New geometric concepts in the foundations of physics

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There has been considerable renewed interest in the foundations of physics in recent years. The reasons for this are manifold, and each researcher involved has his or her own motivation. But why bother with the foundations of physics at all? Our current physical theories, the standard model of particle physics and general relativity, are extremely successful. Experiments and observations, most notably at the Large Hadron Collider (LHC) at CERN and in the form of greatly refined astrophysical observations, have so far failed to provide any clear hints of new physics that would point to the need for new theories. The predictions of the standard model and of general relativity have been confirmed again and again. Yet, despite the great phenomenological successes, there remain serious conceptual problems. Finding a unified theory, or at least a quantum theory of gravity, has proved to be an extraordinarily thorny task. Moreover, the relevant quantum field theories in themselves are plagued by a host of mathematical difficulties which still prevents a mathematically and conceptually clear formulation, and general relativity quite directly predicts its own breakdown in the form of singularities. Many speculative extensions of the standard model that were proposed in the last 20 years have already been ruled out by LHC. The fact that many suggestive and ingenious, yet ‘conservative’ modifications and extensions of the existing theories failed to provide correct experimental predictions suggests—at least to some researchers—that a radical rethinking of the foundations of physics and the basic structure of physical theories is required if we want to overcome the problems of our existing theories and make progress towards a unified theory of physics. Hence the motivation to (re)consider the foundations of physics.

Historically, geometry has always played a key role in the formulation and understanding of physical
theories, and the concepts of what a physical geometry is have extended considerably over time. Newton’s and Leibniz’s theories unified celestial and terrestrial mechanics and geometry, which was still firmly rooted in everyday spatial and temporal experience. These theories required a serious treatment of infinitesimals, which laid the foundations for differential geometry that underlies Hamiltonian and Lagrangian mechanics and many further physical theories. Maxwell’s theory of electrodynamics is another example of a theory with a very strong geometric flavour, describing interacting fields on space–time. Electrodynamics revealed that the geometry of space–time is not the classical, Euclidean one, but Lorentzian geometry, as was properly recognized by Einstein, who applied this insight in the formulation of special relativity. An even more radical step was taken by Einstein when proposing general relativity, which is a geometric theory through and through, although this is sometimes obscured by rather inelegant presentations. In general relativity, the geometry of space–time itself, in the form of the metric, becomes dynamical and interacts with matter. As is well known, Einstein failed to realize his dream of finding a unified, geometric theory of all of physics, but this ambitious goal has influenced many researchers subsequently. Quantum theory did not start out as a mainly geometric theory, but when the step to quantum field theories was taken, it was realized gradually that gauge theories, which are fundamentally geometric and encode internal symmetries, are needed to describe all the non-gravitational interactions. While a global gauge symmetry was already known from electrodynamics, local gauge symmetries are a new ingredient in Yang–Mills theories, which underlie our successful quantum field theories. (The step from classical Yang–Mills theories to their quantized versions, which describe nature so well, contains many mathematical challenges.) Apart from the gauge principle, locality principles, which encode the constraints imposed by Lorentzian geometry, play an important role in the formulation of quantum field theories, and many further geometric ideas enter into the formulation of our current best theories.

In November 2013, the Royal Society funded a workshop on ‘New geometric concepts in the foundations of physics’ at Chicheley Hall. This workshop, organized by Andreas Döring and Tim Palmer, brought together a number of researchers, some very well established and some young, who are all interested in the foundations of physics, and who mostly incorporate new geometric ideas into their foundational work. A broad range of approaches was represented at the workshop, from the mostly mathematical to the phenomenological. Many contributors presented novel, non-mainstream ideas, and lively discussions ensued. Subsequently, we suggested to *Philosophical Transactions A* to publish a themed issue with the same title as the Chicheley workshop. This issue is not a ‘mere’ proceedings, though, but also contains a number of contributions by researchers who were not present at the workshop.

Calmet [1] considers a potential energy dependence of Planck’s constant $\hbar$ and the corresponding modified quantization rules from a phenomenological point of view. The fact that Newton’s ‘constant’ $G_N$ is energy-dependent and becomes renormalized (in a tentative quantum theory of the gravitational interaction) suggests considering a potential energy dependence also of $\hbar$. It turns out that at low energy of $ca$ 100 MeV, the anomalous magnetic moment of the muon puts a rather strict limit on the energy dependence of Planck’s constant, while arguments from inflation show that also at very high energies of $ca$ $10^{16}$ GeV, $\hbar$ must be non-zero.

Cruz Morales & Zilber [2] present progress towards a reformulation of key aspects of quantum mechanics using model theory. This approach pays particular attention to (mathematical) structures, number systems and approximations that physicists use, mostly implicitly, in the formulation of quantum theory, and aims to make them explicit and to axiomatize them in model-theoretic terms. Zilber and Cruz Morales develop the outlines of a duality theory between rational Weyl algebras and Zariski geometries and the extension of this duality to algebras that can be approximated by rational Weyl algebras. Here, sheaf theory plays an important role.

Dietrich [3] considers geometric formulations of gauge theories and the effect of introducing mass. While in suitable formulations the massless theory is structurally very similar to Einstein gravity, an additional mass term introduces torsion and the shift to a theory of Einstein–Cartan–Schiama–Kibble type. After introducing the main concepts in a three-dimensional example,
Dietrich also considers four-dimensional theories. In both cases, a mass term leads to an additional antisymmetric part of the connection. Some relations to higher gauge theory are discussed.

