Introduction

Feedback may have a tremendous impact on the behaviour of physical systems, reveal new effects and phenomena, provide tools for optimal planning of physical experiments and even create models of complex physical systems. Cybernetical physics is the area on the borderland between physics and control studying possibilities of changing properties of the physical systems by means of control, particularly by means of feedback. In this introductory paper, the development of the area during the last decade is outlined. The paper may be of interest for a rather broad audience as it provides a guide across this emerging interdisciplinary field.

Cybernetical physics is understood as the science of studying physical systems by cybernetical means. And cybernetical means are assumed to include, in addition to control, such areas as estimation, filtering, optimization, identification, information theory, pattern recognition, etc.
and other related areas born within cybernetics in the middle of the twenty-first century. The term was coined in 1999 [1,2] to name an emerging research area on the borderland between physics and control. A panoramic view of this new area was presented in the survey paper [3] and in the book published by Springer in 2007 [4]. Since then cyberphysical research has been actively developing and numerous publications have appeared. The main goal of this paper is to outline the main research directions during the last decade and introduce the other papers of this theme issue.

After exposing a brief history of cybernetical physics (§2), its methodology (models, algorithms and results) is introduced in §3. Then the main areas are briefly outlined: control of oscillations and chaos (§4), controlled synchronization (§5), control in thermodynamics (§6), control of networks (§7), quantum control (§8).

2. Cybernetical physics: history

A real demand for control algorithms actually emerged in physics in the early 1990s. One source of such a demand was the discovery by Ott et al. [5] of a possibility to transform a chaotic trajectory into a periodic one by means of a properly chosen controlling action. This discovery triggered an avalanche of publications aimed at the investigation of such possibilities for different physical (and not only physical) systems. Similarly, a possibility of synchronizing two nonlinear systems with chaotic behaviour discovered in the paper by Pecora & Carroll [6] initiated numerous publications discovering more and more cases and methods for synchronizing chaotically moving systems of various origin. Both papers were published in physical journals and just in a few years the number of the papers published in physical journals in this area was counted in hundreds. In 2016, after a quarter of century the paper by Ott et al. has more than 4000 citations, while the paper by Pecora and Carroll has more than 6000 citations. Note that the ideas and methods of cybernetics (control, estimation, identification, etc.) were in many cases reopened and rediscovered.

Another area where control and estimation problems play an important role is that of quantum control. Progress in this area was stimulated by the invention of the femtosecond lasers at the end of the 1980s. The duration of pulses generated by such lasers is comparable with the period of natural oscillations of single molecules which allows easier manipulation by single molecules and nanodimensional systems and offers new possibilities for chemistry, nanotechnologies and quantum computing.

Mutual interest of researchers in physics and control led to organizing conferences dedicated to this new area. The 1st International Conference ‘Physics and Control’ (Physcon 2003) took place in 2003 in St Petersburg, Russia, with 250 experts from 32 countries participating. During the 2nd International Conference ‘Physics and Control’ (Physcon 2005) that took place in 2005, again in St Petersburg, a number of researchers had decided to organize an International Society for better promotion of this emerging field. The foundation meeting took place right after the conference where the International Physics And Control Society (IPACS) [7] was founded. Since then the IPCS conferences take place biannually in different countries: Potsdam, Germany (2007); Catania, Italy (2009); Leon, Spain (2011); San Lois Potosi, Mexico (2013); Istanbul, Turkey (2015). The 8th conference Physcon 2017 is planned in Florence in July, 2017. In 2012 a new Cybernetics And Physics (CAP) journal was launched [8].

It is important that most publications are published in physical journals, i.e. the area is growing as a new part of physics. Some tutorials on control for physicists are published in physical journals too [9,10]. The importance of control theory in physics is currently better understood by physicists [9, p. 783]:

Feedback and its big brother, control theory, are such important concepts that it is odd that they usually find no formal place in the education of physicists.
3. Cybernetical physics: methodology

Recall that the cybernetic approach is based on methodology of mathematical modelling. It means that application of cybernetical methods is based on building, analysing and using mathematical models. In particular, studies related to control of physical systems should deal with mathematical models of the controlled physical system and a mathematical model of the control goal. Below, a brief account of such models is provided, following [4].

