Watching surface waves in phononic crystals

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In this paper, we review results obtained by ultrafast imaging of gigahertz surface acoustic waves in surface phononic crystals with one- and two-dimensional periodicities. By use of quasi-point-source optical excitation, we show how, from a series of images that form a movie of the travelling waves, the dispersion relation of the acoustic modes, their corresponding mode patterns and the position and widths of phonon stop bands can be obtained by temporal and spatio-temporal Fourier analysis. We further demonstrate how one can follow the temporal evolution of phononic eigenstates in k-space using data from phononic-crystal waveguides as an example.

1. Introduction

Phononic crystals—periodic structures that scatter acoustic waves—have been the subject of intensive investigation owing to their use in filtering and control of sound [1–13]. In order to characterize phononic crystals and their derivative devices, the measurement of the acoustic field evolution in space is an attractive goal. This allows one to access fundamental properties of phononic crystals such as dispersion relations, phononic stop bands and their eigenstate field distributions, for example.

Following on from a variety of phonon imaging studies [14], time-resolved optical imaging of acoustic waves in solids has been demonstrated using photoelastic techniques [15–18], beam deflection [19,20], holography [21] and interferometry [22–29]. Time-domain imaging of acoustic wave propagation in phononic crystals was first demonstrated in millimetre-scale structures and liquid/solid systems without the use of optics by means of focused transducers at megahertz frequencies [30–32]. More recently, the versatility of time-domain optical interferometry for ultrafast measurements of surface acoustic waves (SAWs) on micrometre-scale solid and solid/air phononic crystals has been demonstrated up to approximately 1 GHz [33–39].
In this paper, we review results obtained using such ultrafast time-domain imaging of the acoustic field on the surface of microscopic phononic crystals in two spatial dimensions in the range around 100 MHz–1 GHz. The imaged area is typically around $100 \times 100 \mu m$, and the lateral spatial resolution, limited by optical diffraction, is approximately $1 \mu m$. We first describe how the spatio-temporal acoustic field can be Fourier analysed to yield the acoustic dispersion relations, in this case the phononic band structure. To this end, we extend our results to one- and two-dimensional surface phononic crystals exhibiting phononic stop bands, including phononic-crystal waveguides, and also demonstrate how time-domain acoustic imaging may be extended to $k$-space.

2. Obtaining the acoustic dispersion relation from time-resolved imaging

Ultrafast time-resolved imaging \cite{33,34,36–39} allows the velocity field representing the out-of-plane surface motion $v(r, t)$ as a function of two-dimensional position $r$ and time $t$ to be obtained. One can relate this to the surface displacement field $u(r, t)$ using Fourier analysis. The corresponding Fourier amplitudes $V(k, \omega)$ and $U(k, \omega)$ are obtained as follows:

$$V(k, \omega) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} v(r, t) \exp\{-i(k \cdot r - \omega t)\} \, d^3r \, dt,$$

where $\omega = \omega(k)$ is the angular frequency of the Bloch harmonic (BH) specified by $G$. The integration over the 1st BZ and the summation over all $G$ can be extended to an integral over $k$-space.

$$u(r, t) = \int_{-\infty}^{\infty} U(k, \omega) \exp\{i(k \cdot r - \omega t)\} \, d^2k \, d\omega.$$

Comparing time derivative of equation (2.4) with equation (2.2), one obtains

$$U(k, \omega) = \frac{i}{\omega} V(k, \omega).$$

In general, the displacement field $u(r, t)$ can be expressed as a superposition of normal modes of the medium. For a sample exhibiting periodicity, the displacement (more rigorously, a component of the displacement vector) corresponding to a normal mode, which can be specified by branch index $j$ and wavevector $k$ in the first Brillouin zone (1st BZ), can be expressed according to Bloch’s theorem as

$$u_{k,j}(r, t) = \text{Re} \left[ \sum_{G} C_j(k + G) \exp\{i((k + G) \cdot r - \omega_j(k)t)\} \right],$$

where $G$ is a reciprocal lattice vector (consisting of a linear combination of integer multiples of unit reciprocal lattice vectors), and $C_j(k + G)$ is the amplitude of the corresponding Bloch harmonic (BH) specified by $G$. The summation covers all possible $G$’s. The angular frequency $\omega_j(k)$ refers to the mode specified by $j$ and $k$, and represents part of the dispersion relation. We have assumed completeness and orthogonality of the eigenmodes. The spatio-temporal displacement field can therefore be expanded as follows:

$$u(r, t) = \frac{1}{2} \sum_{j} \int_{1st \, BZ} \left\{ A_j(k) \sum_{G} C_j(k + G) \exp\{i((k + G) \cdot r - \omega_j(k)t)\} \right\} \, d^2k.$$
all \( \mathbf{k} \)-space:

\[
    u(\mathbf{r}, t) = \frac{1}{2} \sum_j \int_{\text{all } \mathbf{k}} \left\{ A_j(\mathbf{k}') C_j(\mathbf{k}) \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega_j(\mathbf{k})t)] + A_j^*(\mathbf{k}') C_j^*(\mathbf{k}) \exp[-i(\mathbf{k} \cdot \mathbf{r} - \omega_j^*(\mathbf{k}')t)] \right\} \, \text{d}^2 \mathbf{k}
\]

\[
    = \frac{1}{2} \int_{\text{all } \mathbf{k}} \sum_j \left\{ A_j(\mathbf{k}') C_j(\mathbf{k}) \exp[-i\omega_j(\mathbf{k}')t] + A_j^*(\mathbf{k}') C_j^*(\mathbf{k}) \exp[i\omega_j^*(\mathbf{k}')t] \right\} \exp(i\mathbf{k} \cdot \mathbf{r}) \, \text{d}^2 \mathbf{k},
\]

