Error analysis of tomographic reconstructions in the absence of projection data

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Error estimates for tomographic reconstructions (using Fourier transform-based algorithm) are available for cases where projection data are available. These data are used for reconstructions with different filter functions and the reliability of these reconstructions can be checked as per guidelines of those error estimates. There are cases where projection data are large (in gigabytes or terabytes) so storage of these data becomes an issue. It leads to storing of only the reconstructed images. Error estimation in such cases is presented here. Second-level projection data are calculated from the given reconstructed images ('first-level' images). These ‘second-level’ data are now used to generate ‘second-level’ reconstructed images. Different filter functions are employed to check the fidelity of these ‘second-level’ images. This inference is extended to first-level images in view of the characteristics of the convolution operator. This approach is validated with experimental data obtained by the X-ray micro-CT scanner installed at IIT Kanpur. Five specimens (of same material) have been scanned. Data are available in this case thus we have performed a comparative error estimate analysis for the ‘first-level’ reconstructions (data obtained from CT machine) and second-level reconstructions (data generated from first-level reconstructions). We observe that both approaches show similar outcome. It indicates that error estimates can also be applied to images when data are not available.

1. Introduction

Two- and three-phase flow systems are commonly studied in various engineering applications. Void-fraction is one of the important parameters to characterize the behaviour of such flow systems. A detailed analysis is important for design and maintenance aspects in several
industries. Several invasive techniques have already been used to determine the phase distributions in pipes having multi-phase flows [1,2]. These techniques disturb the flow field, thus it is advantageous to use non-invasive techniques for flow measurements. Computerized tomography (CT) is one of such technique. It is commonly used in medical diagnostic areas and it also has a large number of applications in other industries. This technique is used for identification of flow patterns and various other parameters [3–8].

Seshadri et al. [9] have done a sensitivity analysis on filter functions to improve the reconstruction of a two-phase flow cross section. This analysis includes conversion of fan-beam data to parallel-beam by rearranging it. Munshi et al. [10] have successfully implemented a direct fan-beam reconstruction algorithm to get reasonably good results. These results are further enhanced with improved higher order Fourier filters [11]. Flow cross section changes with time of measurement. A computerized tomographic scanner rotates around the flow channel to collect data for several views. Dynamic bias (DB) error arises in reconstructions if the speed of the scanning system is slower than the flow speed. Accurate measurements can be obtained by eliminating this DB error. Jayakumar & Munshi [12] have done a study on statistical uncertainty and DB error for a liquid metal magneto hydrodynamic (LMMHD) system. It was shown there that DB error increases with air fraction and the maximum error is observed when the air fraction ranges from 0.5 to 0.6. This DB error decreases with further increase in air fraction. A DB correction scheme is proposed by Shakya and colleagues [13,14]. This scheme was tested on simulated and real data (three-phase bubble column reactor). The suggested scheme is compared with another two existing correction methods [15]. Precise reconstructions, thus, can be obtained from a time-dependent projection data.

The DB-corrected CT images can now be used for characterization of flow cross sections, and help in predicting flow distributions and patterns. Such a characterization step, based on first the Kanpur theorem (KT-1) approach, is performed on a three-phase bubble column reactor [16]. This KT-1 approach is based on the Sobolev space concept [17], and was proposed by Munshi and colleagues [18–20]. The bubble column reactor and CT set-up was installed at Leibniz University, Hannover. The three phases used in the study were air, water and PVC particles. The selected cross section was at a height of 1.7 m from the sparger. This KT-1 approach helped in identifying ‘good’ projection data and it also helped in predicting the transition from a homogeneous to a heterogeneous flow regime. Atha et al. [21] have revisited this bubble column for a different height of 3.2 m. Two different approaches, ‘KT-1’ and ‘fractals’, were used to characterize the flow cross sections at that height. Both approaches led to a similar outcome. A comparative analysis for two levels, 1.7 and 3.2 m, was performed by Shakya et al. [22] by using a global ‘KT-2’ approach. This approach was proposed by Munshi et al. [23]. It helps in characterizing cross sections in a ‘global’ manner in comparison to ‘local’ formulation of the KT-1 approach.