(a) Models of controlled systems

The most common class of controlled system models consists of continuous-time systems with finite-dimensional state space described by differential equations

\[ \dot{x} = F(x, u), \]  

(3.1)

where \( x \) is \( n \)-dimensional vector of the state variables; \( \dot{x} = \frac{d}{dt} \) stands for the time derivative of \( x \); \( u \) is \( m \)-dimensional vector of inputs (control variables). Components of the state vector \( x \) are denoted as \( x_1, x_2, \ldots, x_n \), while the components of controlling vector \( u \) are denoted as \( u_1, u_2, \ldots, u_m \). Apparently, (3.1) is a compact notation for the system of ordinary differential equations

\[ \frac{dx_i}{dt} = F_i(x_1, x_2, \ldots, x_n, u_1, u_2, \ldots, u_m), \quad i = 1, 2, \ldots, m. \]

(3.2)

Vector–function \( F(x, u) \) is usually assumed continuously differentiable to guarantee the existence and uniqueness of the solutions of (3.1) at least at some time interval close to the initial point \( t = 0 \). However, a time interval where the model (3.1) is considered is usually not predefined and some additional requirements may be needed to guarantee that solutions of (3.1) are well defined for all \( t \geq 0 \).

In the presence of external disturbances, more general time-varying models are studied

\[ \dot{x} = F(x, u, t). \]

(3.3)

On the other hand, many nonlinear control problems can be described using more simple affine in control models:

\[ \dot{x} = f(x) + g(x)u. \]

(3.4)

The model should also include the description of measurements, i.e. the \( l \)-dimensional vector of output variables (observables) \( y \) should be defined, for example, as a function of the system state:

\[ y = h(x). \]

(3.5)

If the outputs are not defined explicitly, it will be assumed that all the state variables are available for measurement, i.e. \( y = x \). Using notation \( y = h(x, u) \) means that some input variables are also available for measurement.

In many cases, discrete-time state-space models are used:

\[ x_{k+1} = F_d(x_k, u_k), \]

(3.6)

where \( x_k \in \mathbb{R}^n, u_k \in \mathbb{R}^m, y_k \in \mathbb{R}^l \) are state, input and output vectors at the \( k \)th stage of the process. Then the model will be defined by the mapping \( F_d \). Using a discrete-time model may be convenient even if the process \( x(t) \) is a continuous-time one, but the measurements are taken at discrete time instants (sampling times) \( t_k, \ k = 1, 2, \ldots \) Then \( x_k = x(t_k), \ u_k = u(t_k), \ y_k = y(t_k) \).

1Hereafter the following notations are used: \( \mathbb{R}^n \) is real \( n \)-dimensional vector space; \( x = \text{col}(x_1, x_2, \ldots, x_n) \) stands for a column vector with the components \( x_1, x_2, \ldots, x_n \); the Euclidean norm of the vector \( x \in \mathbb{R}^n \) is denoted as \( |x| = (x_1^2 + x_2^2 + \cdots + x_n^2)^{1/2} \); if \( X \) is the vector (matrix), then \( X^T \) stands for the transposed vector (matrix). In particular, if \( X \) is the column vector, then \( X^T \) is the row vector. Notation \( I_n \) stands for \( n \times n \) unity matrix; \( \square \) marks the end of definition, example or remark.
(b) Models of control goals

Control goals appearing in cybernetics and cybernetical physics may be classified into five categories [4].

(i) Regulation

Regulation (often called stabilization or positioning) is the most common and simple control goal. Regulation is understood as driving the state vector \( x(t) \) or the output vector \( y(t) \) to some equilibrium state \( x^* \) (respectively, \( y^* \)). Owing to the presence of various uncertainties, it is convenient to eliminate time from formulation of the control goal and to consider an idealized control goal as the limit relation

\[
\lim_{t \to \infty} x(t) = x^* \quad (3.7)
\]

or limit relation

\[
\lim_{t \to \infty} y(t) = y^*. \quad (3.8)
\]

In the presence of additive disturbances, achievement of the control goals (3.7) and (3.8) is impossible in general and one should replace them by the limit relations for the upper limit (maximum limit over all subsequences) of the error:

\[
\lim_{t \to \infty} |x(t) - x^*| \leq \Delta \quad (3.9)
\]

or

\[
\lim_{t \to \infty} |y(t) - y^*| \leq \Delta, \quad (3.10)
\]

where \( \Delta \) is the maximum value of admissible error. In the presence of stochastic disturbances, the errors in the left-hand sides should be replaced by their averaged values.