(2.8)

where \( \mathbf{k}' \) is the wavevector reduced to the 1st BZ using the relation \( \mathbf{k} = \mathbf{k}' + \mathbf{G} \) with an appropriate \( \mathbf{G} \).

Rewriting equation (2.4) as

\[
    \int U(\mathbf{k}, \omega) \exp(-i\omega t) \, \text{d}\omega
\]

and comparing it with equation (2.8), one obtains

\[
    \int U(\mathbf{k}, \omega) \exp(-i\omega t) \, \text{d}\omega
    = \frac{1}{2} \sum_j \left\{ A_j(\mathbf{k}') C_j(\mathbf{k}) \exp[-i\omega_j(\mathbf{k}')t] + A_j^*(\mathbf{k}') C_j^*(\mathbf{k}) \exp[i\omega_j^*(\mathbf{k}')t] \right\}.
\]

(2.10)

When the absorption of the acoustic waves is negligible over the imaged region, \( \omega_j(\mathbf{k}') \) is real, that is \( \omega_j(\mathbf{k}') = \omega_j^*(\mathbf{k}') \). Equation (2.10) can then be further simplified by multiplying both sides by \( \exp(i\omega'/t)/(2\pi) \) and integrating with respect to \( t \):

\[
    U(\mathbf{k}, \omega') = \frac{1}{2} \sum_j \left\{ A_j(\mathbf{k}') C_j(\mathbf{k}) \delta(\omega' - \omega_j(\mathbf{k}')) + A_j^*(\mathbf{k}') C_j^*(\mathbf{k}) \delta(\omega' + \omega_j(-\mathbf{k}')) \right\}.
\]

(2.11)

The above equation satisfies the relation

\[
    U(\mathbf{k}, \omega') = U^*(-\mathbf{k}, -\omega'),
\]

(2.12)

as expected for the Fourier transform of the real function \( u(\mathbf{r}, t) \) (see equations (2.3) and (2.4)).

The time-reversal symmetry of the acoustic wave equation implies \( \omega_j(\mathbf{k}') = \omega_j(-\mathbf{k}') \), and also ensures that the amplitudes of the BHs can be chosen as \( C_j(\mathbf{k}) = (\mathbf{k}) \). This can be seen by letting \( t \rightarrow -t \) in equation (2.6) and comparing it with \( u_{-\mathbf{k}j}(\mathbf{r}, t) \). Equation (2.11) can therefore be written as

\[
    U(\mathbf{k}, \omega') = \frac{1}{2} \sum_j \left\{ A_j(\mathbf{k}') C_j(\mathbf{k}) \delta(\omega' - \omega_j(\mathbf{k}')) + A_j^*(\mathbf{k}') C_j^*(\mathbf{k}) \delta(\omega' + \omega_j(\mathbf{k}')) \right\}.
\]

(2.13)

Equations (2.11) or (2.13) indicate that the Fourier amplitude \( U(\mathbf{k}, \omega') \) only takes a finite value when the combination \( (\mathbf{k}', \omega') \) satisfies the relation \( \omega' = \omega_j(\mathbf{k}') \) for the branch \( j \). This shows that the dispersion relation can be determined from a temporal series of time-resolved two-dimensional images of the acoustic field.

If \( \omega_j(\mathbf{k}') \) is chosen to be positive, the first term in the sum of equation (2.11) or (2.13) corresponds to \( \omega' > 0 \), whereas the second term corresponds to \( \omega' < 0 \). Since the Fourier amplitudes for positive and negative \( \omega' \) are related by equation (2.12), it suffices to consider only the first term:

\[
    U(\mathbf{k}, \omega') = \frac{1}{2} \sum_j A_j(\mathbf{k}') C_j(\mathbf{k}) \delta(\omega' - \omega_j(\mathbf{k}')) \quad (\omega' > 0).
\]

(2.14)

For \( (\mathbf{k}', \omega') \) satisfying the dispersion relation, that is \( \omega' = \omega_j(\mathbf{k}') \), \( U(\mathbf{k}' + \mathbf{G}, \omega') \) with any \( \mathbf{G} \) can take a finite value, and the ratios among \( C_j(\mathbf{k}' + \mathbf{G}) \) with different \( \mathbf{G} \) are equal to the corresponding ratios among \( U(\mathbf{k}' + \mathbf{G}, \omega') \). We shall see in the next sections how this equation can reveal the acoustic dispersion relation of surface phononic crystals and phononic-crystal waveguides.
3. One-dimensional phononic crystals

