Mixing and transport in urban areas

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Concern over terrorist releases of harmful material has generated interest in short-range air-borne dispersion in urban areas. Here, we review the important fluid dynamical processes that control dispersion in the first kilometre, the neighbourhood scale, when much of the material remains within the urban canopy. Dispersion is then controlled by turbulent mixing and mean flow transport through the network of streets. We consider mixing and transport in a long straight street, street intersections and then a network of streets connected by intersections. The mixing and transport in these systems are illustrated with results from recent fine-resolution numerical simulations and laboratory models, which then inform simpler scaling estimates and modelling schemes. Finally, we make some tentative steps to pull the process studies together to begin to understand results from full-scale observations. In particular, it is shown that the positions of ‘shear layers’ and ‘dividing streamlines’ largely control the patterns of mixing and transport. It is also shown that neighbourhood-scale dispersion follows one scaling in the near field and another in the far field after passage through many intersections. The challenge for the future is to bring these threads together into a coherent mathematical model.

Keywords: fluid dynamics; atmospheric dispersion; urban areas

1. Introduction

Recent events have focussed the world’s attention on the possibilities of terrorist attack. Urban areas are particularly vulnerable with their concentrated populations. This recognition has motivated efforts to understand mixing and transport of air-borne pollutants within urban areas. In the case of an emergency in which material is released at a point, rapid decisions need to be made about where the material is being transported by the wind and how widely it is being spread and diluted by mixing with uncontaminated air. Scientific understanding of these processes is also motivated by concerns over the health impacts of poor air quality, again particularly in urban areas. In this case, the sources of the pollutants, for example car exhausts, are spread over a wide area, and so the accumulation of material at any particular location is the sum of releases from many sources.

If we are to gain understanding of mixing and transport in urban areas, then we need to face the bewildering complexity and variability that drives New Yorkers to complain that ‘In the city the wind is always in your face’. The air flow is almost always turbulent; it is driven by a combination of synoptic scale
wind, surface heating and moving traffic, all within a complex network of streets and buildings. New possibilities in computing turbulent flow and in new observational techniques, together with wind-tunnel measurements, have led to rapid developments in the last few years. The aim here is to give a personal view of these developments, with emphasis on the fundamental fluid dynamical processes that control the mixing and transport in simplified urban geometries.

First, we identify how this problem relates to previous work on dispersion in the atmosphere. The pressure gradients associated with the synoptic-scale weather systems drive winds. The wind speed goes to zero at the Earth’s surface, so that the wind profile in the lowest kilometre of the atmosphere is highly sheared, leading to random, turbulent, motion. This part of the atmosphere is known as the atmospheric boundary layer. If smoke is released in a large flat field on a windy day, then the smoke is transported by the wind. At the same time turbulence mixes the plume both across the wind and upwards. A snapshot reveals a highly irregular boundary to the plume, caused by the irregular, and fundamentally unpredictable, turbulent motions. But time-lapse photographs reveal that the averaged plume boundary takes a more predictable form. It grows roughly linearly with distance close to the source and as the square root of distance further away. In a seminal paper, Taylor (1921) first described the fundamental theory of this ‘diffusion by continuous movement’. As we shall see later dispersion in street networks has analogous regimes.

This dispersion from a source over flat ground, when the concentration across the plume has a Gaussian variation, is now well understood. Good predictions can be made, for example, of the average spread of the plume and the strength of the concentration fluctuations (see Pasquill & Smith 1983; Arya 1999). Now, such flat ground is invariably covered with ‘roughness elements’, such as grass, crops or forests. Above about three to five times their heights, buildings can also be represented as roughness (Rotach 1993a,b). The Gaussian plume methods work well when the plume height is large compared with the size of the roughness elements, which can then be parameterized simply.

Similarly, dispersion in the wake of a single large building is also reasonably well understood motivated by concerns over safety, particularly of nuclear power stations (e.g. Hosker 1985; Vincent 1977, 1978). As reviewed by Robins & McHugh (2001), the main effects are associated with the wake behind the building. Extending two to three building heights downwind is a recirculating flow region of slowly circulating air in the sheltered lee of the building. Beyond, in the far wake, the wind speed recovers to its upwind value after 10 to 30 building heights. Material released upwind of the building is swept over or around and can then be entrained into the recirculating flow region. It is then released slowly. Meanwhile, the remainder of the plume is carried downstream and is rapidly mixed across the far wake by the strong turbulence generated by the building.

The new challenge, posed particularly by a possible terrorist release, is to understand the variation of average and peak concentrations within an urban area in the first kilometre or so of a point source release (Britter & Hanna 2003). The material has then dispersed around several buildings, and the plume depth is comparable to the building heights. It is this neighbourhood scale that is the focus here. The characteristic of this scale is that much of the released material is still within the building canopy, and so it is the mixing and transport actually through the network of streets that is important in determining exposure at
ground level. Further downwind, most of the material is above the buildings, and the ground-level concentrations are determined by the transport and mixing in the boundary layer above.

