Coherence resonance in a chemical excitable system driven by coloured noise

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We investigate how the temporal correlation in excitable systems driven by external noise affects the coherence of the system’s response. The coupling to the fluctuating environment is introduced via fluctuations of a bifurcation parameter that controls the local dynamics of the light-sensitive Belousov–Zhabotinsky reaction and of its numerical description, the Oregonator model. Both systems are brought from a highly incoherent regime to a coherent one by an appropriate choice of the correlation time and keeping noise variance constant. This effect has been found both for an Ornstein–Uhlenbeck process and for a dichotomous telegraph signal. In the latter case, we are able to connect the optimal correlation time, for which the system behaviour is most coherent, with a characteristic time scale of the system.

Keywords: excitable media; coherence resonance; coloured noise

1. Introduction

That noise can help rather than hinder the performance of a nonlinear system is well known nowadays, not only in the physical community (Roberts 1998). Fluctuations of suitable intensity can enhance the regularity of the system’s response to a periodic input, a phenomenon known as stochastic resonance (SR; Gammaitoni et al. 1998). Originally proposed in the early 1980s to account for the periodic occurrence of the ice ages on the Earth (Benzi et al. 1981), SR has been offering clues and ideas for a huge variety of works that range from weapon pointing systems (Freund et al. 2001) to the food detection mechanism in the crayfish (Wiesenfeld & Moss 1995; Hänggi 2002).

The question has been addressed whether a noise of particular intensity induces coherent behaviour in a nonlinear system without the need of an external periodic input. First encountered in oscillatory systems as ‘stochastic resonance without periodic forcing’ (Gang et al. 1993) and later as ‘internal stochastic resonance’ (Hou & Xin 1999), it is with the work of Pikovsky & Kurths (1997) on the FitzHugh–Nagumo (FHN) model that this phenomenon got its present name.
coherence resonance (CR) and that it was associated with excitable systems. In this work, the authors showed that the regularity at which an excitable element fires under white noise driving has a non-monotonic resonant dependence on the noise intensity, and that there exists an optimal noise intensity at which a sequence of noise-induced excitations is most regular. This phenomenon has been studied for more than a decade and has been observed in a huge variety of systems of quite different nature, such as, for example, anti-CR in excitable systems with feedback (Lacasta et al. 2002), CR in coupled chaotic oscillators (Zhang et al. 2002), internal CR in variable size patches of a cell membrane (Schmid et al. 2001, 2003), system size CR in globally coupled FHN elements (Toral et al. 2003), array-enhanced CR in a model for Ca$^{2+}$ release (Coombes & Timofeeva 2003), CR at the onset of a saddle-node bifurcation of limit cycles (Lee et al. 1998) and of period-doubling bifurcations (Neiman et al. 1997) and spatial CR in an extended system near a pattern-forming instability (Carrillo et al. 2004). For a comprehensive review see Lindner et al. (2003).

What has been reported so far are features and effects that excitable systems display when driven by white external fluctuations, that is, those characterized by the $\delta$-correlation function,

$$\langle \xi(t)\xi(t') \rangle = 2D\delta(t-t'),$$

(1.1)

$D$ being the noise intensity. The idealization of $\delta$-correlated white noise is sometimes hardly realized in the real world. Indeed, the correlation of a real noise source is never going to be strictly equal to zero. Generally in the modelling of a real system it is desirable to take into account a few variables to describe it; so often a coarsening is done over a hidden set of fast variables (adiabatic elimination). Thus, realistically, noise in the simplified low-dimensional system exhibits finite correlations. Time-correlated fluctuations are often modelled through an Ornstein–Uhlenbeck (OU) stochastic process (Van Kampen 1981) generated by the differential equation

$$\frac{d\eta(t)}{dt} = -\frac{\eta(t)}{\tau} + \frac{\xi(t)}{\tau},$$

(1.2)

with $\xi(t)$ being characterized by (1.1), and by the exponential correlation function the

$$\langle \eta(t)\eta(t') \rangle = \sigma^2 \exp\left(-\frac{|t-t'|}{\tau}\right),$$

(1.3)

where the parameter $\sigma^2 = D/\tau$ is the variance of the process given by its second moment $\langle \eta^2 \rangle$ and $\tau$ is the characteristic correlation time.

