strong sheets, which is responsible for large-scale turbulent reconnection. Note that the study of Eyink et al. [126] showed that standard magnetic flux-freezing is violated at general points in turbulent MHD, not just at the most intense, sparsely distributed sheets. Thus, large-scale reconnection in the solar wind is a very promising area for LV99.

Preliminary comparisons between such events in MHD turbulence and in the high-speed solar wind have yielded very promising results [135]. Criteria can be employed that are designed specifically to look for large-scale reconnection. For example, the ‘partial-variance of increments’ criterion recently proposed by Osman et al. [136] can be adapted for this purpose, by considering magnetic increments over inertial-range separation distances rather than ion-scale distances and, possibly also, with coarse-graining of the magnetic field to eliminate smaller scale features. A similar modification may be made to the criterion of Gosling [130], which identifies reconnection events by roughly Alfvén-jetting plasma bounded on one side by correlated changes in the antiparallel components of \( \mathbf{u} \) and \( \mathbf{B} \) and by anti-correlated changes in those components on the other side. Here the criterion may be modified by requiring that the two large changes must be separated spatially by inertial-range lengths, i.e. essentially by conditioning on a broad outflow jet.

Examples of some events yielded by this latter Gosling-type criterion are shown in figure 10. Figure 10a,b shows a typical event selected from the Johns Hopkins University (JHU) turbulence database, which archives the output of a \( 1024^3 \) pseudo-spectral simulation of the incompressible MHD equations. Figure 10c,d shows a similar event obtained from a study of a fast solar wind stream, 14 January 2008 04:40:00–21 January 2008 03:20:00, using 3 s resolution Wind spacecraft observations from the Magnetic Field Investigation (MFI) and 3D Plasma Analyzer (3DP) experiments. Figure 10a,c shows magnetic field components, and figure 10b,d shows velocity components, both rotated into the local minimum-variance frame [137] plotted versus space for one-dimensional cuts through the MHD simulation and versus time for the spacecraft data. The JHU MHD data are in the arbitrary units of the simulation, for which the r.m.s. magnetic field strength \( b' = 0.24 \), the magnetic integral length \( L_b = 0.35 \) and the resistive dissipation length \( n_b = 0.0028 \). The units for the Wind data are nanotesla for the magnetic field, kilometre per second for velocity and minutes for time. Average solar wind conditions were speed \( u = 660 \text{ km s}^{-1} \), magnetic field strength \( B = 4.4 \text{ nT} \), proton number density \( n_p = 2.4 \text{ cm}^{-3} \), Alfvén speed \( V_A = 62 \text{ km s}^{-1} \) and proton beta \( \beta_p = 1.2 \). The outer scale of the turbulent inertial range (boundary with the \( 1/f \) spectral range) is 33 min and the inner scale (a few ion gyroradii) around 10 s.

The event from the MHD database was found by searching for ‘Gosling events’ that show opposing changes in \( \mathbf{u} \) and \( \mathbf{B} \) within a distance of 0.196, about half an integral length. The event from the high-speed solar wind was found by applying the same criterion for separation of 400 s.

Interestingly, neither of these events show the ‘double-step’ structure, with an intermediate plateau of reversing magnetic field component, which often characterizes the events identified by Gosling [130], although other events we have found do show this structure. Most importantly, both events show the features expected of large-scale reconnection, with a sizable magnetic reversal over an inertial-range length and with a corresponding outflow in the same direction and of the same width. This makes both events likely candidates for turbulent reconnection. In the case of the MHD database event, this interpretation can be verified from the simulation data. A detailed study [35] shows that the MHD event presented in figure 10a,b accords well with the predictions of the LV99 theory and has the expected morphological features: a wide (inertial-range scale) outflow jet, a distribution of small-scale current sheets rather than a single dominant sheet, turbulent wandering of magnetic field lines and Richardson dispersion of field lines normal to the reversal direction. It is therefore natural to identify the similar events in the solar wind as turbulent reconnection as well. This identification is strengthened by the similar statistical rates of occurrence of such events at corresponding scales, as observed also in previous studies of inertial-range magnetic increments in MHD turbulence and the solar wind [138]. The high-speed solar wind is presumably full of such turbulent reconnection events, across its broad spectrum of inertial-range length scales.
Figure 10. Candidate events for turbulent reconnection. MHD turbulence simulation (a,b) and high-speed solar wind (c,d). Both (a,c) magnetic field components and (b,d) velocity components rotated into a local minimum-variance frame of the magnetic field. The component of maximum variance in red is the apparent reconnecting component, the component of medium variance in green is the nominal guide-field direction and the minimum-variance direction in blue is perpendicular to the reconnection layer.

We note that the situation for applicability of LV99 generally gets better with increasing distance from the Sun, because of the great increase in scales. For example, reconnecting flux structures in the inner heliosheath could have sizes up to 100 AU, much larger than the ion cyclotron radius around $10^3$ km [139]. A detailed comparison of the results of solar wind measurement with the results of MHD simulations testifies that the features of the large-scale solar wind reconnection are consistent with the turbulent reconnection that we discuss here [48].

(c) Parker spiral and heliospheric current sheet

More recently, Eyink [35] discussed some implications of LV99 for heliospheric reconnection, in particular for deviations from the Parker spiral model of interplanetary magnetic field. Note that the spiral model [141] of the interplanetary magnetic field is one of the most famous applications in astrophysics and space science of the ‘frozen-in’ principle for magnetic field lines. The model has been shown to be approximately valid when taking into account solar cycle variations in source magnetic field strength and latitude/time variation in solar wind speeds. Nevertheless,
Parker [141] concluded his paper with a ‘warning to the reader against taking too literally any of the smooth idealized models which we have constructed in this paper’.

Burlaga et al. [142] had studied the magnetic geometry and found ‘notable deviations’ from the spiral model. They studied daily averages of magnetic field observations of Voyager 1 and 2 in the ecliptic plane at solar distances \( R = 1–5 \) AU during a period of increasing solar activity in the years 1977–1979. In contrast to the Parker predictions for radial magnetic field component radial dependencies \( B_R \sim R^{-2} \) and azimuthal component \( B_T \sim R^{-1} \), Burlaga et al. [142] found \( B_R \sim R^{-1.56} \) and \( B_T \sim R^{-1.20} \), and they attributed the observed deviations to ‘temporal variations associated with increasing solar activity, and to the effects of fluctuations of the field in the radial direction’. These early observations were recently confirmed by Khabarova & Obridko [143], who presented evidence on the breakdown of the Parker spiral model for time- and space-averaged values of the magnetic field from several spacecraft (Helios 2, Pioneer Venus Orbiter, IMP8, Voyager 1) in the inner heliosphere at solar distances 0.3–5 AU and in the years 1976–1979. Khabarova & Obridko [143] interpret their observations as due to ‘a quasi-continuous magnetic reconnection, occurring both at the HCS and at local current sheets inside the IMF sectors’. They present extensive evidence that most nulls of BR and BT, where reconnection may occur, are not associated with the HCS. They as well observe a rapid disappearance of the regular sector structure at distances past 1 AU, which they attribute to turbulent processes in the inner heliosphere. Eyink [35] estimated the magnetic field slippage velocities and related the deviation from Parker original predictions to LV99 reconnection.