This review builds upon the comprehensive surveys of Britter & Hanna (2003) and Hunt & Carruthers (2003). The particular focus here is on the fluid dynamical processes, which will be illustrated on simple urban geometry. In §2, we examine mixing and transport in some simple elements of urban geometry. Sections 3 and 4 describe fluid dynamics of the flow and dispersion on the neighbourhood scale. Finally, in §5 some conclusions are drawn and priorities for future work identified.

2. Mixing and transport at street scale

To begin, we identify the mixing and transport patterns in simple geometrical elements that make up an urban area, namely a simple long straight street and simple street intersections.

(a) Mixing and transport in a simple street

The simplest element of an urban area is a simple street of length $L$, with rows of straight buildings, height $H$, separated by a road of width $W$, which encloses a street canyon, see figure 1. Consider first a symmetrical configuration with $H/W \approx 1$. Figure 1a shows a side view of the flow when the wind blows perpendicular to the street. There is a layer of strong shear (where the wind speed increases rapidly with height) at roof level across the top of the street...
canyon. Louka et al. (2000) showed that the position of this roof top shear layer intermittently flaps up and down. When the shear layer moves down, fast moving air from above roof level impinges upon the downwind building and drives an intermittent mixing circulation around the street canyon, mixing air across the street canyon (e.g. DePaul & Sheih 1986; Hoydysh & Dabbert 1988). When the wind blows parallel to the street, there is strong transport as the wind is channelled along the street (e.g. Nakamura & Oke 1988; Johnson & Hunter 1999), as shown schematically in figure 1b.

Figure 1c shows schematically the flow when the wind at roof level, $u_H$, blows at an angle $\theta$ to the street. The flow in the street can be considered to be a superposition of channelling and a mixing circulation, leading to helical paths of the air parcels (e.g. Nakamura & Oke 1988; Johnson & Hunter 1999). Dobre et al. (2005) analysed measurements taken in the DAPPLE (Dispersion of Air Pollution & Penetration into the Local Environment) observational campaign in London (see www.dapple.org.uk) to show that this superposition works quantitatively. Flow in the street is the vector superposition of a channelling flow that scales on the parallel component of the above-roof wind, $u_{||} = u_H \cos \theta$, and a mixing circulation that scales on the perpendicular component of the above-roof wind, $u_{\perp} = u_H \sin \theta$. An interesting consequence of this pattern of mixing and transport is that air may be transported along the street at a large angle, up to nearly 90°, to the above-roof wind direction. This observation gives the first clue that dispersion through street networks can be very different from dispersion over level ground. A number of practical dispersion models are based on this simple street flow (e.g. Berkowicz 2000; CERC 2000). However, there is a simpler, and intuitively appealing, method initiated by Soulhac (2000) that is based on the idea that the mixing circulation across the street canyon leads to concentrations being nearly well mixed and so uniform across the street canyon. The time taken to mix across the street is $T_1 \approx (H+W)/u_{\perp}$. Over this time an air parcel is transported along the street a distance of the order of $u_{||} T_1$, which is shorter than the length of the street if $L$ is much greater than $(H+W)u_{||}/u_{\perp}$. So for a 45° wind, when $u_{||} = u_{\perp}$, for a square cross-section, $H=W$, the street needs to be only about four times longer than the building height for the air to be thoroughly mixed. The concentration of well-mixed material, $\bar{C}$, can then be calculated from a budget integrated over the volume of the street, as indicated schematically in figure 1c and which can be written mathematically as

$$V \frac{d\bar{C}}{dt} = Q - F_V - \Delta F_T.$$  

(2.1)

Here, $V$ is the street volume, $Q$ is the source rate, $F_V$ is the flux ventilated from the top of the street and $\Delta F_T$ is the rate of loss due to differences in transport across the two ends of the street. The ventilation flux and the loss by transport differences are considered separately.

(b) Ventilation flux from a simple street

The ventilation flux has been obtained from numerical simulations of the flow (e.g. Sini et al. 1996), but as discussed later these models have significant uncertainties in the way they represent the turbulent mixing. Recently, at Reading, we have developed a wind tunnel technique to measure the ventilation...
flux (Barlow & Belcher 2002; Barlow et al. 2004). The idea is to use a volatile material as a proxy for the pollutant dispersing in a scale model placed in a wind tunnel. We have used napthalene, the material that our grandmothers used to keep moths away from clothes! The napthalene sublimes from solid to gas at room temperature, forming a thin layer of saturated vapour just above the coated surface of the model. The model is placed in a wind tunnel, and the wind is blown over the model. The saturated vapour then mixes through the street and is ventilated out into the boundary layer above. Further solid napthalene sublimes quickly to maintain the saturated layer just above the surface. After about 20 min the lost mass of napthalene is measurable, from which the ventilation flux, \( F_V \), can be calculated. The results can be generalized to any pollutant as follows. On dimensional grounds

\[
F_V = A w_t \Delta C,
\]

where \( A = W \times L \) is the area of the street and \( \Delta C \) is the difference in napthalene concentration between the saturated layer and above the roofs (where it is taken to be zero). Equation (2.2) is used to evaluate \( w_t \), the transfer velocity or the average speed of ventilation of any pollutant.

