Large- and very-large-scale motions in channel and boundary-layer flows

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Large-scale motions (LSMs; having wavelengths up to 2–3 pipe radii) and very-LSMs (having wavelengths more than 3 pipe radii) have been shown to carry more than half of the kinetic energy and Reynolds shear stress in a fully developed pipe flow. Studies using essentially the same methods of measurement and analysis have been extended to channel and zero-pressure-gradient boundary-layer flows to determine whether large structures appear in these canonical wall flows and how their properties compare with that of the pipe flow. The very large scales, especially those of the boundary layer, are shorter than the corresponding scales in the pipe flow, but otherwise share a common behaviour, suggesting that they arise from similar mechanism(s) aside from the modifying influences of the outer geometries. Spectra of the net force due to the Reynolds shear stress in the channel and boundary layer flows are similar to those in the pipe flow. They show that the very-large-scale and main turbulent motions act to decelerate the flow in the region above the maximum of the Reynolds shear stress.

Keywords: hairpin-vortex packets; large-scale motions; very-large-scale motions; long-wavelength motions; travelling wave solutions; channel flow

1. Introduction

The friction Reynolds number, $Re_f = \frac{d_o}{\nu}$, can be interpreted as a ratio of the outer (large) scale, $d_o$, to the inner (small) scale, $\nu/u_f$ (where $d_o$ is the half-height of a canonical channel, $h$, the boundary-layer thickness, $\delta$, or the pipe radius, $R$; $\nu$ is the kinematic viscosity; and $u_f$ is the friction velocity). While satisfying at first glance, this ratio offers no information about the contributions or the importance of scales of motion larger than $d_o$. Recent experiments have shown that substantial portions of the kinetic energy and shear stress in turbulent flows are carried by these large motions across a wide range of Reynolds numbers and in

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a variety of flows. Insufficiently careful interpretation of Townsend’s (1961, 1976) idea that such large-scale motions (LSMs) are ‘inactive’ has sometimes led to the misleading notion that such long structures make negligible contributions to the Reynolds stress (Jimenez 1998). Further, computational and experimental difficulties encountered in the study of the larger wavelengths, owing to the requirements of a long computational box and long time records from a statistically stationary flow, respectively, have contributed to the marginalization of the LSMs. However, recent experimental results of Jimenez (1998), Kim & Adrian (1999) and Guala et al. (2006) at moderate Reynolds numbers, e.g. pipe Reynolds numbers up to 200 000, and developments and observations of travelling wave solutions to the Navier–Stokes equations in relatively low Reynolds number flows (Waleffe 2001; Faisst & Eckhardt 2003; Hof et al. 2004; Wedin & Kerswell 2004), and the analyses showing instability at very long wavelengths (e.g. del Alamo & Jimenez 2006), have rekindled an interest in the LSMs and very-large-scale motions (VLSMs), owing to the surprisingly large amounts of kinetic energy and Reynolds stress they carry, and their important ramifications for direct numerical (DNS), large-eddy (LES) and Reynolds-averaged (RANS) simulations modelling.

Guala et al. (2006), henceforth referred to as GHA06, focused on motions in fully developed pipe flows that were larger than the pipe radius. The motions were termed ‘LSMs’ if their lengths range between $0.1 \pi R$ and $\pi R$, and VLSMs if their lengths were greater than $\pi R$, nominally. Motions shorter than $0.1 \pi R$ were attributed to the range of active turbulent motion or, in Townsend’s terms, the ‘main turbulent eddies’. This work followed the observations that the turbulent kinetic energy in a pipe flow was contained in surprisingly large scales (Jimenez 1998; Kim & Adrian 1999), a fact observed, but not emphasized in several earlier investigations of the pipe flow (Perry & Abell 1975; Bullock et al. 1978; Perry et al. 1986). GHA06 found, in particular, that the VLSMs carried not only more than half the kinetic energy, but also more than half of the Reynolds shear stress. This finding seemed at odds with the works of Grant (1958) and Townsend (1961), who, while discovering large eddies and bringing attention to their significant energy content, had concluded that the large scales carry very little Reynolds shear stress in the near-wall region and, hence, called them ‘inactive’.