In addition, Eyink [35] analysed the data relevant to the region associated with the broadened HCS, noticed its turbulent nature and provided arguments on the applicability of the LV99 magnetic reconnection model to HCS. This seems to be a very promising direction of research to study turbulent reconnection in action using in situ spacecraft measurements.

7. Implications

(a) Magnetic flux freezing in the presence of turbulent reconnection

The concept of flux freezing was first proposed by Hannes Alfvén in 1942, and the principle of frozen-in field lines has provided a powerful heuristic [2,144]. However, if strictly valid, flux would forbid magnetic reconnection, because field lines frozen into a continuous plasma flow cannot change their topology. If magnetic reconnection were a phenomenon isolated to regions of special magnetic flux topology or other special conditions, then it would be possible to use flux freezing for generic magnetic field conditions. However, the LV99 model suggests that magnetic reconnection happens everywhere in magnetized turbulent fluids. This means the ubiquitous violation of flux freezing in magnetized turbulence.

Standard mathematical proofs of flux-freezing in MHD always assume, implicitly, that velocity and magnetic fields remain smooth as \( \eta \to 0 \). However, MHD solutions with small resistivities and viscosities (high magnetic and kinetic Reynolds numbers) are generally turbulent. These solutions exhibit long ranges of power-law spectra corresponding to very non-smooth or ‘rough’ magnetic and velocity fields. Fluid particle (Lagrangian) trajectories in such rough flows are known to be non-unique and stochastic (see [145–150] and for reviews [102,151]). In view of the above, it is immediately clear as a consequence that standard flux-freezing cannot hold in turbulent plasma flows. After all, the usual idea is that magnetic field lines at high conductivity are tied to the plasma flow and follow the fluid motion. However, if the latter is non-unique and stochastic, then which fluid element will the field line follow?

For a laminar velocity field, this diffusion effect is small. It is not hard to see that a pair of field lines will attain a displacement \( r(t) \) apart under the combined effect of advection and diffusion obeying

\[
\frac{d}{dt} \langle r^2 \rangle = 12\lambda + 2\langle r \cdot \delta u(r) \rangle,
\]

where \( \lambda \) is the dissipation rate and \( \delta u \) is the turbulent fluctuation velocity field.
where $\delta u(r)$ is the relative advection velocity at separation $r$. Thus,
\[
\frac{d}{dt} \langle r^2 \rangle \leq 12\lambda + 2\|\nabla u\| \langle r^2 \rangle,
\]
where $\|\nabla u\|$ is the maximum value of the velocity gradient $\nabla u$. It follows that two lines initially at the same point, by time $t$ can have separated at most
\[
\langle r^2(t) \rangle \leq 6\lambda \frac{e^{2\|\nabla u\|} - 1}{\|\nabla u\|}.
\]
(7.1)

If we thus consider a smooth laminar flow with a fixed, finite value of $\|\nabla u\|$, then $\langle r^2(t) \rangle \to 0$ as $\lambda \to 0$. Under such an assumption, magnetic field lines do not diffuse a far distance away from the solution of the deterministic ordinary differential equation $dx/dt = u(x, t)$, and the magnetic line diffusion becomes a negligible effect. In that case, magnetic flux is conserved better as $\lambda$ decreases.

However, in a turbulent flow, the above argument fails! The inequality (7.1) still holds, of course, but it no longer restricts the dispersion of field lines under the joint action of resistivity and advection. As is well known, a longer and longer inertial range of power-law spectrum $E(k)$ occurs as viscosity $\nu$ decreases and the maximum velocity gradient $\|\nabla u\|$ becomes larger and larger. In fact, energy dissipation $\epsilon = \nu \|\nabla u\|^2$ is observed to be non-vanishing as $\nu \to 0$ in turbulent flow, requiring velocity gradients to grow unboundedly. Estimating $\|\nabla u\| \sim (\epsilon/\nu)^{1/2}$, the upper bound (7.1) becomes
\[
\langle r^2(t) \rangle \leq 6\lambda \left(\frac{\nu^3}{\epsilon}\right)^{1/2} \left[ \exp \left( 2t \left(\frac{\epsilon}{\nu}\right)^{1/2} \right) - 1 \right].
\]
(7.2)

This bound allows unlimited diffusion of field lines. Consider first the case $\lambda = \nu$ or $Pr = 1$, for simplicity, where Richardson’s theory implies that
\[
\langle r^2(t) \rangle \sim 12\lambda t + \epsilon t^3.
\]
(7.3)

The rigorous upper bound always lies strictly above Richardson’s prediction and, in fact, goes to infinity as $\nu = \lambda \to 0!$ The case of large Prandtl number is just slightly more complicated, as previously discussed in §4c. When $Pr \gg 1$, the inequality (7.2) holds as an equality for times $t \ll t_{trans}$ with
\[
t_{trans} = \frac{\ln(Pr)}{2(\epsilon/\nu)}.
\]
(7.4)

This is then followed by a Richardson dispersion regime
\[
\langle r^2(t) \rangle \sim 6 \left(\frac{\nu^3}{\epsilon}\right)^{1/2} + \epsilon (t - t_{trans})^3, \quad t \gg t_{trans},
\]
(7.5)

assuming that the kinetic Reynolds number is also large and a Kolmogorov inertial range exists at scales greater than the Kolmogorov length $(\nu^3/\epsilon)^{1/4}$. Once again, the upper bound (7.2) is much larger than Richardson’s prediction and, at times longer than $t_{trans}$, the dispersion of field lines is independent of resistivity.