We have used the results here with dynamic bias error correction. The Fourier transform-based convolution back-projection (CBP) algorithm [24] is used for image reconstruction at X-ray energies of 60 and 200 keV. This CBP algorithm uses different filter functions which lead to different reconstructions. Munshi [25] have presented a selection criterion to choose the right solution from the large number of mathematically correct solutions based on KT-1 signatures. We have shown results with two filter functions, one defined as sharp and the other defined as smooth. First-level KT-1 is applied to check noise in the data. We have also investigated the behaviour of second-level KT-1 signatures for cases when projection data are not available. Such cases arise due to the extreme requirement of storage space for projection data which are a few gigabytes when compared with a few megabytes jpegs for reconstructed images. Another application would be to cross-check the fidelity of multi-phase flow CT images without having access to raw projection data.

2. Experimental details

An experiment was performed at Leibniz University Hannover, Germany [14,16,21]. It involved the study of a bubble column reactor which had air, PVC and water as three phases.
Another experiment (for two-phase flow simulation) was performed recently on a mini-CT scanner (Procon X-ray GmbH) installed at IIT Kanpur (http://www.procon-x-ray.de/; http://www.iitk.ac.in/net/ct_mini_webversion.swf). This CT system uses a micro focus X-ray tube of 7 µm focal spot, and it provides three-dimensional projection data. An attenuated X-ray radiation beam is detected on a flat panel detector of 1024 × 1024 photo-diodes. The detector area is 12.1 × 12.1 cm² and it makes a cone beam angle of ±7.9° on the X-ray source. The source-to-detector distance is 46.64 cm. The source-detector system is fixed and the object rotation facility is available in this CT system. A photograph of this set-up is shown in figure 1. More details are given in appendix A.

Figure 2a–e shows photographs of five specimens which are made of Perspex. Specimens 1–3 have 17 holes with different distributions (figure 2a–c). Specimen 4 has 16 holes (figure 2d) and Specimen 5 has 14 holes (figure 2e). There are some manufacturing issues that show up in these photographs. The markers p1–p8 represent the spots that are not holes. Diameters of these specimens and holes are 2 cm and 1 mm, respectively. Different distributions of holes, in
five specimens, represent the flow cross sections at different times of measurement. We have presented here the case of nearly uniform distribution of holes across the cross section. The cross-sectional area of voids is nearly the same for all specimens. It is assumed that bubbles are moving in a vertically upward direction and they appear at different positions (on the cross section) at different times. Specimens were kept at a distance of 10.7 cm (pixel resolution of 25 µm) from the source. The peak energy of the X-ray source is fixed at 110 keV and detector time is selected as 450 ms. The intensity of X-ray radiation is measured with 16-bit data resolution. Data are collected for 400 projections views.

We now perform the sensitivity analysis for the choice of filter function in §3. Section 4 will include the first- and second-level KT-1 analyses to check the fidelity of projection data or reconstructions.

3. Sensitivity analysis for the choice of filter

Reconstruction with the CBP algorithm involves the use of filter function. The choice of filter function plays an important role in the reconstruction step. The selection procedure of filter function depends upon the density distribution across the cross section. It also depends upon the features which the user wants to extract from the reconstructed image. We have done a sensitivity analysis on the choice of filter function. Images have been reconstructed with different filters listed in table 1. We have done this exercise for simulated data in this section.

We have chosen the following three cyber phantoms having different natures of density distribution:

(i) Cyber phantom 1. It is similar to the specimens shown in figure 2. It has a density of 0.2 g cm\(^{-3}\) and 16 holes of density 0 g cm\(^{-3}\) as shown in figure 3a.

(ii) Cyber phantom 2. It is a circular cross section with diameter 256 units (density value of 1 g cm\(^{-3}\) which is equivalent to density value of water) having a large number of small holes of diameter ranging from 3 to 4 units (density value of 0.0012 g cm\(^{-3}\) which is
Figure 4. Reconstructions of (a) cyber phantom 1, (b) cyber phantom 2 and (c) cyber phantom 3 using the H54 filter. (Online version in colour.)