Two different interpretations of the white noise limit $\tau \to 0$ arise depending on the parametrization of the OU noise (Jung et al. 2005). In the first one, the noise intensity $D$ is kept fixed and the effect of noise is studied as the correlation time $\tau$ is changed; while in the second one, the effects induced by changes in the colour of the fluctuations are explored keeping the variance $\sigma^2$ constant. Note that, while under the latter scaling the total power of the noise (the area under the spectral density of the process) is conserved on varying the correlation time, the variance $\sigma^2$ of the noise grows to infinity as $\tau \to 0$ for the former interpretation. As this last scenario, of course, never occurs in a real physical system, we adopt in this work the approach of a time-correlated noise source, keeping its variance constant and studying its effects as a function of the correlation time.

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Stochastic systems with coloured noise sources have been attracting much attention recently. An example in materials science is crackling noise emitted by materials with hysteresis when they get magnetized (Sethna et al. 2001). This effect can be explained by models that take into account correlated forces acting on the magnetic domain walls (Zapperi et al. 1998, 2005). Another interesting example is offered by granular media. The velocity distribution of a granular particle is non-Gaussian. This is due to inelastic collisions between particles, which finally result in correlated random forces acting on the particles (Baldassarri et al. 2002). Furthermore, recently in neuroscience models for cortical neurons with correlated synaptic noise sources have been discussed in great detail (Fourcaud & Brunel 2002; Middleton et al. 2003; Lindner 2004). These studies are of particular relevance for cortical neurons. Interestingly, it has been shown that leaky integrate and fire (LIF) models with coloured synaptic noise can successfully reproduce the statistical properties of spiking cortical neurons taken in the prefrontal cortex of a monkey (Sakai et al. 1999).

Nonlinear excitable systems have been poorly analysed in the case of coloured noise, but the examples given above clearly indicate how relevant can be the role played by correlations in real phenomena. We start to fill the lack of understanding about correlation-induced effects in an excitable system by looking at the phenomenon of CR (to be introduced in §2), considered so far as a white noise phenomenon, in the case of exponentially correlated noise. We present new results on the role played by the correlation time in the control of excitations coherence. The robustness of our findings is checked, considering the two-component Oregonator model (§3) and the paradigmatic Belousov–Zhabotinsky (BZ) reaction (§4) driven by different exponentially correlated noise signals.

2. CR–σ² phenomenon in excitable systems

An excitable system is characterized by its response to a perturbation: a small disturbance causes merely an equally small response; a perturbation above a certain threshold, on the contrary, excites a quiescent element that performs a large excursion in the phase space before reaching the rest state again. Thus, such a system responds nonlinearly to perturbations. We consider systems where the parameter that controls the excitation threshold fluctuates, so that random excitations take place due to noise. The phenomenon by means of which the degree of regularity of those noise-induced events is maximal by tuning the noise amplitude is called coherence resonance (CR).

The typical oscillation period of the system is given by the mean of the interspike time interval (ISI) \( <t_p> \) between two successive noise-induced excitations, over many realizations. If the system fires regularly, say for simplicity periodically, then the error associated with \( <t_p> \) is zero and consequently the ratio of the standard deviation s.d.(\( t_p \)) to its mean value \( <t_p> \), i.e. the normalized fluctuations

\[
R_p = \frac{\sqrt{\langle (t_p - <t_p>)^2 \rangle}}{<t_p>},
\]  

(2.1)
is also equal to zero. On the other hand, if the firing is incoherent and takes place at random times, then the error associated with $t_p$ is of the same order as the mean ISI, so that $R_p(x_1)$. Thus, $R_p$ is a measure of the coherence of the system’s response and the minimum of $R_p(\sigma^2)$ characterizes the optimal noise intensity.