(ii) Tracking

State tracking is driving a solution \( x(t) \) of (3.1) to the prespecified function of time \( x^*(t) \), i.e.

\[
\lim_{t \to \infty} [x(t) - x^*(t)] = 0 \quad (3.11)
\]

for any solution \( x(t) \) of (3.1) with initial conditions \( x(0) = x_0 \in \Omega \), where \( \Omega \) is given set of initial conditions. Similarly, output tracking is driving the output \( y(t) \) to the desired output function \( y^*(t) \), i.e.

\[
\lim_{t \to \infty} [y(t) - y^*(t)] = 0. \quad (3.12)
\]

The desired output function \( y^*(t) \) may be interpreted as the command or reference signal. It may be either given explicitly before the system starts functioning or it may be measured online by some measurement device. Alternatively, \( y^*(t) \) may depend on the motion of some auxiliary system called the reference model or the model of the goal.

For example, a typical problem of chaos control can be formulated as tracking of an unstable periodic solution (orbit). In this case, \( x^*(t) \) is the \( T \)-periodic solution of the free (uncontrolled, i.e. \( u(t) = 0 \)) system (3.1) with initial condition \( x(0) = x^0 \), i.e. \( x(t + T) = x^0(t) \) for all \( t \geq 0 \).

(iii) Generation (excitation) of oscillations

The third class of control goals corresponds to the problems of excitation or generation of oscillations. Here, it is assumed that the system is initially at rest. The problem is to find out if it is possible to drive it into an oscillatory mode with the desired characteristics (energy, frequency, etc.). In this case, the goal trajectory of the state vector \( x(t) \) is not prespecified. Moreover, the goal trajectory may be unknown, or may even be irrelevant to the achievement of the control goal. Such problems are well known in electrical, radio engineering, acoustics, laser and vibrational technologies—wherever it is necessary to create an oscillatory mode for the system. Such a class of control goals can be related to problems of dissociation, ionization of molecular systems, escape
from a potential well, chaotization and other problems that may be related to growth of the system energy and its possible phase transition. Sometimes such problems can be reduced to tracking, but the reference trajectory \( x_*(t) \) in these cases are not necessarily periodic and may be unstable.

To formalize such kind of control goals, one can introduce a scalar goal function \( G(x) \) and specify the goal as achieving the limit equality

\[
\lim_{t \to \infty} G(x(t)) = G_*
\]

or the following inequality for the lower limit of the goal function value:

\[
\lim_{t \to \infty} G(x(t)) \geq G_*.
\]

In many cases, the total energy of mechanical or electrical oscillations can serve as the goal function \( G(x) \).

(iv) Synchronization

The fourth important class of control goals corresponds to synchronization (more accurately, controlled synchronization as distinct from autosynchronization or self-synchronization). Generally speaking, synchronization is understood as concurrent change of the states of two or more systems or, perhaps, concurrent change of some quantities related to the systems, e.g. equalizing of oscillation frequencies [11–13].

A simple way to formulate the control goal, corresponding, e.g. to asymptotic synchronization of the two system states \( x_1 \) and \( x_2 \) is to express it as the limit relation:

\[
\lim_{t \to \infty} [x_1(t) - x_2(t)] = 0.
\]

In the extended state space \( x = \{x_1, x_2\} \) of the overall system, relation (3.15) implies convergence of the solution \( x(t) \) to the diagonal set \( \{x : x_1 = x_2\} \).

(v) Modification of the limit sets (attractors) of the systems

The last class of the control goals is related to modification of some quantitative characteristics of the limit behaviour of the system. It includes such specific goals as

— changing the type of the equilibrium (e.g. transformation of an unstable equilibrium into a stable one or vice versa);
— changing the type of the limit set (e.g. transformation of a limit cycle into a chaotic attractor or vice versa, changing fractal dimension of the limit set, etc.);
— changing the position or the type of the bifurcation point in the parameter space of the system.