Figure 5. Reconstructions of (a) cyber phantom 1, (b) cyber phantom 2 and (c) cyber phantom 3 using the H99 filter. (Online version in colour.)

Table 2. Root mean squared error for reconstructions with filters H54 and H99.

<table>
<thead>
<tr>
<th>Phantom</th>
<th>RMSE H54</th>
<th>RMSE H99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cyber phantom 1</td>
<td>0.0247</td>
<td>0.0261</td>
</tr>
<tr>
<td>Cyber phantom 2</td>
<td>0.2196</td>
<td>0.2325</td>
</tr>
<tr>
<td>Cyber phantom 3</td>
<td>0.2285</td>
<td>0.2420</td>
</tr>
</tbody>
</table>

equivalent to the density value of air). This phantom represents a two-phase flow system (air–water), and it is shown in figure 3b.

(iii) Cyber phantom 3. It is also a circular cross section with diameter 256 units (density value of 1 g cm\(^{-3}\) which is equivalent to the density value of water) with small circles (two different densities of 0.0012 g cm\(^{-3}\) and 1.38 g cm\(^{-3}\)) of diameter ranging from 3 to 4 units. Chosen density values are the densities of air (0.0012 g cm\(^{-3}\)), water (1 g cm\(^{-3}\)) and polyvinyl chloride (1.38 g cm\(^{-3}\)). This three-phase flow system is depicted in figure 3c.

We, here, have shown the results for two filters only, H54 (smooth) and H99 (sharp). These filters are the popular Hamming filter and the approximate Ram-Lak filter. These two filters represent the two extremes of the filtering scheme. Other images are not shown here but they are included in a representative form in KT-1 graphs. Projection data are generated for 1024 rays and 400 views. Figure 4a–c is the reconstructions of cyber phantom 1, cyber phantom 2 and cyber phantom 3 by using the H54 filter. The corresponding reconstructions using the H99 filter are displayed in figure 5a–c. Table 2 shows root mean squared errors (RMSEs) for the reconstructions of H99 and H54. It is noted that the cross section, having uniform density distribution and small density variance (0 for holes and 0.2 for base matrix), shows less error as in case of cyber phantom 1. The other two cyber phantoms show comparatively large errors. This is due to large variations in density profiles (0.0012, 1 and 1.38) and non-uniform distributions of particles.
Figure 6. Cyberphantom 1 (a) original, (b) reconstruction with H99 filter and (c) second-level reconstruction with H99. (Online version in colour.)

4. Second-level Kanpur theorem-1

The first Kanpur theorem was used earlier for characterization/quantification of reconstructed images. It is applied when projection data are available. These data are used to reconstruct the image with different filter functions. Reciprocals of maximum grey level values of these reconstructed images are used as an error indicator. These values are plotted against the corresponding second derivatives of window functions at Fourier space origin. A good linear fit indicates that projection data are good.

The second level of the KT-1 signature is applied when the image is available, but projection data are not available. It is calculated from the available CT images. These ‘second-level’ projection data are now used to reconstruct ‘second-level’ CT images with different filter functions. The results obtained from these images are referred to as second-level KT-1 signatures. Images reconstructed from the sharp Hamming filter (H99) are used to calculate the ‘second-level’ projection data in this study. All the ‘second-level’ CT images displayed in this study are reconstructed with the H99 filter.

(a) Simulation

First- and second-level KT-1 signatures are obtained for the above-defined three cyber phantoms. Figure 6a depicts the original cyber phantom 1 which represents a linear combination of a limited number of approximate Dirac-delta functionals. Its first-level reconstruction with the H99 filter is shown in figure 6b. Figure 6c is the second-level reconstruction from the projection data calculated by using the image given in figure 6b. It is noted that the second-level reconstruction is close to the original phantom. The reconstructed density values do not show much difference in first- and second-level reconstructions. This is because the phantom does not have a large number of density values. First- and second-level KT-1 signatures of this phantom are shown in figure 7. The x-axis of these plots is the second-order derivative of the window function at the Fourier space origin, and y-axis is the reciprocal of maximum grey level value ($n_{\text{max}}$) of the reconstructed image. This $n_{\text{max}}$ value is used here as a characterization tool for error [26]. The parameters of KT-1 signatures are given in table 3. Slope value is doubled for the second-level KT-1 but the linear fits are good for both the signatures. We note that the H99 filter is chosen here for the second-level case because the nature of bubbles is inherently ‘sharp’, implying sudden change in density from 0 to 1 g cm$^{-3}$.