The phenomenon of CR is due to the presence of two different characteristic time scales in an excitable system which are affected by noise in different manners. One is the activation time $t_a$, which is the time during which the system just fluctuates around the stationary state before an excitation is induced. The second time scale, the excursion time $t_e$, is the typical duration of an excitation loop. Therefore, the ISI $t_p$ is given by the sum of these two times, $t_p = t_a + t_e$, and the value of $R_p$ depends on the statistics of both $t_a$ and $t_e$ under the fluctuations. Noise of low intensity does not affect $t_e$ (figure 1a,c). Consequently, assuming s.d.($t_e$) small and $t_a \gg t_e$ (excitations are rare events), we can write that $R_p \approx s.d.(t_a)/\langle t_a \rangle$. In this regime, the spikes are completely random events, so that $s.d.(t_a)/\langle t_a \rangle = 1$, and as noise increases, the coherence of $t_a$ increases too. Only at moderate noise intensities, where $t_a \ll t_e$, does the excursion time $t_e$ start to get affected by noise (figure 1b,d). In this regime the excitation loops, which at low noise intensities possess well-defined trajectories, lose regularity, spreading out in the phase space for increasing noise intensity. The transition between these two regimes takes place where the quantity $R_p$ displays its minimum. There, the coherence of the system is the highest.

Figure 1. (a,b) Noise-induced excitations and (c,d) phase portraits in the stochastic FHN model (Pikovsky & Kurths 1997) under white noise driving for (a,c) low noise intensities and (b,d) moderate noise intensities. Time scales $t_p$, $t_e$ and $t_a$ are discussed in the text.
3. CR–τ in the Oregonator model

(a) The model

We consider the two-component Oregonator model for the light-sensitive BZ reaction (Krug et al. 1990), which will be introduced in §4:

\begin{align}
\frac{du}{dt} &= \frac{1}{\varepsilon} \left[ u - u^2 - (fv + \phi) \frac{u - q}{u + q} \right], \\
\frac{dv}{dt} &= u - v.
\end{align}

(3.1)

The variables \( u \) and \( v \) stand for the dimensionless concentration of bromous acid and the oxidized form of the light-sensitive catalyst Ru(bpy)_3, respectively. Here \( \varepsilon \) controls the time-scale separation between the two variables; \( q \) is a scaling parameter; \( f \) is a stoichiometry parameter (Tyson & Fife 1980); and \( \phi \) represents the photochemically induced bromide flow that is assumed to be proportional to the applied light intensity in the BZ reaction. From the recipe concentrations, we obtain \( \varepsilon = 0.0766 \) and \( q = 0.002 \); \( f \) is chosen equal to 1.4. In the absence of noise and for \( \varepsilon, f \) and \( q \) kept fixed, the parameter \( \phi \) controls the kinetics: for small \( \phi \), the kinetics is oscillatory; and for \( \phi > \phi_{\text{HB}} = 4.43 \times 10^{-3} \), it becomes excitable via a supercritical Hopf bifurcation.

(b) OU noise

We are interested in how fluctuations with non-vanishing correlation time in the excitability parameter affect the coherence of the system’s response. Thus, we assume the parameter \( \phi \) to be an exponentially correlated stochastic variable expressed as

\[ \phi \rightarrow \phi_\eta(t) = \phi_0 \cdot \left[ \eta(t) + 1 \right]. \]  

(3.2)

We choose \( \phi_0 \) in the excitable regime and analyse how \( R_p(\tau) \) behaves in the Oregonator model as the correlation time of the OU process \( \eta \), given by equation (1.2), varies. For the numerical integration of equation (1.2), we adopt an integral algorithm proposed by Fox et al. (1988).

In figure 2, we present the dependence \( R_p(\tau) \) for different noise intensities \( \sigma^2 \). We find that the coherence of the noise-induced excitations is strongly affected by the correlation time \( \tau \), and, moreover, that at an optimal value \( \tau_{\text{opt}} \) of the correlation time the system reaches maximal coherence. It is important to stress that the optimal values of the correlation time \( \tau_{\text{opt}} \) are not only smaller than the mean ISI, \( \langle t_p \rangle \), but are also even smaller than the typical values of \( t_e \). Furthermore, the position of the minima in figure 2 varies only slightly as the variance of the noise driving is changed. This suggests that the behaviour of \( R_p \) with \( \tau \) is also due to a resonance with some internal deterministic time scale of the system.

To understand the mechanism responsible for this phenomenon, we analyse in figure 3b,c the phase portraits of the system (3.1) for correlation times much smaller and much larger than the optimal value \( \tau_{\text{opt}} \). Interestingly, since in our scaling the noise intensity \( \sigma^2 \) is the same for different correlation times, in these two limit cases the phase portraits are almost the same (compare figure 3b,c with...
Along the noise-induced excitation loops, the system does not visit increasing sets of states for increasing correlation times, as encountered for CR with white noise upon changes of the noise intensity. This suggests that here the two characteristic time scales $t_a$ and $t_e$ are not differently affected by $\tau$ as described in §2; thus the effect depicted in figure 2 is new and different from CR under white noise driving.