The simplest example of a phononic crystal is one exhibiting one-dimensional periodicity. Such a sample has been investigated using the ultrafast imaging technique mentioned above [33,37,38]. The sample consists of alternating stripes of copper and silicon oxide formed on a silicon (001) substrate of thickness 0.74 mm (figure 1a,b). Each stripe has a width of 2 µm and a thickness of 800 nm. The stripes are oriented along the [110] direction of the substrate, and form a one-dimensional periodic structure with a period \( a = 4 \mu \text{m} \) along the [110] direction. The sample fabrication is based on a damascene process, and the top surface is prepared by chemical–mechanical polishing (CMP) to a flatness better than 10 nm. A 25 nm tantalum layer is formed beneath the Cu stripes as a diffusion barrier. The top surface is then covered by a 30 nm gold film by radio frequency sputtering to produce a sample with uniform optical reflectivity and also to increase the reflectivity of the probe light.

Time-resolved two-dimensional images of the velocity of the out-of-plane surface motion are obtained using ultrafast imaging with micrometre-sized optical spots. The imaged area is 150 × 150 µm. Images are recorded at different delay times in steps of \( \frac{13.1}{40} = 0.33 \) ns within the laser repetition period of 13.1 ns. Figure 1c shows a typical image at delay time 2.62 ns. The centre of the image corresponds to the pump light spot. Concentric rings corresponding to SAWs emerging from the excitation point are visible, as well as complicated wave patterns inside the rings.

The origin of this pattern can be better understood after a spatio-temporal Fourier transform. The upper row of figure 2 shows the modulus of the Fourier amplitude, \( |V(k, \omega)| \), plotted in two-dimensional \( k \)-space at some representative frequencies that are constrained to be integral multiplies of the laser repetition rate (76.3 MHz). (This experimental constraint can be lifted by the use of modulation techniques, [40] but such techniques were not implemented for the results of this paper.) As described in §2, \( |V(k, \omega)| \) only takes a finite value if \((k, \omega)\) satisfies the dispersion relation \( \omega = \omega_j(k) \).

The 458 MHz image (figure 2a) exhibits two concentric rings (red regions) with their centres at the \( \Gamma \) point. The near-circular rings indicate that these waves propagate almost isotropically at this frequency. The outer ring has a phase velocity of \( \approx 4000 \) ms\(^{-1}\), and corresponds to Rayleigh-like waves (RWs), whereas the inner ring has a phase velocity of \( \approx 7400 \) ms\(^{-1}\), and corresponds to Sezawa waves (SWs)—higher order modes of the film/substrate system [41]. Both rings lie within the 1st BZ (\( |k_x| < \pi/a = 0.79 \mu \text{m}^{-1} \), where \( a = 4 \mu \text{m} \) is the period of the phononic-crystal structure). The RW ring is accompanied by two faint rings on the right and left. These are BHs and have
Figure 2. Upper row: amplitude (modulus) images of the spatio-temporal Fourier transform obtained from a set of time-resolved two-dimensional SAW images for a one-dimensional phononic-crystal sample at several representative frequencies. (a) 458 MHz, (b) 534 MHz, (c) 610 MHz and (d) 687 MHz. Arrows indicate the 1st BZ boundaries. Lower row: mode identification diagram. The solid lines represent dominant components in the acoustic dispersion relation, whereas the red and green dotted lines correspond to BHs. Red lines: Rayleigh-like waves (RWs). Green lines: Sezawa waves (SWs). The black dotted lines indicate the 1st BZ. (Online version in colour.)

exactly the same shape as the central RW ring. They are shifted by unit reciprocal vectors \( \pm G = 2\pi i/a \) (\( i \) being the unit vector in the \( x \)-direction), with respect to the dominant rings inside the 1st BZ. The BHs for the SW modes are less prominent. Mode identification is shown schematically in the bottom part of figure 2.

In the 534 MHz image (figure 2b), the radius of the RW and SW rings becomes larger, but some part of the rings are missing. By defining a propagation angle \( \theta \) with respect to the \( k_x \) axis, one can see that there are the openings for \( |\theta| < 15^\circ \) for RW and for \( |\theta| < 20^\circ \) for SW, providing evidence for a directional phononic stop band. The shape of the curves indicate that the formation of the phononic stop bands are caused by the interaction between the SW mode and the RW mode [41]. This gap owes its existence to an crossing (i.e. a degeneracy) of the modes in the dispersion relation for an analogous non-interacting system (i.e. an empty lattice). When the interacting system is considered, there is a repulsion of the modes to produce bonding and anti-bonding modes [42]. This is also referred to as an avoided crossing.