(b) Making Goldreich–Sridhar model self-consistent

Historically, a lot of reconnection research was aimed to obtain the Holy Grail number of reconnection speed, which on the basis of solar flare observations was determined to be $0.1V_A$. This reconnection speed has been recently claimed to be attained in a number of plasma simulations [16]. We claim, however, that to make any model of strong turbulence self-consistent the velocity of $0.1V_A$ is insufficient. Below we show this for the GS95 model by reproducing the arguments in LV99. Magnetic reconnection is required for free mixing of magnetic field lines, which is a part of the GS95 picture of turbulence. In fact, the critical balance that is the cornerstone of the GS95 model can be derived from the equality of times for mixing of magnetic field lines perpendicular to the local direction of magnetic field and the period of the Alfvén wave that this
mixing induces. Therefore, we consider magnetic reconnection within magnetic eddies elongated along the local magnetic field direction.

It is possible to see that within the GS95 picture, the reconnection happens with nearly parallel lines with magnetic pressure gradient $V_A^2/l_\parallel$ being reduced by a factor $l_\perp^2/l_\parallel^2$, as only reversing component is available for driving the outflow. At the same time, the length of the contracted magnetic field lines is also reduced from $l_\perp$ by $l_\perp^2/l_\parallel^2$. Therefore, the acceleration is $\tau_{\text{eject}}^{-2} l_\perp^2/l_\parallel^2$. As a result, Newton’s law gives $V_A^2 l_\perp^2/l_\parallel^3 \approx \tau_{\text{eject}}^{-2} l_\perp^2/l_\parallel^2$. This provides the result for the ejection rate $\tau_{\text{eject}}^{-1} \approx V_A/l_\parallel$. The length over which the magnetic eddies intersect is $l_\perp$ and the rate of reconnection is $V_{\text{rec}}/l_\perp$. For the stationary reconnection, this gives $V_{\text{rec}} \approx V_A l_\perp/l_\parallel$, which provides the reconnection rate $V_A/l_\parallel$. The latter rate is exactly the rate of the eddy turnovers in GS95 turbulence, which shows that it is fast magnetic reconnection that makes the GS95 picture self-consistent. In the case of trans-Alfvénic turbulence, this means that the reconnection velocity should be of the order of $V_A$. This sort of reconnection rate has never been reported to be attainable within plasma reconnection simulations [16]. However, this is the reconnection rate that is expected for trans-Alfvénic turbulence within the LV99 model.

(c) Reconnection diffusion and star formation

As we have argued earlier at length, standard flux-freezing breaks down at every point and time in a turbulent plasma. In that case, the only objectively meaningful way to give a magnetic field line an identity over time is by tagging it with a certain plasma fluid element. As suggested by Axford [152], we understand the crucial feature of magnetic reconnection to be the ‘disconnection’ of fluid elements that start on the same field line. Figure 11a uses data from the JHU MHD turbulence database archive to illustrate how an initial magnetic field line changes its connections to plasma fluid elements over time. The figure shows an initial magnetic field line, in black, decorated with 11 plasma fluid elements, indicated by various colours. The plasma elements are then evolved with the fluid velocity for about one large-eddy turnover time ($t = 2.00$ in units of the simulation). The magnetic field lines threading these latter plasma elements are drastically different. Indeed, the plasma has ‘drifted’ to distinct lines separated by distances of the order of the magnetic integral length (0.35 in the units of the simulation). This drift occurs even though the conductivity of the simulation is high and the Ohmic electric fields are tiny, because their

Figure 11. (a) Turbulent splitting of a magnetic field line. An initial magnetic field line in black is decorated with plasma elements, which are allowed to move with the fluid velocity. The magnetic field lines that thread these plasma elements at the later time are drastically different, effectively ‘splitting’ the original line. (b) Reconnection event within three-dimensional-driven turbulence. The structure of the event corresponds to the LV99 picture and is consistent with reconnection events studied in solar wind.
small direct effects are greatly magnified by the turbulence. The complexity of three-dimensional reconnection in a turbulent flow is illustrated in figure 11b.

The violation of flux freezing means that the astrophysical theories based on the concept of flux freezing must be revised. In particular, the standard star formation theory assumes that flux freezing is being violated in the partially ionized gas only due to the relative drift of neutrals and ions. As we discussed in §4a,b in the partially ionized gas, the important magnetic flux violation arises from magnetic diffusion induced by turbulence. This process that was termed ‘reconnection diffusion’ was identified and described in [153] (see also [110]) and successfully tested in subsequent publications for the case of molecular clouds and protostellar discs (e.g. [72,154–157]).

A comprehensive review dealing with reconnection diffusion is presented in [90].

The theory of transporting matter in turbulent magnetized medium is discussed at length in [90,158] and we refer the reader to those publications. The process was termed ‘reconnection diffusion’ to stress the importance of reconnection in the diffusive transport.

The peculiarity of reconnection diffusion is that it requires nearly parallel magnetic field lines to reconnect, while the textbook description of reconnection is usually associated with anti-parallel description of magnetic field lines. One should understand that the configuration shown in figure 2 is just a cross section of the magnetic fluxes depicting the anti-parallel components of magnetic field. Generically, in three-dimensional reconnection configurations the sheared component of magnetic field is present. The process of reconnection diffusion is closely connected with the reconnection between adjacent Alfvénic eddies (figure 12a,b). As a result, adjacent flux tubes exchange their segments with entrained plasmas and flux tubes of different eddies get connected. This process involves eddies of all the sizes along the cascade and ensures fast diffusion which has similarities with turbulent diffusion in ordinary hydrodynamic flows.

Figure 12c illustrates the evolution of magnetic flux during the process of reconnection diffusion of magnetic flux out of the circumstellar accretion disc. The magnetic field lines are smoothed in the picture to illustrate the evolution of the mean magnetic field.