Figure 2 shows results from such measurements for transfer from the road surface of a simple street with a perpendicular wind. Transfer from the road into the street air volume largely controls the absolute value of the transfer. Harman et al. (2004) developed a model that shows how the factor two variation with \( H/W \) is the result of two effects of geometry on the flow. Firstly, the transfer velocity is proportional to the above-roof wind speed, which is determined by the roughness of the array of streets. As \( H/W \) increases so the roughness of the array of streets increases, and the above-roof wind speed decreases. The mixing and ventilation flows are then also reduced, and so is the transfer velocity. This explains the drop in transfer velocity as \( H/W \) increases from 0 to about 0.35. Secondly, the flow within the street canyon changes with geometry. Caton et al. (2003) showed that when \( H/W = 1 \) ventilation can be quantified by understanding the mixing across the roof top shear layer described in §2a. Harman et al. (2004) show that as geometry changes so does the behaviour of the roof top
shear layer and hence ventilation. The roof top shear layer is deflected downwards by pressure forces as it moves across the street canyon. When $W$ is greater than about $3H$, it reaches the street surface before impinging upon the downwind building, and so above-roof air sweeps down and efficiently ventilates the street. When $W$ is less than about $1.5H$, the roof top shear layer impinges onto the downwind building setting up the mixing circulation in the street canyon. As $H$ further increases, the circulation is slowed by friction of the building wall before it reaches the street surface, thus reducing the transfer velocity from the street for $H/W > 2/3$.

The organizing principle of the dynamics of the roof top shear layer, together with measurements using the naphthalene sublimation method, should help quantify ventilation from more complex street geometries, such as pitched roofs (Rafailidis 1997), unequal building heights (Longley et al. 2004) and realistic geometries (Kastner-Klein & Rotach 2004; Kastner-Klein et al. 2004).

(c) Flow patterns in street intersections

As first recognized by Scaperdas et al. (2000) and Soulhac (2000), street intersections are a second basic element of urban geometry that are critical in dispersion. The volume-averaged model of the concentration in the street network (equation (2.1)) emphasizes the transport fluxes into and out of streets, and these fluxes originate from intersections. There are two generic intersections: a four-way intersection, where two simple streets cross at $90^\circ$, and a T-junction, where two simple streets meet at $90^\circ$ and the entrance street terminates.

Figure 3 shows typical transport patterns in the vicinity of these two types of intersection, when the above-roof flow is aligned along an entrance street. Boddy et al. (2005) identified the T-junction, shown in figure 3a, as a strong force for lateral and vertical dispersion. The flow emerges from the entrance street as a jet.
This jet then impinges on the downwind wall, leading to a recirculation region where the centreline of the entrance street impinges upon the downwind wall. The centreline is then a dividing streamline, shown with dashed lines in figure 3a. Air parcels that originate above the dividing streamline are forced upwards and over the building. Air parcels to the left or right of the dividing streamline are deflected into either the left- or right-hand exit streets. The flow in the exit streets combines with the mixing circulation forced by the above-roof wind, as in a simple street, to generate helical paths for the air parcels. Far enough away from the intersection, the component of motion along the exit street decays, and the flow reverts to a simple street flow with a perpendicular wind, with no along street transport. For this reason a flux of air equal to all the incoming jet must get ventilated out of the street into the boundary layer above. At present we do not know how long the exit streets have to be to reach this state; the estimate developed in §2a suggests \( L \) must be greater than about \( 4H \).

Kastner-Klein et al. (2004) and Brown et al. (2004) measured the effects of a four-way junction on the flow in a simple street and identified the circulation sketched in figure 3b. Perpendicular flow over a simple street generates a mixing circulation, with its axis aligned horizontally along the street (see figure 1a). Similarly, at a four-way junction the jet that emerges from the entrance street generates a mixing circulation, this time with its axis aligned vertically. In effect this corner vortex is just the end part of the mixing circulation in the body of the street: within a distance of the order of \( H \) the axis of the mixing circulation tilts into the vertical. The corner vortex is thus driven by similar shear layer processes, and has similar dimensions, to the mixing circulation in a simple street. Importantly there is no dividing streamline at a perfectly aligned four-way junction. But if the junction is not perfectly aligned, or if the above-roof wind direction is not perfectly aligned with the entrance street, then there is a dividing streamline, because the jet impinges upon a downwind building, and we expect enhanced dispersion as observed by Scaperdas et al. (2000).

The discussion here suggests that two principles can organize the complex flow patterns found in streets and intersections. These are: the dynamics of the shear layers shed from roofs and corners, which generate mixing circulations in street canyons and their end manifestation of corner vortices; and the topology of dividing streamlines in the flow, a matter we return to in later sections.

3. Dynamical processes at the neighbourhood scale

Over longer length scales, air is transported along several streets and over and around several buildings. This is the neighbourhood scale (Britter & Hanna 2003). There is complexity in the neighbourhood scale arising from interactions of the flow around more than one building and along more than one street. Nevertheless, it is also possible to simplify analysis when thinking on the neighbourhood scale, as we shall see.