Figure 8a–c is the original cyber phantom 2, first- and second-level reconstructions with the H99 filter, respectively. This phantom is a linear combination of several Dirac-delta functionals. The reconstructed density values are close to the original phantom for both first- and second-level reconstructions. This phantom also has only two density values, thus the projection data generated from this image are more accurate and close to the original. First- and second-level KT-1 signatures corresponding to this cyber phantom 2 are displayed in figure 9. The corresponding
Figure 7. KT 1 signatures of cyber phantom 1. (Online version in colour.)

Figure 8. Cyber phantom 2 (a) original, (b) reconstruction with the H99 filter and (c) second-level reconstruction with the H99. (Online version in colour.)

Figure 9. KT 1 signatures of cyber phantom 2. (Online version in colour.)

Table 3. Parameters of first- and second-level KT-1 signatures of cyber phantom 1, cyber phantom 2 and cyber phantom 3.

<table>
<thead>
<tr>
<th>phantom</th>
<th>slope</th>
<th>intercept</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>cyber phantom 1</td>
<td>0.3078</td>
<td>0.6661</td>
<td>0.0031</td>
</tr>
<tr>
<td>cyber phantom 2</td>
<td>0.0853</td>
<td>0.0861</td>
<td>0.0015</td>
</tr>
<tr>
<td>cyber phantom 3</td>
<td>0.0915</td>
<td>0.2065</td>
<td>0.0004</td>
</tr>
</tbody>
</table>
parameters are provided in table 3. Slope and intercept values do not show much difference and the linear fits are good.

Cyber phantom 3, representing a three-phase flow, is displayed in figure 10a. Its first- and second-level reconstructions are given in figure 10b,c, respectively. We observe that the second-level reconstructed density values are comparatively smaller than the original and the first-level reconstruction. Its first- and second-level KT-1 signatures are displayed in figure 11. It is noted that instrumentation noise is absent in the simulation case, thus all the KT-1 signatures show a good linear fit.

(b) In-house data for Perspex specimen representing two-phase flow

This section includes the first- and second-level KT-1 analyses for real-world data. Figure 12a–e represents the first-level reconstructions of five specimens (given in figure 4). Second-level
projection data are now calculated from these first-level reconstructions. These data are now used for second-level reconstructions and these images are shown figure 13a–e for five specimens.

First- and second-level KT-1 signatures for five specimens are displayed in figure 14a–e. The corresponding parameters are given in table 4. All show a good linear fit; therefore, these
signatures can be used for characterization purposes. It is noted that the change in slope and intercept values (for five specimens) is similar in both the first and the second levels. The values, however, are quite different due to the difference in reconstructed density values at second level. A small dust particle appears in specimen 1 at the boundary of one of its hole. This particle has a larger density (approx. 0.8) and it leads to different slope values in comparison to other specimens. This dust particle ‘separates’ specimen 1 from the other four specimens. The first-level KT-1 check, in the absence of projection data, cannot be performed but second-level analysis is possible in such cases. We see that second-level KT-1 slope is also able to ‘separate’ specimen 1. This second-level analysis, therefore, can be used to identify the presence of some unknown material/particle in the cross section whenever CT scanner software does not permit extraction of projection data.

We note that intercept values, in the case of second-level KT-1 signatures, are smaller and this behaviour is similar to the simulation case. This information supplements the inferences drawn on the basis of slope values.