Reconnection diffusion should not necessarily be understood as a concept that makes the earlier theories of star formation invalid. In fact, turbulence in dark cores giving birth to stars may be reduced and this may make the traditional ambipolar diffusion, i.e. the drift of neutrals in relation to ions, important. However, reconnection diffusion must be a part of the star formation paradigm. In fact, it can successfully explain many pieces of observational data that are completely puzzling within the ambipolar diffusion paradigm. This includes, for instance, the famous ‘magnetic breaking catastrophe’ for accretion discs which is the inability of removing magnetic flux from accretion discs fast enough to enable formation of such discs. Similarly, the poor correlation of density and magnetic field in ISM is also impossible to explain on the basis of ambipolar diffusion. A comprehensive review dealing with reconnection diffusion is presented in [90]. Closely related is the recent development of the ‘turbulent general magnetic reconnection’ theory in [35]. The starting point of this theory is the understanding that magnetic field line ‘motion’ can be objectively defined only relative to plasma fluid elements and their magnetic connectivity. In star formation for example, the magnetic field lines threading the protostar will appear to ‘slip’ relative to the ambient ISM, whereas the field lines embedded in the ISM will likewise appear to ‘slip’ through the collapsing magnetic cloud. Neither picture is more correct than the other and, indeed, one cannot uniquely define a ‘motion’ of the field lines. However, it has been shown in [35] that the lines wandering between the protostar and the surrounding ISM acquire a unique slip velocity per unit arc-length of field line, which is completely independent of which end of the line is regarded to be the ‘foot-point’ tied to the plasma. This so-called slip-velocity source is given by the expression

$$\Sigma = -\frac{(\nabla \times R)_\perp}{|B|},$$  \hspace{1cm} (7.6)

where $R$ is the non-ideal electric field in the generalized Ohm’s law $E + u \times B = R$ for the plasma, and ‘$\perp$’ denotes the component perpendicular to $B$. It is only by spatially wandering/intersecting a region with non-vanishing $\Sigma$ that a field line can evade the ‘frozen-in’ condition. Furthermore,
in a turbulent plasma, the slip source $\Sigma$ is enormous, even though the electric fields $R$ are tiny and the plasma nearly ideal. This approach is essentially a refinement of the LV99 idea that field lines must wander into microscopic ‘current sheets’ in order to break the flux-freezing constraint. Eyink [35] applied this theory to explain observed deviations from the Parker spiral model of the interplanetary magnetic field in our own solar system, owing to ‘slippage’ of the field lines through the turbulent solar wind.

(d) Solar flares and $\gamma$-ray bursts

The picture of flares of reconnection described in §4d is broadly supported by current observations and numerical simulations of solar flares and CMEs. For example, simulations by Lynch et al. [160] of the ‘breakout model’ of CME initiation show that an extremely complex magnetic line structure develops in the ejecta during and after the initial breakout reconnection phase, even under the severe numerical resolution constraints of such simulations. In the very high Lundquist number solar environment, this complex field must correspond to a strongly turbulent state, within which the subsequent ‘anti-breakout reconnection’ and post-CME current sheet occur. Direct observations of such current sheets [128,161] verify the presence of strong
turbulence and greatly thickened reconnection zones, consistent with the LV99 model. In the numerical simulations, the ‘trigger’ of the initial breakout reconnection is numerical resistivity, and there is no evidence of turbulence or complex field structure during the eruptive flare onset. This is very likely to be a result of the limitations on resolution, however, and we expect that developing turbulence will accelerate reconnection in this phase of the flare as well.

While the details of the physical processes discussed above can be altered, it is clear that LV99 reconnection induces bursts in highly magnetized plasmas. This can be applicable not only to the solar environment but also to more exotic environments, e.g. to γ-ray bursts. The model of γ-ray bursts based on LV99 reconnection was suggested by Lazarian et al. [119]. It was elaborated and compared with observations by Zhang & Yan [36]. Currently, the latter model is considered promising and it attracts a lot of attention of researchers. Flares of reconnection that we described above can also be important for compact sources, like pulsars and black holes in microquasars and active galactic nuclei [120]. We note that LV99 reconnection is getting more applications related to emission of astrophysical objects. For instance, recently it has been discussed to explain the radio and γ-ray emission arising through accretion on black holes [162] as well as for describing the radiation of microquasars [163].

(e) Turbulent reconnection and particle acceleration

Turbulent reconnection provides a way of the first-order Fermi acceleration as illustrated in figure 13. The efficiency of the process is ensured by LV99 model being the volume-filling reconnection.8

Figure 13 illustrates a situation when the particle anisotropy arises from particles preferentially accelerated in direction parallel to magnetic field. Similarly, Lazarian et al. [164] showed that the first-order Fermi acceleration can also happen in terms of the perpendicular to the magnetic field component of particle momentum. This is illustrated in figure 13b. There the particle with a large Larmour radius is bouncing back and forth between converging mirrors of reconnecting magnetic field systematically getting an increase of the perpendicular component of its momentum. Both processes take place in reconnection layers.

Disregarding the backreaction, one can get the spectrum of accelerated cosmic rays [120,153]

\[ N(E)dE = \text{const.} E^{-5/2}dE. \]  

(7.7)

This result is the result of acceleration in the absence of compression [165]. The first-order acceleration of particles entrained on the contracting magnetic loop can be understood from the Liouville theorem. In the process of the magnetic tube contraction, a regular increase of the particle energies is expected. The requirement for the process to proceed efficiently is to keep the accelerated particles within the contracting magnetic loop. This introduces limitations on the particle diffusivity perpendicular to the magnetic field direction. The process in figure 13a was discussed in [11] in relation to the acceleration of particles in collisionless reconnection. There by accounting for the backreaction of particles the authors obtained a more shallow spectrum.

Testing of particle acceleration in turbulent reconnection was performed in [34] and its results are presented in figure 14. Figure 14 shows the evolution of the kinetic energy of the particles. After injection, a large fraction of test particles accelerates and particle energy growth occurs (see also the energy spectrum at \( t = 5 \) in the detail at the bottom right). This is explained by a combination of two effects: the presence of a large number of converging small-scale current sheets and the broadening of the acceleration region due to the turbulence. The acceleration process is clearly a first-order Fermi process and involves larger number of particles, since the size of the acceleration zone and the number of scatterers naturally increase by the presence of turbulence. Moreover, the reconnection speed, which in this case is independent of

8 We stress that figure 2 exemplifies only the first moment of reconnection when the fluxes are just brought together. As the reconnection develops the volume of thickness \( \Delta \) becomes filled with the reconnected three-dimensional flux ropes moving in the opposite directions.
Figure 13. (a) Cosmic rays spiral about a reconnected magnetic field line and bounce back at points A and B. The reconnected regions move towards each other with the reconnection velocity $V_R$ (adapted from [153]). (b) Particles with a large Larmor radius gyrate about the magnetic field shared by two reconnecting fluxes (the latter is frequently referred to as ‘guide field’). As the particle interacts with converging magnetized flow corresponding to the reconnecting components of magnetic field, the particle gets energy gain during every gyration (adapted from [164]).