(a) Numerical simulations of flow around arrays of buildings

Increased computer power is just making it possible to compute explicitly the atmospheric boundary-layer flow through streets and around arrays of buildings. Different approaches have been taken to handling the turbulence. In the so-called
Reynolds averaged approach, the effects of turbulence are parameterized, and a steady, time-mean, flow is calculated. Commercial software packages based on this approach are available that can handle the very complex geometry of real urban areas. But the parameterizations of the turbulence have been developed for simple flows, and there are concerns that this approach does not capture important aspects of these complex flows, especially in the unsteady wakes behind the buildings. Consequently, there is increasing effort directed towards simulation methods that resolve explicitly the unsteady turbulence.

The range of scales of motions present in a turbulent flow is a function of the Reynolds number, defined to be $Re = UL/v$, where $U$ and $L$ are characteristic velocity and length scales of the flow and $v$ is the molecular viscosity. In the real atmospheric boundary layer the Reynolds number is so large (being of the order of $Re = 10 \times 30/1.5 \times 10^{-5} = 2 \times 10^7$) that even present computers cannot compute the full range of length scales in an unsteady calculation (so-called direct numerical simulation or DNS). Instead the technique of large-eddy simulation (known as LES) is used, where the large-scale overturning motions are computed explicitly, but the small scales are parameterized (e.g. Cui et al. 2004; Kanda et al. 2004).

In wind tunnel flows the Reynolds number is smaller, of the order of $Re = 4 \times 0.2/1.5 \times 10^{-5} = 5000$. Although this Reynolds number is much smaller than in real urban areas, careful measurements made in wind tunnels suggest that it is high enough to be in the high-Reynolds number regime, so that further increases change only slightly the important properties of the flow (Castro & Robins 1977). It is just beginning to be possible to compute DNS of the whole spectrum of turbulent fluctuations over simplified urban geometry at this Reynolds number. Recently Coceal et al. (submitted) have reported DNS of boundary-layer flow over a surface covered with regular arrays of cubes and shown excellent agreement with detailed wind tunnel measurements of Cheng & Castro (2002).

Figure 4 shows snapshots of the spanwise vorticity\(^1\) and illustrates the exquisite detail obtained in these simulations. The flow is from left to right and the cubes, shown in black, are in a staggered arrangement. The domain is periodic in the streamwise and spanwise directions. Figure 4a shows contours of spanwise vorticity on the vertical plane at $y = 0.5$ (in units where the cube height is one). The arrows indicate the direction of the mean flow and show a separated wake region extending about 2.5 units downstream of the cube and another ahead of the downwind cube (which in the periodic domain has its front face at $x = 4$). Notice how high vorticity is generated on the roof of the upwind cube and shed downstream, forming a shear layer, of high spanwise vorticity, and high turbulence, at roof level. This shear layer inhibits interaction between the flows above and below roof level. Below roof level both the intensity and the length scale of the vorticity and turbulent fluctuations are smaller than above roof level (see also Louka et al. 2000). Animations of the DNS results show that flows within and above the roof level evolve largely independently for surprisingly long periods (corresponding to perhaps 10 min when scaled up to real urban areas) but with occasional bursts of fluid from below roof level into the boundary layer aloft. The mechanism

\(^1\) Spanwise vorticity is a measure of the spin of an air parcel around an axis in the spanwise direction, computed from the spatial gradient of velocity.
controlling these bursts is not clear at present. It would be interesting to know how the picture changes with different patterns of buildings and with buildings of different heights.

Figure 4b shows contours of spanwise vorticity on the horizontal plane at $z=0.5$. Again the arrows indicate the mean flow and show the separated wake region behind the cubes. The dotted lines at $y=1.5$ and $y=3.5$ indicate the dividing streamlines for the mean flow at the T-junction ahead of the cube: air parcels starting either side of this line pass, on average, either side of the downstream cube, a powerful dispersion mechanism, which is discussed further in §4. The arrows in the wake of cube one in figure 4b also indicate how mean streamlines are brought together downstream of the wake. In this way air parcels are on average subject to periodic stretching and squashing, which distorts the turbulence (as in flow beneath a water wave, see Teixeira & Belcher 2002). Hunt & Eames (2003) have argued that the resulting strain in the wake tends to annihilate vorticity there, so destroying the wake. There is a suggestion in the snapshot that the high vorticity generated on the front face of the cube is substantially reduced once it has reached about halfway between cubes three and four.

Simulations like these offer tremendous opportunities to evaluate processes and mechanisms in these complex turbulent flows. New simulation methods such as detached eddy simulation (Spalart et al. 1997), which aims to combine the best features of LES and Reynolds averaged models, and immersed boundary methods (Tseng & Ferziger 2003), which can handle very complex geometries, would be well-suited to this problem. Recently, J. Boris and colleagues have developed advanced computational fluid dynamics methods that have been used to simulate flow around several square kilometres of Chicago and Manhattan resolving all the main buildings (see Pullen et al. 2005).