Light is shed on this phenomenon by analysing the time evolution of variable $u$ (see figure 3). We recognize that, for small values of the correlation time, $\tau < \tau_{\text{opt}}$, noise-induced excitations are very rare events, and the resulting sequence of excitations is highly incoherent. When $\tau$ increases, the regularity of the noise-induced spike sequence grows and the excitations happen almost periodically. Here, $\tau = \tau_{\text{opt}}$ and $R_p(\tau)$ reaches its minimum. For higher values of

Figure 2. $R_p(\tau)$ for the Oregonator model (3.1) driven by an OU process. Kinetic parameters here and in the following figures: $\epsilon = 0.0766$ and $q = 0.002$, $f = 1.4$. Excitability parameter $\phi_0 = 7.5 \times 10^{-3}$. Noise variance kept constant: dots, $\sigma^2 = 0.16$; circles, $\sigma^2 = 0.25$; squares, $\sigma^2 = 0.49$.

Figure 3. (a) Noise-induced excitations in the Oregonator model (3.1) at constant noise intensity $\sigma^2 = 0.25$, for different correlation times: (i) $\tau = 0.07$, (ii) $\tau = 0.7$ and (iii) $\tau = 7$. Kinetic parameters as in figure 2. (b, c) Phase portraits corresponding to $\tau = 0.07$ and 7.

figure 1c, d). Along the noise-induced excitation loops, the system does not visit increasing sets of states for increasing correlation times, as encountered for CR with white noise upon changes of the noise intensity. This suggests that here the two characteristic time scales $t_a$ and $t_e$ are not differently affected by $\tau$ as described in §2; thus the effect depicted in figure 2 is new and different from CR under white noise driving.

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the correlation time, \( \tau \gg \tau_{\text{opt}} \), we find clusters of excitations followed by time intervals without any activity. For those large \( \tau \) values, fluctuations can occasionally drive the system for a long time interval close or below the bifurcation value \( \phi_{\text{HB}} \). Consequently, during these clusters the system is effectively oscillatory and fires regularly. The clusters are spaced out by intervals of the same duration in which the system is held at low excitability, \( \phi \ll \phi_{\text{HB}} \), and thus no activity can be triggered. The regularity of such a sequence of spike clusters, alternated by intervals of no activity, is low: in this regime, \( R_p \) increases with \( \tau \). Note that the duration of the bursts of excitations shown in the lower panel of figure 3a is much longer than the correlation time \( t \) of the noise driving.

\[(c)\] Random telegraph noise

In an attempt to get a deeper understanding of the behaviour of the system (3.1) in the presence of coloured noise, we consider in what follows a simpler exponentially correlated stochastic process: a random telegraph signal \( \eta(t) = \eta_\pm(t) \) (Horsthemke & Lefever 1984). This process is dichotomous and assumes only the values \( \pm \Delta \). It can be described by a phase according to

\[
\eta_\pm(t) = \text{sign}[\cos(\gamma_\pm(t))] \cdot \Delta, \tag{3.3}
\]

where the phase increases at each random time \( t_i \) by an angle \( \pi \). Thus, the process has zero mean and variance \( \Delta^2 \). Introducing the Heaviside step function \( \theta \), we can write the phase as

\[
\gamma_\pm(t) = \pi \sum_i \theta(t - t_i), \quad t_i = t_{i-1} + e. \tag{3.4}
\]

We assume the random variable \( e \) to be exponentially distributed with distribution

\[
p(e) = \frac{1}{\tau} \exp\left(-\frac{e}{\tau}\right). \tag{3.5}
\]

With this assumption, the process expressed in equation (3.3) has the correlation function

\[
\langle \eta_\pm(t) \eta_\pm(s) \rangle = \Delta^2 \cdot \exp\left(-\frac{|t - s|}{\tau}\right), \tag{3.6}
\]

where \( \tau \) is the correlation time of the process \( \eta_\pm \) (Horsthemke & Lefever 1984; Hänggi & Jung 1995).