Döring [4] gives a review of some aspects of the topos approach to quantum theory and presents some progress, in particular in the description of time evolution. The topos approach, which aims at a radical reformulation of quantum theory using topos theory, provides a generalized state space for a quantum system, given by the spectral presheaf. In this formulation, non-relativistic quantum theory becomes structurally very similar to classical Hamiltonian mechanics. In particular, time evolution of quantum systems is described by Hamiltonian flows on the quantum state space.

Fewster [5] presents the framework of locally covariant quantum field theories and discusses physical equivalence of such theories. A theory of this kind is a particular functor, and two theories are equivalent if and only if the functors are naturally isomorphic. Criteria for the equivalence within a more restricted class of theories, i.e. those which obey the time-slice axiom, are also discussed, and finally the question of when a (single) theory represents the same physics in all space–times (SPASs) is considered. This intuitive concept is formalized in the SPASs property, and the class of dynamically local theories is shown to have this property.

Hardy [6] presents an operationally motivated formulation of aspects of quantum theory in terms of the so-called bold tensors. Bold operator tensors are lists of operator tensors all acting on the same Hilbert space. They serve to encode physical operations, which can be seen as applications of apparatuses and can be described in a graphical notation, which allows an intuitive way of composition into circuits. Imposing a physicality condition singles out quantum theory among the potential theories that can be described by such circuits, and probability distributions for circuits can be calculated in a straightforward manner.

Hodges [7] gives a review of the recent uses of twistor geometry in the theory of scattering amplitudes. After a beautiful introduction to the original ideas by Penrose, Hodges goes on to discuss the Parke–Taylor amplitude from gauge field theory and its connections with twistor geometry, which were gradually recognized and led to many further developments by Arkani-Hamed, Cachazo and collaborators, and by Hodges himself, in particular from 2008 onwards. Throughout his lucid review, Hodges emphasizes the role of conformal symmetry (and its breaking) in gauge theories and in gravity.

Kent [8] presents progress in his programme of formulating a realist version of relativistic quantum theory. The key new idea consists in considering beables at any point in space–time which are defined using only final outcome data from outside of that point’s future light cone. This is motivated by the intuition that any massive quasi-classical object leaves effective records of its location outside its future light cone by its possible interactions with photons and other massless particles. The proposal is illustrated by two toy models.

Lapidus [9] presents his theory of fractal strings and their complex dimensions and the connections with the Riemann zeta function and the Riemann hypothesis. The so-called spectral operator can be seen as a quantization of the Riemann zeta function. The main result is a new criterion for the Riemann hypothesis: the spectral operator $\alpha_c$ is invertible for every $c \in (0, 1/2)$, if and only if the Riemann hypothesis holds. While the physical interpretation of this result is currently still unclear, Lapidus conjectures that it relates to a phase transition at $c = 1/2$ in a (tentative) underlying quantum field theory.

Palmer [10] discusses his cosmological invariant set postulate and its consequences for the existence of correlations that play a role for Bell’s inequality. In Palmer’s scheme, it is assumed that the state space of the universe is a fractal set, on which the state of the universe moves deterministically. Palmer attempts to show that quantum effects and correlations arise from the constraints given by the fractal geometry of state space. It is shown that a locally causal hidden-variable theory can violate the CHSH inequality without being conspiratorial or retrocausal.

Penrose [11] presents some recent progress on the ‘googly problem’ within the twistor programme. This long-standing problem concerns the description of right-handed interacting massless fields using the same twistor conventions as those giving rise to left-handed fields.
After a very nice introduction to the twistor programme, including twistor cohomology, Penrose develops the idea that the twistor structure is to be defined not on twistor space as usual, but on the non-commutative twistor quantum algebra generated by twistors seen as linear operators. This also leads to an extension of the usual sheaves of holomorphic functions to sheaves including operators of twistor differentiation.

’t Hooft [12] presents aspects of his programme that aims to provide a classical, deterministic, and real universe underlying quantum theory and quantum systems. While a complete description of such a classical universe is out of reach, ’t Hooft discusses some concrete quantum systems and provides classical models that underlie them: the harmonic oscillator, massless chiral fermions and superstrings. He goes on to discuss Bell inequalities and related issues that are hard to capture in a classical model. ’t Hooft argues that by assuming that ontological states in his sense exist and make up the usual wave functions by superposition, seemingly conspiratorial correlations that are needed to circumvent Bell type arguments occur naturally, because altering a few beables on Alice’s side while keeping all beables on Bob’s side is precluded.

The articles collected in this issue represent a variety of mostly non-mainstream approaches to foundational issues of physics. Taken together, they demonstrate the great variety of ideas and techniques entering physics now. Foundations of physics has become a focus of interest not just for philosophers of physics, but also for a number of well-established researchers with a considerable track record and comprehensive technical knowledge. This provides very welcome input for the field. In the articles presented here, there is a clear tendency to go beyond standard quantum theory and to employ a number of mathematical tools that have not been used much in physics so far (category theory, sheaf theory, nonlinear dynamics, etc.). Many contributors use geometric ideas and geometric techniques in a fundamental way. We hope that the results and ideas presented here will inspire other, especially young researchers.

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