In the 610 MHz image (figure 2c), each segment of the RW and SW branches is pushed further out, whereas the openings in the RW and SW branches around \( k_y = 0 \) persist. On further increase of the frequency to 687 MHz (figure 2d), new features appear in the region where the opening is located at 610 MHz. This indicates that the directional phononic stop band persists at 534 MHz and 610 MHz, but is closed at 458 MHz and 687 MHz. The red arcs that appear in figure 2d are Rayleigh-like modes in the 2nd Brillouin zone.

By stacking constant frequency curves such as those shown in figure 2, one may construct a three-dimensional dispersion surface in \((k_x, k_y, \omega)\) space. Figure 3a shows a cross section of such a dispersion surface for a \((k_x, \omega)\) plane that represents the dispersion relation along \( k_x \) with \( k_y = 0 \), whereas figure 3b shows a cross section for a \((k_y, \omega)\) plane that represents the dispersion relation along \( k_y \) with \( k_x = 0 \). Each figure contains lines that originate at \( k_{x,y} = 0 \) with different slopes.
Figure 3. Modulus of the spatio-temporal Fourier transform, representing the experimentally obtained dispersion relations for the one-dimensional phononic crystal (a) along $k_x$ with $k_y = 0$ and (b) along $k_y$ with $k_x = 0$. The downward-pointing arrows indicate the 1st BZ boundary. S.B.: stop band (phononic band gap). (c) Schematic diagram to help with mode identification. Red lines, RW; green lines, SW. (Online version in colour.)

The steeper one corresponds to the SW branch, whereas the shallower one to the RW branch. As expected, for waves propagating along $k_x$ (figure 3a), a directional stop band is observed around 0.5–0.7 GHz as a region with missing bright parts. Figure 3c shows the origin of the formation of the first stop band. It is formed where the Rayleigh-like and Sezawa branches cross, a position slightly removed from the 1st BZ boundary [41]. The plots of figure 3 also allow access to the group velocity from the slope of the curves, the accuracy being limited by the frequency and wavenumber resolution (76.3 MHz and 0.042 $\mu$m$^{-1}$, respectively).

It is also useful to plot real-space images representing the temporal Fourier transform $V(r, \omega)$ of the experimentally obtained spatio-temporal data at constant frequencies. The first row of figure 4 shows images of the Fourier amplitude $A = |V(r, \omega)|$ and the second row shows images of the acoustic phase $\psi = \arg V(r, \omega)$. The spatio-temporal evolution of the acoustic field at a single frequency $\omega$ is given by

$$\text{Re}[V(r, \omega) \exp(-i\omega t)] = \text{Re}[A \exp(i(\psi - \omega t))].$$

Thus, a snapshot of the acoustic field at $t = 0$ is given by $A(r, \omega) \cos \psi(r, \omega)$. Images representing this function are shown in the third row of figure 4. The amplitude images are useful to see the spatial extent of the modes at a given frequency, whereas the phase images are useful to see the wavelength of the modes. The snapshot images are related to both the phase and amplitude images, and show the instantaneous acoustic field at a moment in time during one cycle when exciting at a single frequency.

At 458 MHz (figure 4a), the wavelength for the RW branch for $x$-directed propagation is slightly longer than twice the structure periodicity ($2a = 8 \mu$m), and the phase image displays an approximately circular pattern. At this frequency, the excitation of equal-amplitude counter-propagating waves in the $x$-direction would result in adjacent unit cells vibrating in antiphase [43]. For such $x$-directed propagation this mode is a pure standing wave that satisfies the Bragg scattering condition. At 534 MHz and above (figure 4b–d), high-amplitude regions exist localized along directions symmetrically aligned either side of the $y$-direction, forming an X shape. This is the result of a directional stop band around the $k_x$ axis and the existence of portions of the constant frequency curves with a relatively low-curvature; since the group velocity is given by the gradient of $\omega_j(k)$, a low curvature in a constant-frequency curve results in phonon focusing effects along the direction perpendicular to the low-curvature lines in plots like figure 2. It has been demonstrated [38] that this behaviour produces a double-horn structure in group-velocity space. As the frequency increases, the X shape closes because of the shift of the low-curvature regions in $k$-space, resulting in the vertical beaming effect seen in figure 4c,d at 610 and 687 MHz, respectively.
This temporal and spatio-temporal Fourier transform technique has also been applied to time-domain simulations of the wave fields in this one-dimensional phononic-crystal structure. The simulations were found to agree well with experiment [37,38].

4. Two-dimensional phononic crystals

The acoustic properties of a two-dimensional phononic crystal have also been investigated by ultrafast imaging [34,36]. The sample consists of a two-dimensional square lattice of air-filled holes etched in an Si (100) substrate of thickness 0.46 mm made by a deep reactive ion etching process. The hole diameter is 12 µm and the lattice constant is \( a = 15 \) µm. Figure 5a shows an optical micrograph of the sample over an area of 135 × 135 µm. The hole depth is 10.5 µm, as measured by scanning white light interferometry. The \( x \)- and \( y \)-axes of the crystal are parallel to the [110] and [\( \bar{1}10 \)] directions of the underlying Si substrate. The surface of the phononic-crystal
sample is covered with a 40 nm gold film to enhance the SAW generation efficiency by the pump light absorption and also to increase the reflectivity of the probe light.