Figure 14. Particle kinetic energy distributions for 10 000 protons injected in the fast magnetic reconnection domain. The colours indicate which velocity component is accelerated (red or blue for parallel or perpendicular, respectively). The energy is normalized by the rest proton mass. Subplot shows the particle energy distributions at $t = 5.0$. Model with $B_{0z} = 0.1$, $\eta = 10^{-3}$ and resolution $256 \times 512 \times 256$ is shown.

resistivity [31,33] and determines the velocity at which the current sheets scatter particles, has been naturally increased as well (i.e. $V_{rec} \sim V_A$). During this stage, the acceleration rate is in the range 2.48–2.75.

The process of acceleration via turbulent reconnection is expected to be widespread. In particular, it has been discussed in [139] as a cause of the anomalous cosmic rays observed by Voyagers and in [166] as a source of the observed cosmic ray anisotropies. We expect turbulent reconnection to accelerate energetic particles in relativistic environments, like those related to accretion discs and relativistic jets as well as in $\gamma$-ray bursts (see [21] for a review). The latter process, discussed first in [119], has been given strong observational support in [36].
8. Connection with other ideas of fast reconnection

As we mentioned in the Introduction, the reconnection research is a vast vigorously developing field and is not limited to reconnection in MHD approximation in turbulent fluids that we deal with in this review. A lot of experimental research is done for the Earth magnetosphere and laboratory plasmas, where the MHD description is not valid. Some plasmas may not be turbulent either. Below we briefly outline the connection of our turbulent model with other directions of research.

(a) Plasmoid/tearing mode reconnection

Plasmoid/tearing mode reconnection is currently a vibrant direction of reconnection research [23, 167]. The work in this direction shows that Sweet–Parker reconnection is unstable for sufficiently large Ludquist numbers. What should be kept in mind is that in three dimensions the thicker outflows induced by plasmoid/tearing reconnection inevitably induce turbulence. This corresponds to both PIC and MHD simulations that we discussed earlier. This is also inevitable on theoretical grounds. Indeed, from the mass conservation constraint requirement in order to have fast reconnection one has to increase the outflow region thickness in proportion to \( L_x \), which means the proportionality to the Lundquist number \( S \). The Reynolds number \( Re \) of the outflow is \( \Delta V_A / \nu \), where \( \nu \) is viscosity, which grows also as \( S \). The outflow gets turbulent for sufficiently large \( Re \). It is natural to assume that once the shearing rate introduced by eddies is larger than the rate of the tearing instability growth, the instability should get suppressed.

If one assumes that tearing is the necessary requirement for fast reconnection this entails the conclusion that tearing should proceed at the critically damped rate, which implies that \( Re \) and therefore \( \Delta \) should not increase. This entails, however, the decrease in reconnection rate driven by tearing in proportion \( L_x \sim S \) as a result of mass conservation. As a result, the reconnection should stop being fast. Fortunately, we know that turbulence itself provides fast reconnection irrespective of whether tearing is involved or not. Thus, one may conclude that the tearing reconnection, similar to the Sweet–Parker reconnection, should be applicable to a limited range of \( S \) for realistic magnetized plasmas with low viscosity in the perpendicular to magnetic field direction.

Tearing may be important for initiating turbulence and transiting from the laminar initial state. To what extent tearing is required is not clear from the three-dimensional simulations that we discussed above. Those suggest the importance of Kelvin–Helmholdz instability, but whether tearing plays any role must still be explored. However, if reconnection was excited by tearing/plasmoid instability generically, we expect a transition to the regime of turbulent reconnection.

Another limitation to the applicability of tearing reconnection arises from its speed. The reported rates do not exceed a small fraction of Alfvén speed. However, as we discuss later magnetic turbulence requires reconnection speeds which are substantially larger than that value for the theory of MHD turbulence to be self-consistent (LV99, ELV11). In this situation, the dominance of turbulent reconnection seems inevitable.

(b) Reconnection in two dimensions in the presence of turbulence

Matthaeus & Lamkin [42,168] explored numerically turbulent reconnection in two dimensions. As a theoretical motivation, the authors emphasized analogies between the magnetic reconnection layer at high Lundquist numbers and homogeneous MHD turbulence. They also pointed out various turbulence mechanisms that would enhance reconnection rates, including multiple X-points as reconnection sites, compressibility effects, motional EMF of magnetic bubbles advecting out of the reconnection zone. However, the authors did not understand the importance of ‘spontaneous stochasticity’ of field lines and of Lagrangian trajectories and they did not
arrive at an analytical prediction for the reconnection speed. Although an enhancement of the reconnection rate was reported in their numerical study, the set-up precluded the calculation of a long-term average reconnection rate.

The relation of this study with LV99 is not clear, as the nature of turbulence in two dimensions is different. In particular, shear-Alfvén waves that play the dominant role in three-dimensional MHD turbulence according to GS95 are entirely lacking in two dimensions, where only pseudo-Alfvén wave modes exist. We believe that the question whether turbulence is fast has not been resolved yet if we judge from the available publications. For instance, in a more recent study along the lines of the approach in [168], i.e. in [169], the effects of small-scale turbulence on two-dimensional reconnection were studied and no significant effects of turbulence on reconnection were reported. Servidio et al. [170] have more recently made a study of Ohmic electric fields at X-points in homogeneous, decaying two-dimensional MHD turbulence. However, they studied a case of small-scale magnetic reconnection, and their results are not directly relevant to the issue of reconnection of large-scale flux tubes that we deal with in this review.

Instead of studying bulk reconnection in two-dimensional turbulence as the aforementioned studies did, Loureiro et al. [171] and Kulpa-Dybel et al. [172] studied large-scale reconnection,9 which is advantageous if the determination of the actual reconnection rates is sought. The two groups reached different conclusions. On the one hand, Loureiro et al. [171] had a better resolution but used periodic boundary conditions, which strongly constrain the ability to do averaging of the reconnection rate and the attainment of the steady state for reconnection. They inferred from their data that the two-dimensional turbulent reconnection rate may be independent of resistivity. On the other hand, Kulpa-Dybel et al. [172] used smaller data cubes but longer averaging, which is enabled by their outflow boundary conditions. They concluded that the reconnection does depend on resistivity and therefore is slow.

In the view of the difference of MHD turbulence in two and three dimensions, we do not view the reconnection studies in two-dimensional turbulence as directly relevant in any astrophysical settings. Even if eventually two-dimensional reconnection is proved to be fast, the reconnection rate is expected to have different dependences on turbulent power.