(c) Hannover data for multi-phase flow

We have presented a KT-1 analysis for simulation and static objects in previous sections. We, now, discuss it for dynamic objects. Sections 4c(i),(ii) include the results for a two- and a three-phase bubble column, respectively. Attenuation coefficients of different materials (air, water and PVC), at different X-ray energies, are provided by Hubbell [27].

(i) Two-phase case

Air and water were used as two phases in a bubble column. Water velocity was fixed at 0.07 m s$^{-1}$ and air velocities were 0.06, 0.08, 0.12, 0.14 m s$^{-1}$. The bubble column was scanned at two different heights 1.7 and 3.2 m. Projection data were collected for X-ray energy of 60 keV. We note that the density values are the same and bubble distribution is similar as in the case of cyber phantom 2.

First-level reconstructed phase fractions of air, for the 1.7 m level, are displayed in figure 15a–d. These images are reconstructed with filter H99. First-level reconstructed cross sections, for the 1.7 m level, are used to calculate second-level projection data. These data are now used to reconstruct the second-level images with different filter functions. We show the second-level images reconstructed in figure 16a–d. It is observed that second-level reconstructions show relatively smaller air fractions than first-level images.

Air fractions, for the 3.2 m level, are displayed in figure 17a–d. These images are reconstructed from the experimental data obtained directly from the CT scanner for both heights. Displayed images are reconstructed with H99 filter function. Second-level projection data are calculated from the first-level reconstructions. Reconstructed second-level images are displayed in figure 18a–d. The second-level phase fractions are relatively less but the trend of increasing phase fraction with air velocity is, however, the same.

Second-level reconstructed density values are relatively small in comparison with the original (first-level reconstruction). These reconstructions and corresponding KT-1 signatures.
Figure 16. Two-phase case: second-level phase fractions of air at the 1.7 m level for air velocities of (a) 0.06 m s\(^{-1}\), (b) 0.08 m s\(^{-1}\), (c) 0.12 m s\(^{-1}\) and (d) 0.14 m s\(^{-1}\) reconstructed with the H99 filter. (Online version in colour.)

Figure 17. Two-phase case: first-level phase fractions of air at the 3.2 m level for air velocities of (a) 0.06 m s\(^{-1}\), (b) 0.08 m s\(^{-1}\), (c) 0.12 m s\(^{-1}\) and (d) 0.14 m s\(^{-1}\) reconstructed with the H99 filter. (Online version in colour.)

Figure 18. Two-phase case: second-level phase fractions of air at the 3.2 m level for air velocities of (a) 0.06 m s\(^{-1}\), (b) 0.08 m s\(^{-1}\), (c) 0.12 m s\(^{-1}\) and (d) 0.14 m s\(^{-1}\) reconstructed with the H99 filter. (Online version in colour.)

are, however, presented for characterization of a given first-level image. First- and second-level KT-1 signatures, for the 1.7 m level, are displayed in figure 19a–d. Their corresponding parameters (slope, intercept and goodness of fit) are given in table 5. Slope and intercept values for second levels are comparatively larger. We do not observe a clear pattern in slope values but intercept values show a decreasing trend with increases in air velocity. Phase fraction values are comparatively smaller than the first-level reconstructions, thus the corresponding \( n_{\text{max}} \) values are small. It leads to large values of reciprocals of \( n_{\text{max}} \) and the corresponding intercepts. All signatures (both first and second levels) show good linear fits, which indicates the reliability of first-level reconstructions.

We now present the first- and second-level KT-1 signatures for the upper measuring level, i.e. 3.2 m. These signatures are shown in figure 20a–d for air velocities of 0.06, 0.08, 0.12 and 0.14 m s\(^{-1}\). The corresponding parameters are given in table 6. All slope values of the second level are comparatively large except for the large air velocity of 0.14 m s\(^{-1}\). This indicates that second-level reconstructions show comparatively high-frequency content. Intercept values are also relatively large for all the air velocities. It was reported earlier that intercept values can be used as an indicator for X-ray absorption [18]. Air fraction is reduced in second-level reconstruction
(ii) Three-phase case

Air, water and PVC were used as three phases in this analysis. The concentration of PVC was 5% of the total volume of the bubble column. Flow rates were the same as in the case of two-phase flow and the column was scanned at heights of 1.7 and 3.2 m. Projection data were collected at two different X-ray energies of 60 and 200 keV to solve a three-phase flow case.