Time-resolved two-dimensional images of the velocity of the out-of-plane surface motion are obtained using ultrafast imaging under similar conditions to those used for the one-dimensional phononic crystal. Figure 5b shows an SAW image at delay time 7.4 ns obtained over an area of 150 × 150 µm. The pump light is focused near the centre of the image. The SAWs are strongly scattered by the array of holes.

The SAW modes can again be extracted by spatio-temporal Fourier transforms. The upper row of images of figure 6 shows the Fourier amplitude |V(k, ω)| at some representative frequencies. Below each amplitude image a constant frequency circle to match the data for the x- and y-directed velocities at the corresponding frequency is depicted together with replicas shifted by reciprocal lattice vectors of the sample structure. These circles represent the dispersion relation of an empty lattice having the same periodicity as the phononic crystal, and help to give a rudimentary understanding of the origin of the experimental Fourier amplitude images. The 1st BZ corresponds to the central square defined by the thin straight lines. In the periodic zone scheme for representation of the phononic band structure, any other such displaced square can be reduced to the 1st BZ by shifting by an appropriate reciprocal lattice vector G.

The 153 MHz image (figure 6a) shows a constant frequency curve approximately circular in shape that is just touching the edges of the 1st BZ. This circle corresponds to Rayleigh-like modes with a velocity approximately 4800 ms⁻¹, similar to that of Rayleigh waves (approx. 5100 ms⁻¹) observed on bare Si (100) substrates [44]. BHs are visible in the squares displaced from the 1st BZ (the central square) by G = (±2π/a, 0) and (0, ±2π/a). There is an important proviso for the interpretation of BHs in this sample. In contrast to the case of the one-dimensional phononic crystal, the surface of the two-dimensional phononic crystal is not optically homogeneous owing to the surface-breaking holes. What then is the influence of the spatial periodicity of the optical reflectance R of the sample? It turns out that, even in the absence of acoustic scattering, the effect of the optical spatial modulation in R leads to a periodic structure in k-space for |V(k, ω)| exactly mimicking Bloch harmonics. ¹ One should therefore exercise caution in the interpretation of the amplitude of BHs in the present sample.

In the 229 MHz image (figure 6b), the circle corresponding to the Rayleigh-like mode is larger, and parts of the circle go out beyond the 1st BZ boundary. One can see that the modes survive

¹It might appear counterintuitive that a static hole pattern can affect finite-frequency images in k-space. This effect can be understood from a simple one-dimensional example. If a plane wave, g(x, t) = cos(kx − ωt), is modulated by a fixed structure.
Figure 6. Upper row: amplitude (modulus) images of the spatio-temporal Fourier transform obtained from a set of time-resolved SAW images for a two-dimensional phononic-crystal sample at several representative frequencies. (a) 153 MHz, (b) 229 MHz, (c) 305 MHz, (d) 382 MHz and (e) 458 MHz. The thin lines represent the 1st BZ boundaries at \( k_x, k_y = \pm 0.29 \mu \text{m}^{-1} \) and also at \( k_x, k_y = \pm 3 \times 0.29 \mu \text{m}^{-1} \). Lower row: mode identification diagram based on an empty lattice having the same periodicity as the two-dimensional phononic crystal at the frequencies corresponding to the figures in the upper row. Circles in red correspond to the modes in a non-periodic uniform medium whose SAW velocity is chosen so that the construction mimics the images in the upper row. (Online version in colour.)

near the corner of the 1st BZ, but in the other parts of the zone there is an avoided crossing, as noted in the case of the one-dimensional phononic crystal, and a directional phononic stop band opens for propagation angles near the \( k_x \) and \( k_y \) directions.

In the 305 MHz image (figure 6c), the dominant high-amplitude region lies outside the 1st BZ. This indicates that the modes at this frequency exist in the 2nd or higher order BZs in the extended zone scheme. The Fourier amplitude diminishes for the directions around \( k_x \) and \( k_y \) again owing to the persistence of the directional phononic stop band in these directions.

At 382 MHz (figure 6d), a large square array of dots are superimposed on a rounded-square-shaped constant-frequency curve. These dots are located near the points where four adjacent circles centred at \( \mathbf{k} = (\pm 2\pi/a, \pm 2\pi/a) \) meet in the empty lattice dispersion image. For this reason, the dots have an enhanced intensity.

At 458 MHz (figure 6e), the constant-frequency curve forms a large square having significant amplitude in higher order BZs. As explained before in the context of the one-dimensional phononic crystal, the low-curvature of these constant-frequency curves again results in phonon focusing effects, in this case manifested by self-collimation along the directions \((\pm x, \pm y)\) perpendicular to the low-curvature lines in this plot, as will become evident later.