Investigation of the above problems started at the end of the 1980s with the works on bifurcation control and continued in the works on control of chaos. Ott et al. [5] and their followers introduced a new class of control goals, not requiring any quantitative characteristic of the desired motion. Instead, the desired qualitative type of the limit set (attractor) was specified, e.g. control should provide the system with a chaotic attractor.

(c) Control algorithms

Modern control theory provides various recipes to solve complex control problems; see standard textbooks [14–16] for basic results on linear and nonlinear control and identification. A number of monographs and textbooks are devoted also to adaptive control, stochastic control, time-delayed control, discrete-time control, control of distributed systems and networks, etc. A comprehensive survey on control theory including its history and applications, particularly in physics, is worth mentioning [17].
Below, only one approach, the so-called speed gradient (SG) method will be outlined, since it is used in some other papers of this theme issue. Like the gradient method for discrete-time systems, the SG method is intended for control problems where the control goal is specified by means of a goal function.

Consider a nonlinear time-varying system

\[ \dot{x} = F(x, u, t) \]  (3.16)

and control goal

\[ \lim_{t \to \infty} Q(x(t), t) = 0, \]  (3.17)

where \( Q(x, t) \geq 0 \) is a smooth goal function.

In order to design a control algorithm, the scalar function \( \dot{Q} = \omega(x, u, t) \) is calculated as the speed (rate) of changing \( Q_t = Q(x(t), t) \) along trajectories of (3.16): \( \omega(x, u, t) = \frac{\partial Q(x, t)}{\partial t} + [\nabla_x Q(x, t)]^T F(x, u, t) \). Then it is needed to evaluate the gradient of \( \omega(x, u, t) \) with respect to input variables: \( \nabla_u \omega(x, u, t) = (\partial \omega / \partial u)^T = (\partial F / \partial u)^T \nabla_x Q(x, t) \). Finally, the algorithm of changing \( u(t) \) is determined according to the differential equation

\[ \frac{du}{dt} = -\Gamma \nabla_u \omega(x, u, t), \]  (3.18)

where \( \Gamma = \Gamma^T > 0 \) is a positive definite gain matrix, e.g. \( \Gamma = \text{diag} \{ \gamma_1, \ldots, \gamma_n \}, \ \gamma_i > 0 \). The algorithm (3.18) is called the SG algorithm, since it suggests changing \( u(t) \) proportionally to the gradient of the speed of changing \( Q_t \).

Motivation for the algorithm (3.18) is as follows. In order to achieve the control goal (3.17), it is reasonable to change \( u(t) \) along the direction where \( Q(x(t), t) \) decreases. However, it is not clear how to find such a direction since \( Q(x(t), t) \) does not depend on \( u(t) \) explicitly. Instead we may try to decrease \( \dot{Q} \), in order to eventually achieve the inequality \( \dot{Q} < 0 \), which implies the decrease of \( Q(x(t), t) \). The speed \( \dot{Q} = \omega(x, u, t) \) may explicitly depend on \( u \), thus allowing an application of algorithm (3.18). The above heuristic arguments are transformed into rigorous statements in a number of texts, e.g. [2,18] for finite-dimensional systems, described by the ODE.

The SG algorithm can also be interpreted as a continuous-time counterpart of the gradient algorithm, since for small sampling step size the direction of the gradient is close to the direction of the SG.

To finish the discussion on the methodology, note that plenty of the results in many areas of physics are presented in the form of conservation laws, stating that some quantities do not change during the evolution of the system. However, the formulations in cybernetics are different. Since the results in cybernetics establish how the evolution of the system can be changed by control, they should be formulated as transformation laws, specifying the classes of changes in the evolution of the system attainable by control function from the given class, i.e. specifying the limits of control.

Let us provide a few examples of transformation laws. The first example is related to control of an invariant (constant of motions) for a conservative system. In this case, transformation law should provide an answer to the question: ‘What can be done with an invariant by means of feedback?’ A typical result (e.g. [19]) can be loosely formulated as follows:

The value of any controllable invariant can be changed for arbitrary quantity by means of arbitrarily small feedback.