(b) Canopy dynamics at the neighbourhood scale

For many applications we do not want all the detail of the flow resolved around each building. Imagine a simulation of London, where the unsteady turbulent flow around each building was resolved. The computational effort would be huge and the volume of data produced enormous, and so it is helpful to have a more compact description. Here, one promising approach is described briefly.

The atmospheric boundary layer is almost always turbulent. And in many applications, such as weather forecasting, only quantities averaged in time over the turbulent fluctuations are required. This approach capitalizes on the randomness in the turbulent fluctuations. The canopy approach, which was originally developed for flow through vegetation canopies (e.g. Finnigan 2000), capitalizes on the randomness in space caused by the complex geometry. And so variables, such as wind speed, are averaged over time and space. Prognostic variables, such as the wind components, are then separated into three components

$$ u = U + \bar{u} + u' $$

These three components are:

(i) $U = \langle \bar{u} \rangle$ is the spatially averaged mean wind, obtained by averaging measurements over both space (denoted with the angle brackets) and time (denoted by the overbar).
Figure 4. Snapshot of spanwise vorticity in a direct numerical simulation of turbulent boundary-layer flow over an array of surface-mounted cubes. (a) Vertical plane through a cube centreline; (b) horizontal plane at half cube height. Arrows indicate direction of mean flow; dashed lines are dividing streamlines.
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\[ \frac{\partial}{\partial z} \langle u'w' \rangle = \frac{\partial}{\partial z} \langle \bar{u} \bar{w} \rangle \]

inertial sublayer

\[ \frac{\partial}{\partial z} \langle u'w' \rangle - \frac{\partial}{\partial z} \langle \bar{u} \bar{w} \rangle \]

roughness sublayer

\[ \frac{\partial}{\partial z} \langle u'w' \rangle - \frac{\partial}{\partial z} \langle \bar{u} \bar{w} \rangle + D \]

canopy layer

Figure 5. Role of terms in the canopy momentum balance. Above the layer of influence of the buildings is the inertial sublayer, where only the turbulent stress acts. Closer to the surface is the roughness sublayer, where the inertial and turbulent stresses act. Below roof level is the canopy layer, where the aerodynamic drag of the buildings acts in addition to the turbulent and inertial stresses.

(i) \( \bar{u} = \bar{u} - U \) is the spatial fluctuation, which is the variation of time-averaged wind about the spatial mean and is associated with flow around individual buildings.

(ii) \( u' = u - \bar{U} - \bar{u} \) is the turbulence, which fluctuates randomly in both time and space.

The aim is then to develop a model to predict the spatially averaged mean wind, \( U(x) \).

Dynamical effects of the canopy are found by considering the equation for the spatially averaged momentum. Finnigan (2000) describes the mathematical procedure. Substitution of the triple velocity decomposition into the momentum equation and averaging over space and time yields an equation, which, for the horizontal component (on neglecting horizontal gradients of stress terms, which are smaller than the vertical gradients), becomes

\[ \rho \frac{DU}{Dt} + \frac{\partial P}{\partial x} = -\rho \frac{\partial}{\partial z} \langle u'w' \rangle - \rho \frac{\partial}{\partial z} \langle \bar{u} \bar{w} \rangle - D. \]  

(3.2)

The left-hand side contains the acceleration of an air parcel and the pressure gradient. Three forcing terms appear on the right-hand side as a result of the two averaging procedures. Figure 5 shows a schematic of the roles played by these three terms in different regions of the boundary layer:

(i) \( \rho \langle u'w' \rangle \) is the spatial average of the turbulent stress, which represents transport of momentum by turbulent eddies and is active throughout the atmospheric boundary layer.

(ii) \( \rho \langle \bar{u} \bar{w} \rangle \) is the inertial stress. Lower down, towards the surface, individual buildings disturb the flow, leading to spatial fluctuations \( \bar{u}, \bar{v} \) and \( \bar{w} \) and possibly to their correlation \( \rho \langle \bar{u} \bar{w} \rangle \), which represents transport of momentum by spatial fluctuations. This term also arises for example in boundary-layer flow over gentle hills, where it represents drag exerted by the hill on the air (e.g. Belcher & Hunt 1998).

In the literature on vegetation canopies, this term is called the dispersive stress. Here the term inertial stress is used, partly because it is more descriptive of its dynamical origin and partly to avoid confusion when we discuss dispersion.

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(iii) $D$ is the canopy drag. This term acts within the building canopy, where pressure difference across buildings gives rise to an aerodynamic force on the buildings and an equal and opposite force on the airflow.