Figure 7a shows a cross section (derived from the amplitude \(|V(\mathbf{k}, \omega)|\) of the spatio-temporal Fourier transform) of the three-dimensional dispersion surface corresponding to a \((k_x, \omega)\) plane

\[
h(x, t) = (1 + \cos(Kx))/2, \text{ then the product}
\]

\[
f(x, t) = g(x, t) \times h(x, t)
\]

\[
= \frac{\cos(kx - \omega t)(1 + \cos(kx))}{2}
\]

\[
= \frac{\cos(kx - \omega t)}{2} + \frac{(\cos((k + K)x - \omega t) + \cos((k - K)x - \omega t))}{4}
\]

contains BHs at wavenumbers \(k + K\) and \(k - K\) at angular frequency \(\omega\).
Figure 7. (a) Modulus of the spatio-temporal Fourier transform, representing the experimentally obtained dispersion relations for the two-dimensional phononic crystal along $k_x$ with $k_y = 0$. The black arrows indicate the 1st BZ boundary along $k_x$. (b) Schematic diagram to help with mode identification. (Online version in colour.)

representing the dispersion relation along $k_x$ with $k_y = 0$. It contains lines that originate at $k_{x,y} = 0$, which correspond to the RW branch. The directional stop band is observed around 0.3 GHz, as expected, as a region with missing bright parts. Figure 7b shows a possible interpretation for the origin of the stop band formation. The lower limit of the first stop band is located where the acoustic branch crosses the 1st BZ boundary. Above the upper limit of this stop band BHs are particularly evident near the frequency of 382 MHz where we noted a square array of dots in the constant frequency image. As opposed to predictions of a stop band simply based on the repulsion of branches at the 1st BZ boundary that leads to gap formation at the corresponding $k$-value, the bottom edge of the stop band is close to this BZ boundary (and equivalent points in $k$-space), whereas the top edge of the gap is around the $\Gamma$ point (and equivalent points in $k$-space). A full understanding of the origin of this stop band would need to take the full two-dimensional nature of the phonon dispersion relation into account.

Further information is again obtainable from the real-space images representing the temporal Fourier transform $V(r, \omega)$ of the experimentally obtained spatio-temporal data. The first row of figure 8 shows images of the Fourier amplitude $A = |V(r, \omega)|$ and the second row shows the images of the phase $\psi = \arg V(r, \omega)$. The third row shows a snapshot of the acoustic field at $t = 0$ as calculated from $A(r, \omega) \cos \psi(r, \omega)$.

At 153 MHz (figure 8a), the phase images display an approximately circular pattern, as expected from the results in $k$-space. At 229 MHz, the acoustic wavelength for $x$- or $y$-directed propagation is approximately twice the lattice constant of the sample ($2a = 30 \mu m$). The maxima in amplitude occur at points approximately equidistant between four holes, and adjacent unit cells vibrate in antiphase. For the $x$- and $y$-directions, this mode is a pure standing wave that satisfies the Bragg scattering condition. At both 229 MHz and 305 MHz (figure 8b,c), the high-amplitude region is more localized near the excitation point. This is the result of either low group velocity or high attenuation, both characteristics being expected for modes in the vicinity of the phononic stop band: the former explanation is valid for true acoustic modes outside the stop band, whereas the latter is valid for pseudo modes within the stop band.

At 382 MHz, just above the stop band, the wavelength is approximately equal to the period of the lattice ($a = 15 \mu m$), resulting in a characteristic square pattern in the phase image. Because of the nearly square shape of the constant-frequency surface at this frequency, to a good approximation this mode is a pure standing wave that satisfies the Bragg scattering condition for all propagation directions. At 458 MHz (figure 8e), the amplitude image shows a broad cross oriented parallel to the $x$- and $y$-axes, and the phase image again shows a square shape. This is caused by the self-collimation effects discussed in relation to figure 6e.
Figure 8. Images of the temporal Fourier transform obtained from a set of time-resolved SAW images for the two-dimensional phononic-crystal sample at several representative frequencies: (a) 153 MHz, (b) 229 MHz, (c) 305 MHz, (d) 382 MHz and (e) 458 MHz. The first row shows images of the modulus of the Fourier amplitude. The second row shows the argument of the Fourier amplitude. The third row shows snapshot images of the out-of-plane surface velocity for a single frequency component. The imaged area is 150 × 150 µm. (Online version in colour.)

This temporal and spatio-temporal Fourier transform technique has also been applied to time-domain simulations of the wave fields in this two-dimensional Si structure. The simulations were found to agree well with experiment [36].

5. Phononic-crystal waveguides

The acoustic properties of a two-dimensional phononic-crystal waveguides have also been investigated by ultrafast imaging [39]. Such samples are ideal for time-domain analysis in k-space, and the way to achieve this is described in this section.