We choose the parameters \( \phi_0 \) and \( \Delta \) such that \( \phi_0 \) corresponds to the excitable regime and \( \phi_- = \phi_0(1 - \Delta) \) to the oscillatory regime, respectively. The system is excitable for \( \phi(t) = \phi_+ = \phi_0(1 + \Delta) \); therefore, from any initial condition it reaches the stationary state and remains there forever. We analyse how the coherence of the system under random switching between the two kinetic regimes depends on the correlation time of the switching process.

The results for \( R_p(\tau) \) obtained for different values of \( \Delta \) (figure 4) demonstrate that under a random telegraph signal the coherence of noise-induced excitation is enhanced by an optimal choice of the correlation time, as analogously found for the OU noise (figure 2). Here, the optimal correlation time \( \tau_{\text{opt}} \) decreases as the noise amplitude \( \Delta \) increases. Further simulations, not shown here, confirm that this phenomenon holds for a wide range of the bifurcation parameter \( \phi_0 \), covering
almost the whole excitable regime. We emphasize that, for not well-separated time scales, noise-induced excitations are possible even if both $f_K$ and $f_C$ belong to the excitable regime (compare dotted curve in figure 4).

The effect of the random telegraph signal on the Oregonator model can be viewed as perturbations of random duration $\delta T$ and amplitude $\delta \phi = -2\Delta \phi_0$, applied to the system at excitability $f_C$. To clarify this, we first focus on the effect of one single perturbation (figure 5). Initially, we prepare the system in its fixed point $(u_0, v_0)_{\phi_-}$ and apply a perturbation $\delta \phi = -2\Delta \phi_0$. This results in a shift of the $u$-nullcline and, consequently, a shift of the fixed point towards the unstable branch of the $u$-nullcline. The perturbed system is now in the oscillatory regime. Thus, the state $(u_0, v_0)_{\phi_-}$ becomes unstable and the system leaves it to enter the stable periodic orbit (figure 5b). When the perturbation is switched off, the system is in a state $(u, v)_{\delta T}$ beyond the excitability threshold. Therefore, the rest state is recovered by performing an excitation loop (figure 5c). This simplified picture holds only for large correlation times. If the duration $\delta T$ is too short (small correlation times), the perturbed state $(u, v)_{\delta T}$ is not beyond the excitation threshold. Therefore, no excitation takes place but merely a relaxational motion back to the stable state $(u_0, v_0)_{\phi_+}$.
We plot in figure 6 the duration $\delta T$ of the shortest perturbation of amplitude $\delta \phi = -2\Delta \cdot \phi_0$ able to induce an excitation loop. As expected, $\delta T$ decreases for increasing $\Delta$. If we compare this time with the optimal correlation time of the telegraph signal at which the system reaches the highest coherence, we find that the two quantities, $\delta T$ and $\tau_{opt}$, are of the same order of magnitude. Let us now imagine the more realistic situation of a system prepared in a cloud of states around the fixed point $(u_0, v_0)_{\phi_+}$, and let us calculate the mean duration $\langle \delta T \rangle$ of the shortest ‘excitation-inducing’ perturbation. In this case, we find a good agreement with the optimal correlation time $\tau_{opt}$, i.e. $\tau_{opt} \approx \langle \delta T \rangle$. We call the duration $\delta T$ the \textit{excitation time}, which is the time needed by a perturbation of amplitude $\delta \phi$ to induce an excitation. The highest coherence of the system under dichotomous, exponentially correlated fluctuations is reached when the correlation time, and so the typical duration of the two states $\phi_{\pm}$, is of the same order as the excitation time.

4. CR in the BZ reaction

Experimental evidence for the phenomenon of CR with respect to the noise intensity has already been reported in different systems under white noise driving. Some relevant examples are CR in a semiconductor laser subject to optical feedback (Giacomelli \textit{et al.} 2000), in an electrochemical cell (Kiss \textit{et al.} 2003) or in a BZ oscillator stimulated by a noisy electrical field (Miyakawa & Isikawa 2002). Very recently, CR in electronic circuits in the presence of coloured $1/f^2$ noise has been shown in Brugioni \textit{et al.} (2005).