The meaning of the term ‘controllable’ depends on a specific situation and, in principal, includes some conditions ensuring solvability of the problem. Examples will be given in chapter 3.

The second transformation law relates to dissipative systems. The results, presented in chapter 4 demonstrate that the smaller the dissipation in the system the larger efficiency of a small
feedback. A typical quantitative result may be expressed as follows:

The level of energy achievable by means of control of the power \( \gamma \) for controllable Hamiltonian or Lagrangian system with small dissipation of the degree \( \rho \) has the order \( (\gamma/\rho)^2 \).

The third example of transformation law relates to control of chaos. It was first articulated in the seminal paper [5] and can be termed the OGY-law:

Any controllable chaotic trajectory can be transformed into a periodic one by means of an arbitrarily small control.

Let us outline some achievements made during the last decade in the key areas of cybernetical physics: control of oscillations and chaos, quantum control, controlled synchronization, control in thermodynamics and some other topics.

4. Control of oscillations and chaos

(a) Control of oscillations

The most striking impact of the chaos control to physics was the discovery that a significant change of the chaotic system behaviour can be made by means of a small intensity feedback action. A similar phenomenon resembling resonance can be observed in some oscillatory systems, e.g. a nonlinear pendulum swinging up under the influence of a small feedback force. Such a phenomenon was called feedback resonance [1]. A related phenomenon is autoresonance (the term coined by Andronov et al. in 1937 [20]). Autoresonance is observed under excitation with a slowly changing frequency. It was discovered experimentally independently by Veksler and McMillan in the 1940s and is currently used in numerous cyclotron accelerators. Historical notes and some new results on autoresonance are presented by Agnessa Kovaleva [21].

Quite a number of research in mechanics and physics is devoted to the study of possible changes in system behaviour under periodic external disturbance. Such a disturbance may be artificially designed by an experimenter, i.e. should be considered as a version of control. First works of such kind were those of Stephenson and Kapitsa on stabilization of pendula near the upper equilibrium [22–24]. Later, Kapitsa’s approach based on direct separation of slow and fast motion was extended by Blekhman to the whole area of vibration mechanics [11] and recently still further to dynamical systems of more general origin [25]. Synchronization of chaotic systems by weak periodic excitations was considered in the 1980s–1990s in [26,27] and summarized in a monograph [28] where analytical estimates of the parameter ranges of the chaos-controlling excitation for suppression–enhancement of the initial chaos were provided. Later it was demonstrated analytically (by Melnikov analysis) and numerically that a judicious choice of the excitation’s wave form greatly improves the effectiveness of the escape-controlling excitations while keeping their amplitude and period fixed [29,30]. It would be interesting to compare the above-mentioned results with those of [1,4], where superior efficiency of feedback excitation was demonstrated. A different application related to oscillatory impulse control of chemotherapy is presented in this issue by the group of Celso Grebogi [31].

(b) Control of waves

A number of studies are devoted to various aspects of control of waves [32–37]. Recently, the SG algorithms were successfully applied to control of energy in the distributed systems described by a nonlinear PDE (sine-Gordon equation) [38]. Extensions to localization of nonlinear waves problem are presented in [39,40]. The first rigorous results justifying the SG energy control for PDE are obtained in [41].
(c) Control of chaos

A number of publications are devoted to chaotification of keeping sustained chaos. Such a control goal is important to avoid singularities in behaviour of physical systems, e.g. to maintain biodiversity [42], or to avoid voltage collapse in electrical networks [43]. A method developed by the groups of Jim Yorke and Miguel Sanjuan termed partial control [44,45] allows keeping transient chaos (the behaviour where trajectories in a certain region behave chaotically for a while before they escape to an external attractor). Three different methods for control of transient chaos are illustrated in the paper [46] by example of the Lorenz system. The results can be applied to keep the biodiversity of ecological systems. Related problems of using control to keep the biodiversity [47] and to prevent noise-induced phase transition [48] were attacked by the SG method based on energy-like goal functions.

Among other themes of active research are control of chaos under communication constraints, control of hyperchaotic and spatio-temporal (distributed) chaotic systems.