To form a predictive model these three terms need to be modelled. Belcher et al. (2003) show how $D$ can be modelled. Suppose that the urban area covers a total area $A_t$, and consists of $N$ buildings, all of height $H$, of the same shape and each presenting an area $A_f$ to the wind and of plan area $A_p$. The surface is then characterized by the so-called lambda parameters (Britten & Hanna 2003)

$$\lambda_p = \frac{NA_p}{A_t}, \quad \lambda_f = \frac{NA_f}{A_t},$$

(3.3)

which are equal for cubes. The pressure force on a single building is $(1/2)\rho U^2 c_d A_f$ (where $c_d$ is a sectional drag coefficient and $A_f$ is the frontal area of the building). Assume for simplicity that the drag is distributed uniformly through the volume of air in the urban canopy, $(1 - \lambda_p)(H \times A_t)$, then the force per unit volume becomes

$$\rho D = \frac{N \times (1/2)\rho U^2 c_d A_f}{(1 - \lambda_p)(H \times A_t)} = \frac{c_d \lambda_f}{2H(1 - \lambda_p)} \rho U^2 = \rho \frac{U^2}{L_c}.$$  

(3.4)

Here, the canopy drag length scale is defined to be

$$L_c = \frac{2H(1 - \lambda_p)}{c_d \lambda_f} \approx \frac{H(1 - \lambda_p)}{\lambda_f},$$

(3.5)

where the last step uses the value of $c_d=2$ recommended by Coceal & Belcher (2004). The canopy drag length scale is important as it sets a length scale for both vertical and streamwise variation in the canopy.

In a homogeneous canopy, the inertial stresses are negligible above the top of the canopy (Cheng & Castro 2002). There is then a balance within the canopy between turbulent transport of horizontal momentum downwards from the boundary layer above and loss of momentum from the airflow onto the obstacles. (For sparse or shallow canopies, the obstacles do not take up all the momentum supplied from above; the remainder is taken up by the ground surface, a complication ignored here.) Bentham & Britter (2003) have shown how integrating this balance over the depth of the canopy yields a characteristic wind speed for the canopy, $U_c$, which we write here as

$$U_c = \left( \frac{L_c}{H} \right)^{1/2} u_*.$$  

(3.6)

(Here, $u_*$ is the friction velocity, defined by $u_* = \sqrt{\tau_v/\rho}$, where $\tau_v$ is the turbulent momentum flux in the inertial sublayer above the buildings.) This shows clearly that the ratio of the canopy drag length scale to the depth of the canopy, $L_c/H$, gives a measure of the density of the canopy. In vegetation canopies the vegetation typically occupies only a small fraction of the volume, so that $1 - \lambda_p \approx 1$, but the obstacles present a large area to the wind so that $\lambda_f > 1$ (see for example the values quoted in Finnigan 2000). We conclude that vegetation canopies can be regarded as dense in the sense that $L_c/H < 1$ and $U_c$ is small. In contrast urban canopies can be characterized better with cubical obstacles, so that the lambda parameters are equal, $\lambda_p = \lambda_f = \lambda$. This then means $L_c/H \approx (1 - \lambda)/\lambda$ so that $L_c/H$ is greater than one over the realistic range $\lambda < 0.5$, which explains why winds are greater in urban
areas than in forests. We can think of urban areas as sparse canopies of large obstacles. For example if the building height is 10 m, the wind speed at twice the building height is 10 m s$^{-1}$, the roughness length is 1 m and the urban area is made up of cubes with $\lambda=0.2$, then $L_c/H=4$, $u_c=1.35$ m s$^{-1}$ and so $U_c=2.7$ m s$^{-1}$.

Belcher et al. (2003) and Coceal & Belcher (2004) analyse the adjustment of a rural boundary layer to an urban canopy and show how the winds within the canopy adjust to the canopy within a distance of $3L_c$ and that the model agrees well with observations of Davidson et al. (1996). For dense forest canopies this distance might be as small as the height of the canopy. For more open urban canopies this distance ranges from $30H\approx300$ m when $\lambda=0.1$ in suburban areas down to $2H\approx50$ m in downtown areas. Since winds in a boundary layer impinging upon an urban canopy are slowed down, there must be an associated vertical motion. An estimate of this vertical velocity can be obtained using the depth-integrated formulation of the momentum balance, which shows that the vertical velocity scales as $H/3L_c$ multiplied by the change in depth-integrated wind speed.

These results, based on the vertically averaged flow, could form a simple representation of transport in urban areas that might well be sufficient for weather forecasting and air quality models. Similar calculations are needed to provide characteristic scales for the turbulence.

4. Dispersion at the neighbourhood scale

We turn now to dispersion in an array of buildings. Many of the processes are illustrated with flow visualization, and here we examine results obtained by R. W. Macdonald in a water flume at the University of Waterloo, Canada (see MacDonald et al. 1998). Figure 6 shows dispersion of dye through a staggered array of cubes with $\lambda=0.16$. Figure 6a is a side view of the experiment and figure 6b is a view from above. Dye is released at the far left of the picture just ahead of the cube array. The aim of this section is to understand these patterns.

Notice how, even in the first street in figure 6, dye is mixed by the wakes up through the depth of the cubes. Now, Macdonald reports that the vertical mixing above the cube arrays follows the usual scaling for flat ground. So the enhanced vertical spread of the plume may be the result of vertical transport out of the array as the winds adjust to the urban canopy (as analysed in §3b). This complication would be avoided if the release were sited in the array beyond the adjustment region; such experiments are needed. Figure 6 suggests two regions of lateral dispersion. In the near field transport along the streets spreads the plume linearly: in the first two or three rows in figure 6 the plume width increases by one column per row traversed. This rate of spread slows in a far field beyond the first few rows. Davidson et al. (1995) report full-scale observations in a similar array, and their photographs show that the horizontal width of the array plume spreads faster than that of a plume over flat ground. Processes that control the lateral spread are now discussed further.