(c) Turbulent reconnection models based on mean-field approach

Guo et al. [44] modified and extended ideas originally proposed in [106] and suggested their model of fast turbulent reconnection. Both papers use mean-field approach, but unlike the study in [106] which concluded that turbulence cannot accelerate reconnection the more recent study obtains expressions for fast reconnection. These expressions are different from those in LV99 and seem to grossly contradict the numerical testing of turbulent reconnection in [33]. Another model of turbulent reconnection based on the mean-field approach is presented in [173], and it also suffers with the problems of using the mean-field approach for reconnection that we describe later.

The mean-field approach invoked in the aforementioned studies is plagued by poor foundations and conceptual inconsistencies, however [32]. In such an approach, effects of turbulence are described using parameters such as anisotropic turbulent magnetic diffusivity experienced by the fields once averaged over ensembles. The problem is that it is the lines of the full magnetic field that must be rapidly reconnected, not just the lines of the mean field. ELV11 stresses that the former implies the latter, but not conversely. No mean-field approach can claim to have explained the observed rapid pace of magnetic reconnection unless it is shown that the reconnection rates obtained in the theory are strictly independent of the length and time scales of the averaging. Naturally, it is impossible to get reliable results applying mean-field approach to reconnection (see more discussion in ELV11).

---

9The enhancement of two-dimensional large-scale reconnection was reported starting from 2007 at a few conferences by Kowel et al. [33], but for them the two-dimensional study was a testing ground for the realistic three-dimensional simulations to test LV99. Thus, these results were never published.
Other attempts to get fast magnetic reconnection from turbulence are related to the so-called hyper-resistivity concept [41,43,174,175], which is another attempt to derive fast reconnection from turbulence within the context of mean-field-resistive MHD. Apart from the generic problems of using the mean-field approach, we point out that the derivation of the hyper-resistivity is questionable from a different point of view. The form of the parallel electric field is derived from magnetic helicity conservation. Integrating by parts, one obtains a term which looks like an effective resistivity proportional to the magnetic helicity current. There are several assumptions implicit in this derivation, however. Fundamental to the hyper-resistive approach is the assumption that the magnetic helicity of mean fields and of small scale, statistically stationary turbulent fields is separately conserved, up to tiny resistivity effects. However, this ignores magnetic helicity fluxes through open boundaries, essential for stationary reconnection, that vitiate the conservation constraint.

As we discuss further, a common misunderstanding is that ‘resistivity arising from turbulence’ is a real plasma non-ideality ‘created’ by the turbulence. However, such apparent non-ideality is strongly dependent on the length and time scales of the averaging. It appears only as a consequence of observing the plasma dynamics at a low resolution, so that the coarse-grained velocity and magnetic field that are observed will no longer satisfy the microscopic equations of motion. This coarse-graining or averaging is a purely passive operation which does not change the actual plasma dynamics but only corresponds to ‘taking off one’s spectacles’. It is clear that one cannot create true, physical non-ideal electric fields by removing one’s eyeglasses! Such apparent non-ideality in a turbulent plasma observed at length scales in the inertial range or larger is a valid representation of the effects of turbulent eddies at smaller scales. However, such apparent non-ideality is not accurately represented by an effective ‘resistivity’, a representation which in the fluid turbulence literature has been labelled the ‘gradient-transport fallacy’ [176]. It is also clear that no mean-field or coarse-graining approach can claim to have explained the observed rapid pace of magnetic reconnection unless it is shown that the reconnection rates obtained in the theory are strictly independent of the length and time scales of the averaging [35,177].

More detailed discussion of the conceptual problems of the hyper-resistivity concept and mean-field approach to magnetic reconnection is presented in [99] and ELV11.

(d) Indirect evidence for turbulent reconnection

A study of tearing instability of current sheets in the presence of background two-dimensional turbulence that observed the formation of large-scale islands was performed [178]. While one can argue that observed long-lived islands are an artefact of the adopted two-dimensional geometry, the authors present evidence for fast energy dissipation in two-dimensional MHD turbulence and show that this result does not change as they change the resolution. More recently, Mininni & Pouquet [179] provided evidence for fast dissipation also in three-dimensional MHD turbulence. This phenomenon is consistent with the idea of fast reconnection, but cannot be treated as a direct evidence of the process. Indeed, fast dissipation and fast magnetic reconnection are rather different physical processes, dealing with decrease in energy on the one hand and decrease in magnetic flux on the other.

For example, works by Galsgaard & Nordlund [66] could also be interpreted as an indirect support for fast reconnection. The authors showed that in their simulations they could not produce highly twisted magnetic fields. One possible interpretation of this result could be the fast relaxation of magnetic field via reconnection. However, in the view of many uncertainties of the numerical studies, this relation is unclear. With low resolution involved in the simulations, the Reynolds numbers could not allow a turbulent inertial range.

In this case, these observations could be related to the numerical finding of Lapenta & Bettarini [180] which shows that reconnecting magnetic configurations spontaneously get chaotic and dissipate, which, as discussed in [181], may be related to the LV99 model.
9. Concluding remarks

(a) Turbulent reconnection and ‘turbulent resistivity’

As we discussed in the review, the violation of flux freezing and diffusivity of magnetic field that contradicts the Alfvén theorem follows from the LV99 model of fast reconnection. This, however, is sometimes misunderstood as our using some sort of ‘turbulent resistivity’. As we mentioned in the review, this confusion is common for many papers. Therefore, we discuss this issue here in more detail. It is possible to show that ‘turbulent resistivity’ description has fatal problems of inaccuracy and unreliability, owing to its poor physical foundations for turbulent flow. It is true that coarse-graining the MHD equations by eliminating modes at scales smaller than some length \( l \) will introduce a ‘turbulent electric field’, i.e. an effective field acting on the large scales induced by motions of magnetized eddies at smaller scales. However, it is well known in the fluid dynamics community that the resulting turbulent transport is not ‘down-gradient’ and not well represented by an enhanced diffusivity. The physical reason is that turbulence lacks the separation in scales to justify a simple ‘eddy-resistivity’ description. As a consequence, energy is often not absorbed by the smaller eddies, but supplied by them, a phenomenon called ‘backscatter’. In magnetic reconnection, the turbulent electric field often creates magnetic flux rather than destroys it.