A lot of activities are observed in control of chaos in fractional-order systems [49], i.e. the continuous-time models with time-derivatives of non-integer order. The history of science says that an interest in fractional-order differentiation and more generally, fractional calculus, originated in 1695 from the response of Gottfried Leibniz to L’Hôpital letter in which he had asked Leibniz about meaning of derivative of order one half [50,51]. Leibnitz responded with the words:

‘This is an apparent paradox from which, one day, useful consequences will be drawn’.

However, applications of fractional calculus to physics and engineering were found only in the recent years. Some examples include the anomalous diffusion phenomena in inhomogeneous media that can be described by fractional-order diffusion equations [52,53], some viscoelastic materials [54], thermoelectric systems [55], etc.

(d) Delayed feedback

The method of time-delay feedback (TDF) proposed by Kestutis Pyragas in 1992 still attracted a lot of interest during the last decade. Among new achievements are adaptive versions proposed different methods of tuning control gain or time-delay [56–59]. A version was proposed allowing the so-called ‘odd numbers limitation’ to be overcome [60,61] and a deeper analysis and refutation of the ‘odd number limitation’ was given in [62–64]. However, the known applicability conditions were only approximate based on the Floquet theory. Probably, the first rigorous conditions were obtained by Leonov, who proposed to use a periodically changing control gain [65]. However for initial, classical Pyragas TDF the problem of finding rigorous sufficient conditions for its stability is still open.

(e) Chaos control: applications

It was shown that a controlled chaotic system can then serve as a versatile pattern generator that can be used for a range of application, e.g. for computing [66]. This idea is further investigated by Kia et al. [67]. A number of existing approaches and applications can be found in the Handbook of chaos control [68].

5. Synchronization

Controlled synchronization became an area of interest among physicists in the 1980s. Study of the controlled synchronization for chaotic systems started in 1990 with the seminal paper by Pecora & Carroll [6] and was growing rapidly in the 1990s. During the last decade, the main contributions were related to synchronization in complex networks. It is understandable since a lot of phenomena that can be observed in networks cannot be observable in a single system of two synchronized systems. One can mention that such a phenomenon as pinning
synchronization attracted a lot of attention owing to the seminal contributions of Ron Chen and colleagues [69–71]. Among other hot topics are cluster synchronization, synchronization by adaptive network topology, synchronization under communication constraints. We discuss them in more detail below when speaking on control of networks.

Many papers are devoted to studying possibilities and limitations of synchronization using restricted classes of control laws, e.g. linear control [72,73], sample-data control [73], simple adaptive control [74], sliding mode control [75,76], etc. A lot of attention was devoted to controlled synchronization of fractional-order systems [75,77,78].

6. Cybernetics and thermodynamics

Thermodynamics is a field of physics where methods of cybernetics (information theory and control theory) have been actively applied for several decades. Even its classical results can be interpreted in the ‘cybernetic spirit’ using the maximum entropy (MaxEnt) principle: a system evolves to a state with maximum value of entropy compatible with all constraints imposed by other physical laws [79,80]. In fact, connection between Second Law of Thermodynamics, information and control was hidden still in the ‘Maxwell’s Demon’ introduced by Maxwell as early as in 1867. Unveiling this connection, this Demon’s puzzle is still continuing even in a century and a half after the Demon’s birth. The modern understanding of this connection starts with the seminal result of Rolf Landauer, who proposed in 1961 a fundamental link between information and energy (Landauer’s principle) [81] (see [86, p. 190601-1]):

Erasing information in a macroscopic or mesoscopic system is an irreversible process that should require a minimum amount of work, \(kT \ln 2\) per bit erased, where \(T\) is the system temperature and \(k\) is Boltzmann’s constant.