\textit{(a) Experimental set-up}

The set-up adopted for our experiments has an open gel reactor as central element, which allows one to maintain constant non-equilibrium conditions during the measurements (figure 7). With it, and by means of computer-based spectrophotometry (Müller \textit{et al.} 1987), we analyse wave activity in the BZ medium with sufficiently high spatial and temporal resolution.
The reactor consists of two separated chambers. In one of them, the BZ reactants are put in contact with the light-sensitive catalyst, the complex \( \text{Ru(dmbpy)} \text{Ru(dmbpy)}^{2+} \) \(^1\), immobilized in a silica–hydrogel matrix, spread on a glass dish fixed in the chamber. Details and remarks on the preparation of the gel and the catalyst are reported in Brandtstädtter \textit{et al.} (2000). The gel is 0.32 mm thick and 18×15 mm in size and no three-dimensional effects are present. The chamber of the reactor is continuously fed with fresh BZ solution ([\( \text{BrO}_3^- \]) = 2 M, \([\text{H}_2\text{SO}_4]\) = 4 M, \([\text{CH}_2(\text{COOH})_2]\] = 1.5 M, and \([\text{Br}^-]\) = 2 M; see Beato \textit{et al.} (2005) at a pumping rate of 100 ml h\(^{-1}\). In addition, a magnetic stirrer keeps homogeneous concentrations of reagents in the whole chamber. The second chamber of the reactor serves as a heat bath for the system, whose temperature is controlled by a thermostat \((T_{\text{H},0} = 25.0 \pm 0.5^\circ\text{C})\).

The control of the reaction through light is realized by employing a video projector, which illuminates it with a computer-generated pattern (light control unit in figure 7). The projector is a Toshiba TLP X20, equipped with a special auxiliary objective that allows one to project sharp patterns at a short distance. The whole projection area is a small rectangle of size 4×3 cm and is focused at

\(^1\) Abbreviated form for ruthenium-4,4′-dimethyl-2,2′-bipyridyl.

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\( \text{BrO}_3^- \) refers to peroxo-species of the bromate ion, \([\text{H}_2\text{SO}_4]\) to sulphuric acid, and \([\text{CH}_2(\text{COOH})_2]\] to oxaloacetic acid. The measure of the light intensity \(I_\text{d}\) arriving at the gel after passing through the blue filter (BF1) and the polarizer versus the GL of a uniform computer-generated pattern. The Hopf bifurcation for the BZ recipe used occurs at a light intensity \(I_{\text{HB}} \approx 80\ \text{GL}\).

**Figure 7.** (a) Sketch of the experimental set-up (Beato 2005). (b) Photograph of the reactor and, behind it, the polarizer, the blue filter and the video projector. (c) Measurement of the light intensity \(I_\text{d}\) arriving at the gel after passing through the blue filter (BF1) and the polarizer versus the GL of a uniform computer-generated pattern. The Hopf bifurcation for the BZ recipe used occurs at a light intensity \(I_{\text{HB}} \approx 80\ \text{GL}\).
a distance $d=21.5$ cm. The light pattern (8 bit grey level; GL) from the video projector reaches the gel after passing through a blue bandpass filter ($\lambda_{BF1}=465$ nm) to reduce heating effects, BF1 in the set-up in figure 7. After the blue filter, we place a polarizer (angle set at 10°) which permits a fine tuning of the light intensity. For each GL (0–255), we measure with a luxmeter the intensity $I_d$ of the light at distance $d$ from the projector (distance at which the gel is placed), as shown in figure 7. We use GL as the unit of measurement of the light intensity illuminating the reaction throughout this section. The light transmitted through the reactor is again filtered with a bandpass blue filter (BF2 with $\lambda_{BF2}=438$ nm) to enhance the contrast of the transmitted image. Using a charge-coupled device (CCD) camera, the transmitted light patterns (8 bit GL bitmap pictures of $512 \times 512$ pixels) are recorded every second and digitized for later processing by the image acquisition unit.

Here we develop an experiment to study the local coherence properties of the BZ medium under temporally correlated noise. To neglect the effect induced by the presence of diffusive coupling, we concentrate on the coherence of nucleation events induced by temporally correlated fluctuations in a small area of the medium (a square of size $l=1.5$ mm, see figure 8a). A fluctuating light spot serves as nucleation source: through light we locally perturb the medium and, occasionally, fluctuations manage to initiate an excitation. This excitation travels in the form of a circular wave through a detector point in the non-fluctuating background and it signals that a noise-induced excitation occurred.