If we know the reconnection rate, e.g. from LV99, then an eddy resistivity can always be tuned by hand to achieve that rate. But this is engineering, not science. While the tuned reconnection rate will be correct by construction, other predictions will be wrong. The required large eddy resistivity will smooth out all turbulence magnetic structure below the coarse-graining scale \( l \). In reality, the turbulence will produce strong small-scale inhomogeneities, such as current sheets, from the scale \( l \) down to the microscale. In addition, field lines in the flow smoothed by eddy resistivity will not show the explosive, super-diffusive Richardson-type separation at scales below \( l \). These are just examples of effects that will be lost if the wrong concept of ‘eddy resistivity’ is adopted. Note that the aforementioned are important for understanding particle transport/scattering/aceleration in the turbulent reconnection zone. Continuing with the list, we can point out that in the case of relativistic reconnection, turbulent resistivities will introduce acausal, faster than light propagation effects. Nevertheless, the worst feature of the crude ‘eddy-resistivity’ parametrization is its unreliability: because it has no sound scientific basis whatsoever, it cannot be applied with any confidence to astrophysical problems. Therefore, it is pointless to talk about ‘turbulent resistivity’ for the problems that we discussed in the review, e.g. solar flares, star formation, \( \gamma \)-ray bursts.

Equivalently, the stochastic flux freezing [32] closely related to the fast turbulent reconnection concept is definitely not equivalent to the dissipation of magnetic field by resistivity. While the parametrization of some particular effects of turbulent fluid may be achieved in models with different physics, e.g. of fluids with enormously enhanced resistivity, the difference in physics will inevitably result in other effects being wrongly represented by this effect. For instance, turbulence with fluid having resistivity corresponding to the value of ‘turbulent resistivity’ must have magnetic field and fluid decoupled on most of its inertia range turbulent scale, i.e. the turbulence should not be affected by magnetic field in gross contradiction to theory, observations and numerical simulations. Magnetic helicity conservation which is essential for astrophysical dynamo should also be grossly violated.\(^{11}\)

The approach advocated by us in discussing turbulent reconnection is quite different. It is not based on coarse-graining. The spontaneous stochasticity of magnetic field lines and of Lagrangian trajectories (plasma fluid element histories) is a real, verified physical phenomenon in turbulent fluids. Whereas ‘eddy-resistivity’ ideas predict that magnetic flux is destroyed by turbulence, our work shows that turbulent spontaneous stochasticity transforms magnetic-flux conservation

\(^{11}\)This is a serious mistake of a number of numerical simulations of galactic dynamos where to ‘simulate’ effects of turbulent diffusion the Ohmic resistivity \( \nu \) of the order of \( V_L L \) is used. Surely, these simulations do not represent the actual fast astrophysical dynamo, but only a slow one.
into a stochastic conservation law. Because spontaneously stochastic world lines in relativistic turbulence must remain within the light-cone, no acausal effects such as produced by ‘eddy resistivity’ will be predicted. Our approach is based on fundamental scientific progress in the understanding of turbulence, not on engineering parametrizations.

(b) Goldreich–Sridhar turbulence and turbulent reconnection

GS95 turbulence is a theory accepted by a substantial part of the astrophysical community (see [93,95] for reviews). Born outside the mainstream community of turbulence experts, it was initially mostly ignored but then was accepted under the pressure of numerical results. As we mentioned, the debates are not settled about the validity of possible modifications of the model. In parallel, some part of the community is still using the so-called two-dimensional plus slab model of turbulence [182] in spite of the fact that it has no support via numerical simulations with isotropic driving. We consider the latter as some parametrization of the actual heliospheric turbulence over a limited range. This parametrization is not physically or numerically motivated and therefore is not considered within the reconnection model.

The LV99 and our subsequent studies mentioned in the review employed GS95 model. However, we stress again that none of our principal results on fast turbulent reconnection and the physics of turbulent reconnection will be changed if instead of GS95 any other existing model of strong MHD turbulence is used, provided that this model satisfies the constraints that are given by the existing numerical simulations. The corresponding expressions for reconnection rates obtained a wide variety of turbulence indexes are provided in LV99. At the same time, we discussed in the review that fast LV99 reconnection makes the GS95 model self-consistent.

(c) Two- and three-dimensional reconnection

Numerical simulations are very demanding in three dimensions and therefore the numerical research attempts initially to attack the problem of reduced dimensions. In terms of reconnection attempts to attack the problem with two-dimensional simulations are widely spread. While two-dimensional simulations can get insight to some processes, the relationship between two- and three-dimensional reconnection is far from trivial. For instance, from general theoretical positions the importance of three dimensions for reconnection was advocated by Boozer [183,184].

In general, a radical change of physics related to the use of two dimensions instead of actual three dimensions is very common for complex physical systems and the problem of obtaining misleading results extrapolating those obtained in two dimensions for the actual three-dimensional systems goes beyond turbulence. Every time when two-dimensional physics is employed, it is essential to prove that the results stay the same in three dimensions. This, for instance, has not been done in the case of two-dimensional turbulent reconnection [42], and we believe that this may not be possible due to fundamental differences of turbulence physics (see ELV11).

Even in the case when two-dimensional reconnection reflects the physics common to three dimensions, e.g. in the case of tearing reconnection (e.g. [23]), we claim that the development of the instability in three and two dimensions may be very different. The three-dimensional configurations are more prone to secondary instabilities and to the development of the fully turbulent state in which the initial instabilities may not be dominant or even important.

(d) Turbulent reconnection and plasma effects

A substantial part of the reconnection research is based on exploring plasma physics effects on reconnection (see [16,185] for reviews). LV99 shows that reconnection rates should not depend on plasma microphysics in the presence of turbulence. This conclusion was supported by a 

\[ a_{\text{Kraichnan}} = \frac{1}{2} \]
numerical study [33], where plasma effects were simulated by introducing anomalous resistivity. The subdominance of reconnection arising from the Hall effect to that arising due to turbulence was shown analytically in ELV11. A more rigorous comparison of the turbulence induced reconnection with that induced by other terms in the generalized Ohm equation was provided in [35], where it was shown that for typical astrophysical parameters turbulence effects are absolutely dominant.

Nevertheless, the studies that show that magnetic reconnection can be fast in the absence of turbulence (see [16] for a review and references therein) pose interesting questions on the actual role of turbulence. There are, for instance, suggestions that tearing of the current sheet may make collisionless plasma effects applicable to magnetic reconnection on large astrophysical scales (see [29] for a review). Clearly in plasma, one should consider both small-scale plasma turbulence as well as large-scale turbulence that obeys MHD treatment. In addition, the very issue of plasma collisionality is frequently unclear. Indeed, apart from Coulomb collisions, ions may be scattered by magnetic inhomogeneities that arise due to a number of instabilities (e.g. firehose, mirror) in collisionless plasmas and this may make plasma effectively collisional [55,73] in agreement with recent simulations [72]. In this situation, the MHD description should be applicable even to plasmas which are formally collisionless.