Apparently, Landauer’s principle looks like one more law of cybernetical physics. Later Bennett [82] and, independently Penrose [83] related Landauer’s principle to the paradox of Maxwell’s demon. To achieve the final clarity in understanding of Maxwell’s Demon, its experimental implementation and precise measurements were needed. The issues of Maxwell’s Demon’s experimental implementation were discussed in [84,85]. Further advances and real high precision experiments were based on the notion of feedback trap [86]. In the paper by Momčilo Gavrilov & John Bechhoefer [87], the results of [86] are improved and extended. It makes the bridge between control theory and thermodynamics stronger. This bridge is further strengthened by Jean-Charles Delvenne & Henrik Sandberg [88], where the dissipativity theory initiated by Jan Willems [89] within control theory is used to model thermodynamical systems at the microscopic and macroscopic level alike.

There are also other ways for using control theory to model thermodynamical systems. One of them is the so-called ‘finite-time thermodynamics’ studying possibilities and limitations of the system evolution for finite times as well as under other types of constraints caused by a finite amount of resources available. The pioneer works devoted to evaluation of finite time limitations for heat engines were published by Novikov in 1957 [90] and Curzon & Ahlborn [91]. It was shown independently in [90,91] that the efficiency at maximum power per cycle of a heat engine coupled to its surroundings through a constant heat conductor is

\[
\eta_{\text{maxP}} = 1 - \sqrt{\frac{T_{\text{cold}}}{T_{\text{hot}}}}. \tag{6.1}
\]

Note that the Novikov–Curzon–Ahlborn process is also optimal in the sense of minimal dissipation. Otherwise, if the dissipation degree is given, the process corresponds to the maximum entropy principle. Later the results of [90,91] were extended and generalized for other criteria and for more complex situations based on the modern optimal control theory. As a result, the whole direction in thermodynamics arose known under the names optimization thermodynamics, finite-time thermodynamics or control thermodynamics, see monographs...
and surveys [92–96]. The state of the art in finite-time thermodynamics is exposed by Stanislaw Sienyutz & Anatoly Tsyrlin [97].

Another way of control theory application in thermodynamics extends the Gibbs-Jaynes MaxEnt principle [79,80] to a non-stationary case. According to the MaxEnt principle the entropy of any physical system tends to increase until it reaches its maximum possible value under the limitations imposed by other physical laws. However, this statement says nothing about how the system approaches its asymptotic. To answer this question, using the SG principle was proposed [4,98]. In the paper by Tatiana Khantuleva & Dmitry Shalymov [99] further extensions of this idea are presented.

Recently, a move in the opposite direction has become notable: thermodynamics is used for design of new control algorithms [100–103]. Mutual influence of control and thermodynamics is growing rapidly.

7. Control of networks

The term ‘networks’ has recently become one of the most usable ones both in everyday speech and in the scientific literature. During last 15 years, control of networks has become a valuable area of research, see surveys and books [104–106] and books [107–111]. Definitely, it can be associated with cybernetical physics. According to the publication rate, the control of networks is leading both in physical and in the control literature. The search over the largest database in the automation and electrical engineering IEEE Xplore (http://www.ieee.org) exhibits doubling of the number of papers with the terms COMPLEX and NETWORKS in the title in 2005–2006 compared to 2000–2001. In 3 years (in 2008–2009), this amount has tripled! An exposure of an interest in this was also observed in physics: the number of such papers published in the APS journals (www.aps.org) during 2005–2006 was 91 against just four papers in 2000–2001 and 2012–2013. Currently, the Web of Science database shows more than 50 journals with the word ‘network’ in the title. Two seminal papers by Albert & Barabasi [112,113] published in 1999 and 2002 symbolized the onset of the Network Science. In 2016, they have more than 11 000 and 8000 citations, respectively.

Among various control goals synchronization (consensus) used to be the most popular one. Controlled synchronization of dynamical networks is motivated by a broad area of potential applications: formation control, cooperative control, control of power networks, communication networks, production networks and so on. Mathematical tools for study are based on Lyapunov functions formed as the sum of Lyapunov functions for local subsystems. The adequate mathematical formalism here is a mix of stability theory (usually Lyapunov or input–output methods) and graph theory (the properties of a system significantly depend on the spectrum of the so-called Laplacian matrix of the graph of connections defined via its adjacency matrix). An efficient tool joining the topological properties of a network with its dynamics has been proposed by Pecora and Carroll in 1998, who introduced the master stability function [114].

A number of recent researches was devoted to output feedback synchronization of nonlinear systems [115–117].