(a) Secondary sources in wakes

Figure 6 shows the secondary sources introduced by building wakes that were mentioned in §1. Dashed lines indicate the recirculating flow regions immediately
behind the cubes. When the plume touches the side of an obstacle material is entrained into the recirculating flow region, as shown in the wake of the cube in row 1, column c (we refer to the rows of cubes by number and the column by letter). Once entrained into the recirculating flow region, material is released slowly, over a time-scale of the order of $3H/U_H$ (Vincent 1977, 1978; Puttock & Hunt 1979). When the urban area is composed of long streets connected by intersections, the whole street can become a secondary source, making this an important mechanism in determining the pattern of material emitted into the boundary layer above the canopy (see figure 7).

(b) Topological dispersion at intersections

Consider now the role of street intersections. The staggered array of cubes shown in figure 6 can be thought of as a network of short streets connected through T-junctions. In this figure we see clearly the important role that the T-junctions play in lateral dispersion, as discussed in §3a. The dashed lines indicate how dividing streamlines impinge upon a building face. Air parcels on different sides of these lines diverge around the cubes, leading to strong dispersion. Even weak turbulence that moves air parcels a modest distance across the dividing streamlines upstream of the obstacles can then catalyse large lateral dispersion. Davidson et al. (1995) and Jerram et al. (1995) call this process...
Figure 7. Dispersion through an array of rectangular buildings, arranged in lines. (a) Wind perpendicular to streets; (b) wind at $60^\circ$ to streets.

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topological dispersion. As discussed in §2b when the lateral streets are longer, the lateral flow decays along the street. So, when the wind is perfectly aligned, we expect topological dispersion to work in streets less than about $4H$ in length.

The lateral spread of the plume in the far field, after many instances of topological dispersion, can be estimated as follows. The plume spreads by increasing the mean square separation of air parcels, and so it is appropriate to consider the increment in the square of the plume width, $d\sigma_y^2$, in travelling a distance $dx$. If $S_x$ is the spacing between T-junctions in the streamwise direction, then the number of T-junctions encountered in this distance is $dx/S_x$, and the lateral spread of air parcels per encounter is $L^2$, where $L$ is the spanwise length of the building. Finally the fraction of streamlines that can experience topological diffusion is the lateral distance mixed by turbulence, $L_t$, divided by the spanwise separation of the dividing streamlines, taken for this scaling to be the spanwise length of the repeating unit, which yields a fraction $L_t/S_y$. Equating $d\sigma_y^2$ to the product of these factors and integrating over distance yields

$$\sigma_y \approx \left( \frac{L^2L_t}{S_xS_y} \right)^{1/2}.$$  

(4.1)

The variation with $x^{1/2}$ arises from assuming passage through many T-junctions: topological dispersion gives rise to Brownian diffusion on the street network. Carefully designed laboratory experiments are required to examine the geometric dependencies in this relation.

At four-way junctions, such as found in an array of buildings arranged in lines, the dividing streamlines do not impinge on buildings and there is no topological dispersion. Figure 7 shows Macdonald’s visualization of dispersion in an aligned array of rectangular blocks. Consider first figure 7a, where the wind is aligned with avenues and perpendicular to streets. There is strong mixing over the length and depth of even the first street, which then acts as a secondary source. Turbulent fluctuations lead to release of material from the ends of the first street, which is then transported along the avenues. Since the dividing streamline does not impinge upon a downwind building when the flow is perfectly aligned, there is no topological dispersion. Instead, turbulence is the only mechanism for lateral mixing. The result is that the plume width grows only slowly beyond the length of the streets. This case of the mean wind aligned perfectly is unusual, and in the real atmosphere the wind direction fluctuates. The effect on dispersion is examined next.

(c) Fluctuations in wind direction

The wind direction fluctuates in the atmospheric boundary layer. For example, in a neutral boundary layer, at the top of 10 m buildings of roughness of 1 m, the variance in wind direction caused by natural turbulence is $\sigma_\theta \approx v'/U_H \approx 18^\circ$. How does the wind direction change dispersion?

Figure 7b shows dispersion from the same source as in figure 7a but now when the mean wind is rotated $30^\circ$ from the avenues. The differences from figure 7a are striking. The plume width grows very rapidly in figure 7b. There are two reasons. Firstly, there is now a strong transport along both avenues and streets, which yields strong near field dispersion. Secondly, topological dispersion is active,
because the dividing streamlines impinge upon buildings, which controls far field dispersion. It is clear that wind direction fluctuations increase the potency of topological dispersion and probably mean it acts even when the streets are longer than $4H$.

Wind direction fluctuations also accentuate the role played by secondary wakes. Suppose the plume meanders one way so that material is entrained into wakes on one side of the array. Then the wind direction drifts and the plume meanders towards the other direction; material continues to be emitted from the earlier wakes. In this way wind direction fluctuations and the secondary sources associated with wakes reduce the fluctuations in concentration within an urban area, when compared with the fluctuations over flat ground, as shown dramatically in the measurements of Davidson et al. (1995).

