Supplementary Methods

Comparison of inverse solutions: exposing the worst

One of the main goals of a tomographic method is localization of function.

For this reason, rather than avoiding the main issue by illustrating other properties, emphasis will be placed here on exposing localization errors.

The general procedure consists of carrying out experiments in which estimated tomographic images (\( \hat{J} \) or \( \hat{L} \)) are produced from EEG/MEG data generated for a known current density (\( J \) or \( L \)). Different measures of mismatch between the estimated and actual current density can then be used for quantifying the degree of failure of the tomography.

1. The localization error

The simplest measure of localization error is defined as the distance between an actual point-test source and the location of the absolute maximum of the estimated current density power (squared current density magnitude). Quantitatively, consider a point-test source with unit strength (and if applicable, random orientation) at the \( \beta \)-th voxel, and let \( r_{\beta} \in \mathbb{R}^{3x1} \) (for \( \beta = 1...N_v \)) denote its position vector. Let \( \hat{r}_{\beta} \in \mathbb{R}^{3x1} \) denote the position vector of the location of the absolute maximum of the estimated current density power. Then the localization error is:

Eq. 1: \( e_{\beta} = ||r_{\beta} - \hat{r}_{\beta}|| \)

From the set of localization errors to all possible point-test sources \( \{ e_{\beta} , \beta = 1...N_v \} \), the mean, median, and standard deviations are reported in this work.

Ideally, under ideal, noiseless conditions, the localization error should be strictly zero.

2. Mislocalized brain volume

Consider the case of a point-test source at some target voxel. A very informative measure of failure of a tomography is the brain volume that has estimated power greater than the estimated power at the actual target voxel. This gives the brain volume that is mislocalized by the tomography, and in this work, it is reported as the fraction relative to the total brain volume.

In quantitative terms, let \( P_{\beta j} \), for \( \beta, j = 1...N_v \), denote the estimated current density power at the \( j \)-th voxel (using some tomographic method), due to an actual point-
test source at the $\beta$-th voxel (the target voxel). Then $P_{\beta j}$ is the estimated power at the target voxel $\beta$, and the mislocalized brain volume is:

$$MBV_\beta = \left[ \sum_{j=1}^{N_V} I(P_{\beta j} \geq P_{\beta j}) \right]^{-1}$$

where $I(\bullet)$ is the indicator function, taking the value 1 if the argument is true, and taking the value 0 if the argument is false.

From the set of mislocalized brain volumes to all possible point-test sources $\{MBV_\beta, \beta = 1...N_V\}$, the mean, median, and standard deviations are reported in this work.

Ideally, under ideal, noiseless conditions, the mislocalized brain volume should be strictly zero.

3. ROC curves for single point-test sources

This type of ROC curve will be defined for the collection of all normalized tomographic images of current density power, due to all possible single point-test sources. This means that the total number of voxels in this collection is $N^2_V$, where $N_V$ is the actual number of true positive voxels, and $(N^2_V - N_V) = N_V(N_V - 1)$ is the actual number of true negative voxels.

For a given actual point-test source at the $\beta$-th voxel, let:

$$\text{max}_\beta = \text{Maximum}_{ij}\{P_{\beta j}\}$$
denote the absolute maximum current density power of the tomographic image.

The normalized tomographic image is defined as:

$$D_{\beta j} = \frac{P_{\beta j}}{\text{max}_\beta}$$

which now has a maximum value of one.

The collection of values $D_{\beta j}$, for $\beta,j = 1...N_V$, can now be used to define the classical ROC curve. Let $0 \leq t \leq 1$ denote the threshold value that defines a voxel as active (i.e. “positive”) if its power is greater or equal than $t$. Then the fraction of true positives is:

$$\text{%TruePos}(t) = \frac{\sum_{\beta=1}^{N_V} I(D_{\beta \beta} \geq t)}{N_V}$$

and the fraction of false positives is:

$$\text{%FalsePos}(t) = \frac{\sum_{\beta=1}^{N_V} \sum_{j=1}^{N_V} I(D_{\beta j} \geq t)}{N_V(N_V - 1)}$$
The ROC curve plots the fraction of true positives as a function of the fraction of false positives, by varying the threshold from zero to one. The quality of a tomography can then be judged by the area under the ROC curve ($0 < AUC \leq 1$), which ideally should be one.

A low AUC value indicates that the tomography gives high power to the wrong voxels, which will certainly lead to incorrect localization.

### 3. ROC curves for pairs of point-test sources

This type of ROC curve will be defined for a collection of normalized tomographic images of current density power, due to pairs of point-test sources. If the number of pairs of point-test sources is denoted as $N_p$, then the total number of voxels in this collection is $N_pN_v$, where $2N_p$ is the actual number of true positive voxels, and $N_pN_v - 2N_p = (N_v - 2)N_p$ is the actual number of true negative voxels.

For a given actual pair of point-test sources at the $\beta$-th and $\delta$-th voxels, let:

$$\text{Eq. 7: } \max_{ij \beta \delta} = \text{Maximum}\{P_{ij \beta \delta}\}$$

denote the absolute maximum current density power of the tomographic image.

Note that in this case $P_{ij \beta \delta}$, for $j = 1...N_v$, denotes the estimated current density power at the $j$-th voxel (using some tomographic method), due to an actual pair of point-test sources at the $\beta$-th and $\delta$-th voxels (the target voxels).

The normalized tomographic image is defined as:

$$\text{Eq. 8: } D_{\beta \delta j} = \frac{P_{\beta \delta j}}{\max_{i \beta \delta}}$$

which now has a maximum value of one.

Let $(\beta_i, \delta_i)$, for $i = 1...N_p$, denote a collection of pairs of point-test sources. Then the collection of values $D_{\beta_i \delta_i j}$, for $j = 1...N_v$ and for $i = 1...N_p$, can now be used to define the classical ROC curve. Let $0 \leq t \leq 1$ denote the threshold value that defines a voxel as active (i.e. “positive”) if its power is greater or equal than $t$. Then the fraction of true positives is:

$$\text{Eq. 9: } \%\text{TruePos}(t) = \frac{\sum_{i=1}^{N_p} \sum_{j=1}^{N_v} I(D_{\beta_i \delta_i j} \geq t) + \sum_{i=1}^{N_p} I(D_{\beta_i \delta_i \beta_i} \geq t)}{2N_p}$$

and the fraction of false positives is:

$$\text{Eq. 10: } \%\text{FalsePos}(t) = \frac{\sum_{i=1}^{N_p} \sum_{j=1}^{N_v} \sum_{j' \neq \beta_i} I(D_{\beta_i \delta_i j} \geq t)}{(N_v - 2)N_p}$$

The ROC curve plots the fraction of true positives as a function of the fraction of false positives, by varying the threshold from zero to one. The quality of a tomography can
then be judged by the area under the ROC curve ($0 < AUC \leq 1$), which ideally should be one.

A low AUC value indicates that the tomography gives high power to the wrong voxels, which will certainly lead to incorrect localization.