Figure 21a–d shows the phase fractions of air for the 1.7 m level. The Hamming filter, H99, is used for these first-level reconstructions. Air bubbles are distributed uniformly for the low air

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**Figure 19.** Two-phase case: first- and second-level KT-1 signatures for phase fractions of air at the 1.7 m level for air velocities of (a) 0.06 m s$^{-1}$, (b) 0.08 m s$^{-1}$, (c) 0.12 m s$^{-1}$ and (d) 0.14 m s$^{-1}$. (Online version in colour.)

**Table 5.** Parameters of first- and second-level KT-1 signatures for phase fractions of air at 1.7 m level for air velocities of 0.06, 0.08, 0.12 and 0.14 m s$^{-1}$.

<table>
<thead>
<tr>
<th>air velocity</th>
<th>slope first level</th>
<th>slope second level</th>
<th>intercept first level</th>
<th>intercept second level</th>
<th>RMSE first level</th>
<th>RMSE second level</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06</td>
<td>0.6501</td>
<td>0.7410</td>
<td>1.9150</td>
<td>2.2170</td>
<td>0.0134</td>
<td>0.0080</td>
</tr>
<tr>
<td>0.08</td>
<td>0.6474</td>
<td>0.8006</td>
<td>1.8000</td>
<td>2.1880</td>
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<td>0.0158</td>
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<tr>
<td>0.12</td>
<td>0.5146</td>
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<td>1.6240</td>
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<td>0.14</td>
<td>0.4714</td>
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<td>1.5280</td>
<td>1.7910</td>
<td>0.0040</td>
<td>0.0063</td>
</tr>
</tbody>
</table>

and the intercept values are increased. We also observe that the linear fits are good; therefore, reconstructed first-level images can be used for interpretation.
velocity case of 0.06 m s\(^{-1}\) while they are more concentrated towards the centre for other high air velocities. Air fraction is reduced here in comparison with the two-phase flow case as a solid fraction is introduced to the bubble column. Second-level reconstructions of air fractions are given in figure 22\(a\)–\(d\). These images are reconstructed from the projection data which are obtained from first-level reconstructions. The difference is comparatively less (in comparison with two-phase flow case) here in first- and second-level reconstructions.

Air fraction, at the 3.2 m level, is comparatively high in comparison with the 1.7 m level. It is seen in first-level reconstructed images shown in figure 23\(a\)–\(d\). The corresponding second-level reconstructions are displayed in figure 24\(a\)–\(d\). Second-level projection data were calculated from the images reconstructed with the H99 filter and the displayed images are also reconstructed with
Figure 21. Three-phase case: first-level phase fractions of air at the 1.7 m level for air velocities of (a) 0.06 m s$^{-1}$, (b) 0.08 m s$^{-1}$, (c) 0.12 m s$^{-1}$ and (d) 0.14 m s$^{-1}$ reconstructed with the H99 filter. (Online version in colour.)

Figure 22. Three-phase case: second-level phase fractions of air at the 1.7 m level for air velocities of (a) 0.06 m s$^{-1}$, (b) 0.08 m s$^{-1}$, (c) 0.12 m s$^{-1}$ and (d) 0.14 m s$^{-1}$ reconstructed with the H99 filter. (Online version in colour.)

Figure 23. Three-phase case: first-level phase fractions of air at the 3.2 m level for air velocities of (a) 0.06 m s$^{-1}$, (b) 0.08 m s$^{-1}$, (c) 0.12 m s$^{-1}$ and (d) 0.14 m s$^{-1}$ reconstructed with the H99 filter. (Online version in colour.)

Figure 24. Three-phase case: second-level phase fractions of air at the 3.2 m level for air velocities of (a) 0.06 m s$^{-1}$, (b) 0.08 m s$^{-1}$, (c) 0.12 m s$^{-1}$ and (d) 0.14 m s$^{-1}$ reconstructed with the H99 filter. (Online version in colour.)