(d) Variation of maximum concentration with distance from source

In an attempt to draw together and quantify some of these ideas, consider now the variation of the surface concentration with distance from the source. On dimensional grounds, the maximum concentration of material at the surface, $C_{\text{max}}$, scales as

$$C_{\text{max}} \approx \frac{Q}{U_c \sigma_y \sigma_z}, \quad (4.2)$$

where $Q$ is the source rate, $U_c$ is a characteristic plume transport speed and $\sigma_y$ and $\sigma_z$ are the characteristic width and height of the plume, which vary with distance from the source. It is not obvious how $C_{\text{max}}$ will vary when compared with that over flat ground. We have seen that the lateral spread of an urban plume, $\sigma_y$, increases more rapidly, tending to decrease $C_{\text{max}}$. But, according to the canopy model discussed in §3b, the wind speed within an urban canopy, $U_c$, is smaller, tending to increase $C_{\text{max}}$.

We saw in figures 6 and 7b that in the near field the lateral spread of the plume is nearly linear. When the street and avenues run perpendicular to one another, and the wind is at an intermediate angle, material is transported along both perpendicular streets at speeds of the order of the above-roof wind speed projected onto the streets. This reasoning suggests $\sigma_y \approx R$ and $U_c \approx U_H$ in the near field, where $R$ is distance from the source. We argued above that vertical spread follows rules established for standard Gaussian plumes, so that in the near field $\sigma_z \approx R/7$ (Pasquill & Smith 1983). Combination of these results yields

$$\frac{C_{\text{max}} U_H}{Q} \approx \frac{K_N}{R^2}, \quad (4.3)$$

where the dimensionless coefficient in the near field, $K_N$, is approximately 7, with a range of say 5–20.

In the far field regime, when topological dispersion is acting, the lateral spread is given by equation (4.1), and an estimate for the plume speed is given by equation (3.6). Tentatively, we assume $\sigma_y$ remains unchanged and ignore wind direction fluctuations. In the far field $C_{\text{max}}$ then varies as

$$\frac{C_{\text{max}} U_H}{Q} \approx \frac{K_F}{L_t^{1/2} R^{3/2}}, \quad (4.4)$$
somewhat more slowly than in the near field. The dimensionless coefficient in the far field, $K_F$, is estimated to be

$$K_F \approx 7 \left( \frac{H}{L} \right)^{1/2} \frac{\ln(H/z_0)}{(1-\lambda_f)^{1/2}}. \quad (4.5)$$

The surprise is that $\lambda_f$ drops out of this relation: its effect in decreasing the wind speed cancels its effect in topological dispersion. The far field dispersion is then insensitive to array geometry, particularly for arrays of cubes, the geometry most studied in the laboratory.

New methods have recently been developed to measure dispersion on the neighbourhood scale (e.g. Britter et al. 2002). Neophytou & Britter (2004) have analysed measurements of the variation in $C_{\text{max}}$ with distance from the source obtained from full-scale measurements in St Louis, Salt Lake City, Birmingham and London, with $R$ ranging up to a kilometre or so. The measurements are reasonably well approximated by equation (4.3) with $K$ in the range 10–20. There is also a suggestion particularly in the Salt Lake City data that $C_{\text{max}}$ decays more slowly at larger $R$. This is encouraging agreement.

5. Conclusions

The processes that control dispersion in an urban area over the first kilometre or so are beginning to emerge through laboratory modelling, numerical simulations and theoretical reasoning. At the same time quantitative measurements are beginning to become available. In this review, we have seen how the elements can come together to explain aspects of the observations. The dynamics of the shear layers and dividing streamlines have been highlighted here as useful organizing principles. Dividing streamlines lead to topological dispersion, which has two regimes: a near field with a linear lateral plume spread and a far field with a Brownian lateral dispersion on the street network.

To date many of the laboratory studies have considered only arrays of cubes, when the geometric scales degenerate to a single length and the street lengths are short. As discussed by Kastner-Klein et al. (2004), carefully designed laboratory studies with more general building shapes are needed. For irregular building configurations and street networks that occur in real cities, the governing processes need to be integrated into a computational model, perhaps along the lines described in §2b for the average street concentration. Soulhac (2000) has made an impressive start to this approach, but much remains to be done. Alternatively, computational work such as Pullen et al. (2005) shows how advanced computational fluid dynamics might be used to simulate in detail the flow around urban areas. Finally, this review has highlighted the role played by mean transport within the urban street network; mixing by turbulence has been shown to be weaker than for example topological dispersion. So in real emergencies, where a rapid response is required, there is a great deal to be learnt from observation of the local wind direction. Clever use of the motion of flags, leaves on trees, etc. could give valuable information in such emergencies.

I hope I have conveyed some of the fascinating complexity of mixing and transport in urban areas. The diverse range of physical processes involved in a problem of such practical importance makes this a rewarding area to work on.
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