The samples consist of linear defects in two-dimensional phononic crystals formed from microscopic holes in crystalline silicon. Round holes with spacing \(a = 6.2 \mu m\) are milled to square lattices by dry reactive ion etching on (100) silicon substrates, leaving single rows with no holes to produce straight or L-shaped waveguides (figure 9a,b). The silicon crystal-axis orientation with respect to the phononic lattice is the same as for the previous two-dimensional phononic-crystal sample. The hole diameter is 5.6 \(\mu m\) at the top and the depth is approximately 100 \(\mu m\). The 50 nm polycrystalline chromium coating does not significantly affect SAW propagation. The hole walls taper outwards and intersect with neighbouring holes at a depth of 3 \(\mu m\) (figure 9d). The taper angle is measured from oblique scanning electron microscope (SEM) images (figure 9c). Because of the subsurface structure, the surface waves inside the phononic-crystal regions of the sample
straight waveguide  L waveguide

Figure 9. (a,b) SEM images of the straight and L-shaped phononic-crystal waveguides. Marked regions are for Fourier analysis. (c) Oblique SEM image (30° to vertical). (d) Simulated structure, (e) its horizontal cross section at a depth greater than 7 µm and (f) its vertical cross section. (Online version in colour.)

Figure 10. Experimentally obtained real-space time-domain images for phononic-crystal waveguides. (a,b) and (c,d) Snapshot images of SAWs in the straight and L waveguides, respectively, at 4.1 ns intervals. (Online version in colour.)

(as opposed to inside the waveguides) are more accurately described by Lamb waves rather than Rayleigh waves.

Time-resolved two-dimensional images of the velocity of the out-of-plane surface motion are obtained by ultrafast imaging of SAWs under similar conditions to those used for the one- and two-dimensional phononic crystals. Figure 10a,b shows SAW snapshot images from the straight waveguide. The optical pump beam is focused just in front of the left-hand entrance of the waveguide. Figure 10c,d shows similar snapshots for the L waveguide.

Time-resolved images do not allow the interaction between the acoustic eigenstates to be properly investigated. However, k-space-monitoring of phononic eigenstates directly reveals the physical processes at work in such systems. One would like to follow the evolution of these eigenstates in time when analysing the relevant acoustic scattering processes. Although
time-domain imaging in k-space has been achieved in photonics [45], until recently such k-space temporal imaging had not been applied to phononic eigenstates. However, this was shown to be possible by simple Fourier analysis [39]: one calculates the k-t-space amplitude \( V_k(k, t) \) using the so-called analytic signal [46], given by

\[
V_k(k, t) = 2 \int_0^\infty V(k, \omega) e^{-i\omega t} d\omega,
\]

where \( V(k, \omega) \) is the spatio-temporal Fourier transform of the SAW field \( v(r, t) \). A direct two-dimensional spatial Fourier transform only yields \( \frac{1}{2}(V_k(k, t) + V_k^*(-k, t)) \) that mixes positive and negative acoustic propagation, hence we apply equation (5.1).

Figure 11a,b shows \( |V_k(k, t)| \) obtained from equation (5.1) for the straight waveguide by integrating to find \( V(k, \omega) \) over the phononic-crystal region, a 92 \( \times \) 92 \( \mu \)m square, as well as over the region corresponding to the waveguide alone, a 92 \( \times \) 7 \( \mu \)m rectangle in the waveguide (figure 9a). The horizontal curves show the profiles for \( k_y = 0 \), dominated by the contributions from the waveguides. In figure 11a, at 0 ns, the freshly generated SAW pulse has not yet entered the waveguide. A small residual peak at \( k_x a/\pi = 0.8 \) is visible—this is the main transmission wavenumber—arising from the previous SAW pulse. In figure 11b, at 4.1 ns, the pulse is just
Figure 12. Modulus of the spatio-temporal Fourier transform, representing the experimentally obtained dispersion relations for the phononic-crystal waveguides along their propagation axes. (a) For the straight waveguide, and (b,c) for the L waveguide. The insets show the analysis regions. Upward-pointing white arrows: 1st Brillouin zone. (d–f) Profiles of the spatio-temporal Fourier transforms taken along the dispersion relation. (Online version in colour.)

entering the waveguide. This produces an intense broad peak centred at $k_y a / \pi \approx +2$. The $k_y = 0$ profile at this time approximately corresponds to the broadband spectrum of SAW source ($k_x a / \pi \approx 0.2$–3 or wavelength $\lambda = 2\pi / k_x \approx 4$–60 µm). By examining a series of frames in k-space, the height of this initial peak is observed to decrease, and transmission peaks emerge and decay, particularly in the waveguide at $k_y a / \pi \approx 0.8$, 1.6 and 2.2 (not shown here). A relatively high, approximately 45\% of that at $k_y a / \pi = +1$, is visible in figure 11 for the guide near $k_x a / \pi = +1$, a wavenumber close to the Bragg scattering condition from the phononic lattice: the shift from $+k_x$ to $-k_x$ is a unit reciprocal lattice vector $G$. This shows that this peak is a BH arising from the periodic waveguide boundaries [1].