This filter. It is noted that the air fraction, for air velocity of 0.06 m s$^{-1}$, is negligible in the second-level reconstruction. It is, however, significantly high and bubbles are distributed uniformly (figure 23a) in the first-level reconstruction. Only a few bubbles are visible in the second-level case. The air fraction, however, suddenly rises and it is comparable to the first level for 0.08 m s$^{-1}$ air velocity. The trend of increasing air fraction with increases in air velocity remains the same for both first- and second-level reconstructions.
Figure 25. Three-phase case: first- and second-level KT-1 signatures for phase fractions of air at the 1.7 m level for air velocities of (a) 0.06 m s$^{-1}$, (b) 0.08 m s$^{-1}$, (c) 0.12 m s$^{-1}$ and (d) 0.14 m s$^{-1}$. (Online version in colour.)

Table 7. Parameters of first- and second-level KT-1 signatures for phase fractions of air at the 1.7 m level for air velocities of 0.06, 0.08, 0.12 and 0.14 m s$^{-1}$.

<table>
<thead>
<tr>
<th>Air Velocity</th>
<th>Slope first level</th>
<th>Slope second level</th>
<th>Intercept first level</th>
<th>Intercept second level</th>
<th>RMSE first level</th>
<th>RMSE second level</th>
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<td>0.06</td>
<td>1.0827</td>
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<td>2.1949</td>
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<td>0.0491</td>
<td>0.0459</td>
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</tbody>
</table>

We now present the first- and second-level KT-1 signatures for phase fractions of air for different air velocities. These signatures, for the 1.7 m level, are depicted in figure 25a–d. Their parameters are given in table 7. The slope values are large for the second level as was the case for two-phase flow systems. These values are also large in comparison to the slope values of the air fraction in the two-phase flow case. It indicates that high frequencies are introduced due to the presence of a third phase as solid particles. The air fraction is reduced from first-level reconstructions. An increase in intercept values, for the second level, is also observed. Linear fits for all signatures are good; therefore, the presented results are acceptable in the given form for any type of analysis.

Phase fractions of air, in first-level cases, are comparatively high at the 3.2 m level for all air velocities. A similar trend is observed here also in second-level cases except for an air velocity
of 0.06 m s$^{-1}$. We see (figure 24a) that a few bubbles were observed for this case. It leads to a large KT-1 slope for the second level as shown in figure 26a. Figure 26b–d shows the first- and second-level KT-1 signatures for air velocities of 0.08, 0.12 and 0.14 m s$^{-1}$. It is noted that the range for the $y$-axis is the same for all these three air velocities. It is, however, different for an air velocity of 0.06 m s$^{-1}$ due to a comparatively large slope. Table 8 represents the parameters of KT-1 signatures. It is clearly noticeable that the fit is also not good for an air velocity of 0.06 m s$^{-1}$. We cannot make any conclusions about the flow cross section for this case and we neglect it. We see that, out of several real datasets, based on second-level KT-1 signatures (16 for two/three-phase flow data and five for IITK micro-CT data), only one case (approx. 4%) is found where the results are observed to be very far from the first level. All other linear fits are good and the results are acceptable as per the KT-1 criterion.

![Figure 26. Three-phase case: first- and second-level KT-1 signatures for phase fractions of air at the 3.2 m level for air velocities of (a) 0.06 m s$^{-1}$, (b) 0.08 m s$^{-1}$, (c) 0.12 m s$^{-1}$ and (d) 0.14 m s$^{-1}$. (Online version in colour.)](http://rsta.royalsocietypublishing.org/)

Table 8. Parameters of first- and second-level KT-1 signatures for phase fractions of air at the 3.2 m level for air velocities of 0.06, 0.08, 0.12 and 0.14 m s$^{-1}$.