More recently, a lot of interest was attracted by pinning control where only a fraction of the nodes are affected by control. Important contributions in pinning control were made by Chen and co-workers [69–71,118]. In fact, feasibility question for the pinning control is an issue of controllability of the network. Such a question may be generalized and one may ask when the specified structure of the network is controllable. Such a question got a broader attention after the paper [119]. Chen and co-workers [120] provide review of and further insight into the controllability of the networks.

Recent studies led also to discovery of more complex synchronization-related phenomena like cluster synchronization [121–123], chimaera states [124–127], polarization [128,129], motifs [130–132], etc. Schöll and his group have valuable contributions into development of most of those new concepts. Some of them are surveyed by Schöll and co-workers [133] and in the recent book [134].
8. Quantum control

Control of quantum-mechanical systems has already a serious history. First works in this area can be traced back to the 1970s [135–138] The main difficulties in controlling processes on the atomic and molecular levels are related to the small spatial size of the controlled systems and the high rates of the processes. The average size of molecules (monomers) is around 10 nm, the average distance between the atoms in a molecule is 1 nm, and the average speed of the atoms and molecules at room temperature is $10^2$–$10^3$ m s$^{-1}$ and the period of natural oscillations of the atoms in a molecule is 10–100 fs. Measurement and control at such spatial and temporal scales constitutes an extremely complicated scientific and technological problem. Invention of the femtosecond pulse lasers at the end of the 1980s gave rise to a new interest to quantum control in chemistry since duration of such pulses may be comparable with the period of natural oscillations of single atoms or molecules. It allowed for intervention into the natural course of chemical reactions and led to the creation of femtochemistry and the Nobel prize for Ahmed Zewail in 1999 [139]. An idea of adaptation and learning for quantum control was introduced in the seminal work [140] that has more than 1200 citations in 2016. It was applied in a number of experiments [141] and the results were summarized in a number of special issues of the journals [142,143] and books [144–146].

During first decade of the new century, a number of theoretical advances appeared like controllability study [147], Lyapunov-based control [148–150], optimal control [151,152], stochastic control [153–158], etc.

Surveys of control theoretic achievements can be found, for example, in [159–161].

Optimal control techniques have also been successfully applied to multi-dimensional nuclear magnetic resonance targeting at improving the sensitivity of these systems [162,163].

The next important milestone was the 2012 Nobel prize awarded to Serge Haroche and David Weinland ‘for ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems’ [164,165].

In the second decade of the twenty-first century, the research was going mostly along the same lines [150]. Quantum control has recently also been applied to photons in cavities [166,167]. Most achievements are summarized in a few special issues of the international journals [168–170]. A number of open problems can be found in [171–173].

9. Optimal control

Another important contribution of control theory into modern physics is the idea of discovering limits for changes of system behaviour caused by external action. Such a possibility is based on the concept of optimality and is provided by optimal control theory. A general view on applications of optimal control in science is provided in [174,175]. Sieniutycz & Tsirlin [97] is devoted to finding limiting possibilities of thermodynamic systems by optimization.

10. Conclusion

Physics and cybernetics are very different sciences. The history of Physics counts more than 2000 years. Cybernetics on the contrary is relatively young and has well recognized date of birth: 1948, when the book by Norbert Wiener [176] was published. Therefore, it is not surprising that for a long time not much interaction between theoretical physics and cybernetics (control theory) could be observed. However, situation has changed at the end of the twentieth century and in the beginning of the twenty-first century there was an explosion of such an interest. The number of control-related papers in the physical journals increased dramatically. This interest is still growing and the picture recalls the words of Norbert Wiener [176, p. 2]: ‘For many years Dr. Rosenblueth and I had shared the conviction that the most fruitful areas for the growth of the sciences were those which had been neglected as a no-man’s land between the various established fields’. Wiener was writing those words when Cybernetics was a no-man’s land. However by the
beginning of the twenty-first century Cybernetics has become a well-established field and now even the land between Cybernetics and Physics is not a no-man’s land any more. It is occupied by the Cybernetical Physics and I believe that this area is one of the most fruitful areas for the growth of sciences.

Competing interests. I declare I have no competing interests.

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