Our arguments in the review suggest that plasma effects cannot dominate large-scale astrophysical reconnection in the presence of turbulence. However, we accept that the issue is a subject of interesting debates and more testing is valuable.

(e) Present state of turbulent reconnection theory and outstanding questions

We emphasize that at present the turbulent reconnection theory does not amount to the LV99 model only. It is also ELV11 where the LV99 expressions were reproduced using a very different approach that follows from the recent advances of the Lagrangian description of turbulence. It is also a very recent paper by Eyink [35], where the effects of turbulence were included within the generalized Ohm’s law and were shown to be in agreement with results of the two approaches above. The theoretical foundation of turbulent reconnection got substantial support from the recently developed concept of ‘spontaneous stochasticity’ [32].

The predictions of the turbulent reconnection have been successfully tested both with direct simulations of turbulent reconnection layer [33,34] and through the violation of flux conservation that the turbulent reconnection entailed [126]. As we discuss in the review, more promising numerical tests of reconnection, including turbulence being self-driven are under way.

The turbulent reconnection has shown promise in explaining various astrophysical problems, as well as in addressing problems facing solar physics and heliospheric research. Some of these are discussed in this review, while others are discussed in specialized reviews [21,90,186].

At the same time, a number of questions remain not answered. It is obvious that the LV99 model is a very simplified model. It does not take into account many effects, e.g. effects of plasma compressibility, turbulence intermittency, velocity and magnetic field shear. To obtain analytical results it assumes the turbulence is represented by a single power law and disregards the deviations arising from multiple scales of energy injection, Ohmic and viscous dissipation, etc. It deals with isothermal MHD description of the process and does not account for relativistic effects. The role of collisionless plasma effects in turbulent reconnection is hotly debated.

These limitations of the model are gradually dealt with. For instance, a modification of our understanding of GS95 cascade was discussed in our review in relation to describing magnetically dominated perturbations arising from magnetic reconnection. We also discussed accounting for the partial ionization of plasmas. Role of plasma effects is also being clarified (see [35] and references therein). Nevertheless, we are at the very beginning of our studies of turbulent reconnection and its consequences. Therefore, we expect many surprises and discoveries on the way to fully understanding of the intricate relation of turbulence and reconnection.
(f) Turbulence as a converging point for reconnection research

The LV99 model is the one that describes the dynamics of reconnection within turbulent fluids in MHD regime. By itself, the study of dynamics of magnetic fields in MHD regime, irrespective of any plasma physics, is a well-motivated direction. However, astrophysical environments are filled with turbulent plasmas. Thus for astrophysical applications, it is important to define the domain of applicability of turbulence- versus plasma-based reconnection. First of all, we should stress that there is no single mode of reconnection. Astrophysical and laboratory environments present an extensive variety of conditions for magnetic reconnection.

Numerical experiments show that laminar Sweet–Parker reconnection is feasible in the regime of low Ludquist numbers, after which the laminar picture fails and tearing gets important. As we further increase the length of the reconnection sheet, even without external driving, in three-dimensional low viscosity plasma the transition to turbulence is inevitable. Whether this transfers the reconnection to purely turbulent reconnection or plasma effects are important for determining the reconnection rate when the outflow is fully turbulent is a subject of ongoing debates. In our work, we provided arguments in support of the former solution, i.e. that the transition to the turbulent state when plasma effects do not change the reconnection rate is most relevant for most astrophysical settings. This does not exclude that in some particular circumstances, e.g. for the onset of turbulent reconnection from initially laminar state, for current sheet the thickness of which is comparable with ion inertial length, as is the case of magnetosphere, the plasma effects are important. For other opinions and ongoing debates, we refer the reader to [16]. Below, however, we point out the tendency that the reconnection research has demonstrated.

Recent years have demonstrated the convergence of turbulent reconnection in LV99 and other directions of reconnection research. For instance, models of collisionless reconnection have acquired several features in common with the LV99 model. In particular, they have moved to consideration of volume-filling reconnection [11]. While much of the discussion may still be centred around two-dimensional magnetic islands produced by reconnection, in three dimensions these islands are expected to evolve into contracting three-dimensional loops or ropes [187] introducing stochasticity to the reconnection zone. Moreover, it is more and more realized that the three-dimensional geometry of reconnection is essential and that the two-dimensional studies may be misleading.

The departure from the concept of laminar reconnection and the introduction of magnetic stochasticity are also apparent in a number of recent papers appealing to the tearing mode instability to drive fast reconnection [23,27]. These studies showed that tearing modes do not require collisionless environments and thus collisionality is not a necessary ingredient of fast reconnection. Finally, the development of turbulence in three-dimensional numerical simulations of reconnection (see §4c) clearly testifies that the reconnection induces turbulence even if the initial reconnection conditions are laminar.

All in all, in the last decade, the models competing with LV99 have undergone a substantial evolution, from two-dimensional collisionless reconnection based mostly on Hall effect to three-dimensional reconnection, where the collisionless condition is no more required, Hall effect is not employed, but magnetic stochasticity and turbulence play an important role in the thick reconnection regions. Nevertheless, we want to stress that collisionless reconnection may be suitable for the description of reconnection when the reconnecting flux-structures are comparable with the ion gyro scale, which is the case of the reconnection studied in situ in the magnetosphere. However, this is a special case of magnetic reconnection which makes it very atypical of generic astrophysical set-ups, where reconnection involves scales many orders of magnitude greater than the gyroradius involved. Even in this case, we may expect the development of turbulence, but this would not be MHD turbulence which makes LV99 theory not applicable to it. Conversely, it can be shown by exact analytical estimates [35] that the direct effect of the microscopic plasma non-idealities are negligible for reconnection at scales vastly larger than ion gyroradius.

13Plasma turbulence may be still important for such reconnection, but this type of turbulent reconnection is not described by the LV99 model.
Acknowledgements. A.L. thanks Eric Priest for stimulating exchanges.

Funding statement. A.L.’s research is supported by NSF grant AST 1212096, Vilas Associate Award and also award at the UFRN (Natal). G.K. acknowledges support from FAPESP (project nos. 2013/0473-2 and 2013/18815-0).

References


125. Maron J, Chandran BD, Blackman E. 2004 Divergence of neighboring magnetic-field lines and fast-particle diffusion in strong magnetohydrodynamic turbulence, with application


173. Higashimori K, Hoshino M. 2012 The relation between ion temperature anisotropy and