The equivalent series for $|V_k(k,t)|$ for the L waveguide is shown in figure 11c,d. The two sections of the waveguide are analysed separately (figure 9b). The curves are the profiles for $k_x = 0$ (vertical curves—with the y-axis on the left) and $k_y = 0$ (horizontal curves). The peak near $k_y a / \pi = +0.8$ at 4.1 ns (figure 11c,d) shows that approximately 20\% of the SAW amplitude is reflected at the bend. This effect contributes to a lower amplitude in horizontal section of the waveguide: at $k_y a / \pi = +0.8$ the amplitude is reduced to approximately 40\% of that of the equivalent eigenstate in the vertical section. Such reduced amplitude on transmission around a 90° bend is similar to that calculated for SAW waveguides of related design [47]. The energy loss can be explained by acoustic scattering to the bulk of the sample.

The spatio-temporal Fourier transform for the same waveguide regions is shown by $|V(k,\omega)|$ in figure 12a–c. For the straight waveguide (figure 12a), the $+k_x$-directed eigenstates are dominant. The slope is for the most part linear, as previously predicted in simulations of SAWs in phononic-crystal waveguides [47,48]. This slope corresponds to a phase velocity, $v_p \approx 5 \text{ km s}^{-1}$, governed by RWs on the Si substrate [44]. A BH, shifted by a unit reciprocal lattice vector $(k_x a / \pi = -2.0)$, is also visible at 402 MHz. Figure 12d shows the amplitude profiles along the $\pm k_x$ branches of the corresponding dispersion relations, allowing the quantitative comparison of counter-propagating eigenstates. Waveguiding peaks occur at $k_x a / \pi \approx 0.8$, 1.6 and 2.2 (322, 643 and 964 MHz). The higher frequencies have a greater amplitude, characteristic of the SAW excitation spectrum.

For the L waveguide (figure 12b,c,e,f), the analysis is divided into vertical (before the bend) and horizontal (after the bend) sections. Only one frequency, 322 MHz, is significant after the 90°
Figure 13. Images of the temporal Fourier transform obtained from a set of time-resolved SAW images for the phononic-crystal waveguide samples at three representative frequencies: straight waveguide at (a) 161, (b) 322 and (c) 482 MHz. (b–d) The same for the L waveguide. The first row shows images of the modulus of the Fourier amplitude. The second row shows the argument of the Fourier amplitude. The third row shows snapshot images of the out-of-plane surface velocity for a single frequency component. The imaged area is 125 × 125 µm. The curves in the first row represent the amplitude profiles along the central lines of the waveguides. (Online version in colour.)

bend, as previously noted. As expected, the amplitude reflection and transmission coefficients at the bend are the same as those measured in k-t space.

Real-space images representing the temporal Fourier transform $V(r, \omega)$ of the experimentally obtained spatio-temporal data at constant frequencies are shown in figure 13: rows 1, 2 and 3 show such images for $A$, $\psi$ and $A \cos \psi$, respectively, for each waveguide. Three representative frequencies, 161, 322 and 482 MHz, have been chosen in each case. The superimposed curves in the first row represent the profiles along the central lines of the waveguides. At 322 MHz, both waveguides transmit efficiently; this suggests the presence of a band gap in the phononic crystal, i.e. a stop- or a deaf band [49], where the mode represented by the $\omega$, k combination in the waveguide is evanescent in the phononic crystal or forbidden by the spatial symmetry of the SAW excitation, respectively. By contrast, at 161 and 482 MHz, where it is likely that pass bands exist in the phononic crystal, strong attenuation is evident inside both waveguides.

Acoustic energy is easily transmitted in and out of the waveguides, as previously noted in numerical calculations [48]. The strong frequency dependencies of the transmission and reflection coefficients arise from the periodicity of the surrounding phononic-crystal medium. The relatively low dispersion regions—contrasting with the highly dispersive behaviour of bulk waves in phononic-crystal waveguides exhibiting lateral acoustic resonances [2,9]—are a particular characteristic of solid phononic-crystal SAW waveguides.

In addition to the above experimental work, numerical simulations showed good agreement with the above results [39].
6. Conclusion

In conclusion, we have discussed how the acoustic properties of surface phononic crystals can be characterized using time-domain optical imaging of surface motion at gigahertz-order frequencies. Experimental results for generic samples are presented corresponding to phononic crystals with one- and two-dimensional periodicity, including phononic-crystal waveguides. We showed how the temporal Fourier transforms can be used to access the acoustic mode patterns at specific frequencies, and how spatio-temporal Fourier transforms can be used to access the dispersion relation and phononic stop bands. The role of BHs, related to the scattering potential, in the spatio-temporal Fourier transforms was also analysed mathematically, and we showed how they contribute to the experimental results in k-space.

We have also seen how the evolution of the acoustic wave field can be followed in k-space. The acoustic propagation, damping, reflection and the in- and out-coupling for phononic-crystal waveguides are elucidated. Animating k-space is new to acoustics and should prove useful for characterizing SAW devices in telecommunications, for example. It may also be helpful in the analysis of acoustic metamaterials exhibiting, for example, negative refraction [35], extraordinary transmission [50] or cloaking phenomena [51].

Ultrafast time-domain optical techniques for SAW imaging reveal a wealth of physics because both real space and k-space are easily accessible through Fourier analysis. We look forward to their continued use in a wide variety of studies on phononic crystals and acoustic metamaterials over a broad range of frequencies.

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