Figure 8. (a) Three snapshots of a typical experiment taken at equal time intervals of 15 s. Wave activity is recognizable through the high concentrations of the oxidized catalyst (bright waves). (b) Noise-induced spikes for different correlation times of the fluctuating light: (i) $\tau=5$ s, (ii) $\tau=15$ s and (iii) $\tau=40$ s. Sequences are obtained by tracking the gel brightness at the detector point (labelled with a cross in the snapshots) and decoded in GLs.
some time $\delta t$ before at the fluctuating spot. The temporal evolution of the gel brightness at the detector point shows a spike as a wave passes through it. Thus, by tracking the activity at this point over a long time, we obtain a statistically relevant sample of interspike intervals, and, from their analysis, a measure of the temporal coherence of the medium at the fluctuating spot (see figure 8b).

(b) Results

We illuminate the fluctuating box according to a random telegraph signal, $I(t) = I_0 \pm \Delta I$, as done in §3c. Each illumination state has a random duration, taken from an exponential distribution, and therefore $I(t)$ has an exponentially decaying correlation function. Under constant illumination $I_0$ no wave nucleation occurs, but the medium can support excitation waves. At light intensity $I_0 - \Delta I$, the medium is oscillatory and phase waves are induced, which become trigger waves as they propagate towards the surrounding medium maintained at high light intensity. The system is excitable at light intensity $I_0 + \Delta I$ and supports travelling patterns. In the case of dichotomous fluctuating light, nucleations occur randomly. In this way, we get a series of noise-induced nucleations at random time intervals, as shown in figure 8b for the different correlation times of the stochastic light signal $I(t)$. Already at a rough glance at the noise-induced spikes, it is possible to recognize that the coherence is dependent on $\tau$. The spikes plotted in figure 8b(i) show a better regularity than the other two above and below it.

To go further and get a quantitative estimation of this phenomenon, we perform experiments for correlation times $\tau$ in the range between 2 and 60 s. For each value of $\tau$, we carry out measurements as long as statistics of at least 75 nucleations is reached (only for $\tau = 2$ s was it impossible to reach this).

Figure 9a shows the mean period $\langle t_p \rangle$ between two successive nucleations. At low correlation times, corresponding to quick switching among the two states of the fluctuating pattern, nucleations are rarely generated, and many measurements with different gels are required. We note here that just below the Hopf bifurcation the typical oscillation period at GL 60 is $47 \pm 4$ s. This is the slowest deterministic time scale of the system, much smaller than all the typical mean interspike intervals (figure 9a).

Figure 9. (a) Experimental mean interspike period as a function of the correlation time. The continuous line is obtained with a spline-fitting algorithm, just for illustrative purposes. (b) Coherence resonance with respect to the correlation time in the light-sensitive BZ reaction. The normalized fluctuations of the interspike times $t_p$ are reported versus the correlation time of the random telegraph signal.
In figure 9b, the coherence of the nucleation events is quantified through the normalized fluctuations of $t_p$, $R_p$, defined in equation (2.1). $R_p$ presents a minimum at $\tau = 20$ s, a fingerprint of highest coherence. Hence our experimental data give clear evidence of the existence of an optimal correlation time of the fluctuating light driving the light-sensitive BZ reaction, at which the highest coherence is induced (Beato et al. 2005). Moreover, we note that the optimal value for the correlation time is much smaller than any oscillation period measured in the reaction, as analogously found for the Oregonator model in §3. We can thus conclude that these results are in good agreement with the numerical results reported in §4.

5. Conclusions

In conclusion, we find that for exponentially correlated noise driving, the response of an excitable element is highest at an optimal correlation time $\tau_{opt}$. Moreover, in the case of a dichotomous exponentially correlated noise signal, we can connect the optimal correlation time with an internal deterministic time scale of the system.

In addition, in an addressable real excitable medium, we show a new phenomenon induced by the temporal correlations of a fluctuating environment interacting with it. We analysed the dependence of the local coherence of the light-sensitive BZ reaction on the correlation time of a random telegraph signal that illuminates it. We show that, by adjusting the correlation time of the noise signal adequately, we can bring the system into a highly coherent regime.

References