<table>
<thead>
<tr>
<th>air velocity</th>
<th>slope intercept</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>first level</td>
<td>second level</td>
</tr>
<tr>
<td>0.06</td>
<td>0.8275</td>
<td>9.0808</td>
</tr>
<tr>
<td>0.08</td>
<td>1.0990</td>
<td>1.6510</td>
</tr>
<tr>
<td>0.12</td>
<td>1.2041</td>
<td>1.6280</td>
</tr>
<tr>
<td>0.14</td>
<td>0.8600</td>
<td>0.8330</td>
</tr>
</tbody>
</table>
5. Conclusion

A major finding of this work is the suitability of the second-level KT-1 approach for characterizing different flow situations whenever projection data are not available.

Other features of this study are given as follows:

— performance of the IIT Kanpur CT scanner is evaluated using the KT-1 route. Reconstructions with different filters do not show much difference in cases of uniformly distributed cross sections. The RMSE is observed to be approximately 0.02 for simulation cases;
— the difference, for the reconstructions with sharp and smooth filters, is significant in the case of cross sections which have large range of density values with non-uniform distribution. The RMSE is observed to be approximately 0.23 for two- and three-phase simulation cases. Sharp filters are recommended for such flow conditions;
— second-level reconstructions, in the case of IITK data and similar simulated data, show slightly larger density values in comparison with first-level density values;
— air fractions, in second-level cases, are relatively smaller for both two- and three-phase cases; and
— inferences drawn from the second-level KT-1 approach are consistent with the first-level KT-1 approach in cases of simulated data. This fact gives confidence that second-level analysis with real-world data has good reliability.

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Appendix A

An experiment was performed, at the Gottfried Wilhelm Leibniz University Hannover, Germany, to analyse the phase distribution in a three-phase bubble column and this experiment was done and reported by Behling [28]. Some salient features of the data collection process and related instruments are described here for convenience.

An X-ray source was used for scanning purposes and the scanner was selected with variable voltage to provide dual energy X-ray photons. The focal spot size of the X-ray tube was 3 mm, which allows sufficiently fine resolution for this analysis. A 1 mm beryllium filter was mounted on the X-ray tube and a 0.75 mm brass filter was placed in between the X-ray source and the target so that beam-hardening artefacts are minimal.

Scintillation detectors of 384 pixels were used to detect attenuated X-ray radiations. The pixel-width of these detector pixels was 1.6 mm. The distance between source and object centre was 660 mm and the distance between object centre and detector system was 675 mm. This arrangement of source, bubble column and the detector system is shown in figure 27. The height of this bubble column was 7.5 m. The inner and outer diameters \((D)\) were 244 mm and 250 mm, respectively. The wall of the bubble column was composed of PMMA (Perspex, acrylic glass) and the thickness of this wall was 3 mm. The emitted radiation beam was collimated (by lead-based units) to acquire a fan-beam angle of 26°. The detector system was fully illuminated with this fan-beam angle while the bubble column was fully exposed to X-ray radiations with a fan-beam angle of 22° only. The source-detector system was rotated around the bubble column to get projection data for 360 rotations. The total acquisition time to collect all 360 projections was under 75 s. Projection data, obtained from the scanner, are normalized with offset measurements (no X-ray radiations) and gain measurements (empty bubble column exposed to full X-ray radiations). This exercise was done to eliminate the X-ray attenuation due to the presence of air along the data-ray. The intensity of X-ray radiation was measured with a 16-bit data resolution which distinguishes...
65,536 different intensity levels. This fact helps in detecting air bubbles and PVC particles of 0.1 mm and correspondingly low phase fractions in the bubble column.

Projection data were collected for two different X-ray energies: 60 and 200 keV. The attenuation property of an object is significantly different for these two energy regions which help in calculating the three phases separately. The three phases used for this study were water, air and PVC granules. Air supply was via 54 holes of 1 mm diameter at the bottom of the bubble column. These holes were distributed uniformly over the entire cross section. Water supply was via seven nozzles of 16 mm diameter. These nozzles were secured with a wire mesh to avoid falling PVC granules. Individual granular PVC particles were cylindrical in shape with diameter varying from 3 to 3.5 mm and their height ranged from 2 to 3 mm.

References