A number of earlier investigations have also revealed the presence of large eddies (of the order of 2–3δ in streamwise extent) in boundary layers, where the thickness of the boundary layer, δ, sets a scale that is comparable to the pipe radius (Kovasznay et al. 1971; Laufer & Narayanan 1971; Browne & Thomas 1977; Cantwell 1981; Murlis et al. 1982). Many of these large eddies were bulges, but some were longer, as reported by Krogstad et al. (1992). The pre-multiplied streamwise spectra presented by Nagib & Hites (1995) and Hites (1997) show the presence of VLSMs and their non-negligible energy content at various wall-normal locations. The importance of spatially coherent packets for the Reynolds stress and transport in the logarithmic layer was investigated by Marusic (2001). Using particle image velocimetry (PIV), Ganapathisubramani et al. (2003) showed that hairpin packets (which are LSMs) carry more than 30% of the shear stress. Scales of the order of 3δ were found to be the most energetic wavelengths in the inner layer by Nickels et al. (2005). Despite indications of their importance, a comprehensive analysis of the streamwise kinetic energy and the Reynolds shear stress carried by the LSMs and VLSMs throughout the zero-pressure-gradient boundary layer is not yet available.

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Regarding channel flows, the analysis of PIV data by proper orthogonal decomposition by Liu et al. (2001) shows that the first six modes contain about one-third of the turbulent kinetic energy and over half of the Reynolds shear stress. These modes are all 2.4h or longer, corresponding mainly to the very large scales. DNS computations performed in channel flows up to Reₚ = 2320 also provide evidence for the presence of large- and very-large-scale structures in channels (Jimenez 1998; del Alamo et al. 2004; Iwamoto et al. 2005). The recent computations recognize the importance of using very long computational domains (up to 24h) and show the presence of significant kinetic energy and shear stress at large wavelengths. But computational box-size limitations, periodic in-flow/out-flow boundary conditions and the absence of a process for the transition from a laminar to a turbulent flow in the computations are the factors that might influence the very large scales.

LSMs have also been observed at very high Reynolds numbers in the neutrally stable planetary boundary layer (PBL; Hoxey & Richards 1992). Using LES computation, Lin et al. (1996) observed LSMs of the order of 3δ at z/δ = 0.05, but these structures were found to disintegrate by z/δ = 0.583. In field experiments using Doppler lidar, Drobinski et al. (2004) observed low wavenumber motions that are present within the deeper layers of a PBL. Comparing a laboratory boundary layer with a PBL, Metzger & Klewicki (2001) concluded that near-wall mechanisms involved in the turbulent process at low Reynolds numbers (bursts, sweeps, ejections and streaks) also exist in high Reynolds number PBLs. Kunkel & Marusic (2006) showed LSMs in the inner layer of the PBL to be energetically significant and to carry a significant amount of shear stress. Using wavelet cross-scalograms, Hudgins et al. (1993) found structures extending to 2 km in turbulent boundary layers over oceans. These efforts point to the presence of LSMs in flows with very high Reynolds numbers, reaching values as high as Reₚ ~ 10⁶.

Thus, evidence is building for the existence and significance of LSMs and VLSMs in all of the canonical wall flows. However, it is scattered, and the various data are usually not directly comparable owing to differences in the experimental conditions, measurement methods and laboratory conditions to which the large scales might be sensitive, such as inlet flow conditions. The present study performs uniformly consistent measurements in a channel flow and a zero-pressure-gradient boundary-layer (ZPGBL) apparatus at the same laboratory, using the same analysis as the earlier studies of GHA06, and with inlet flows that were generally very well conditioned in each apparatus (low turbulent intensity).

2. Experimental flows and measurement

The results presented in this paper are derived from long records of constant-temperature anemometry (CTA) x-wire measurements performed in a channel and ZPGBL flow.

(a) Channel flow experiments

Three sets of CTA measurements at Reynolds numbers of 531, 960 and 1584 were obtained in a rectangular channel of cross-section 2×20.25 in., with air as the working fluid. Measurements were performed after a development section, 180h in length, to ensure a fully developed turbulent flow. Independent PIV
measurements reported by Balasubramaniam (2005) have confirmed that the mean and turbulence statistics are consistent with the fully developed turbulent channel flow statistics reported in other channels. Pressure measurements along the channel length were obtained from five static pressure taps mounted at x/h = 32, 68, 104, 140 and 212 from the leading edge of the 36-grit sandpaper trip. The gradient of the pressure drop (dP/dx) was calculated using a linear fit on these five measurements. The wall shear stress and friction velocity were thus obtained independent of the velocity measurement using

$$\tau_w = \frac{1}{2} \rho u_*^2 = -h \frac{dP}{dx}. \quad (2.1)$$

Relevant parameters for the channel flow experiments are provided in Table 1, where $f_s$ is the blower frequency and $\delta_p$ is the viscous length-scale.

<table>
<thead>
<tr>
<th>$f_s$ (Hz)</th>
<th>$dP/dx$ (Pa m$^{-1}$)</th>
<th>$\tau_w$ (Pa)</th>
<th>$u_*$ (m s$^{-1}$)</th>
<th>$\delta_p$ (µm)</th>
<th>$\rho$ (kg m$^{-3}$)</th>
<th>$\nu \times 10^5$ (m$^2$ s$^{-1}$)</th>
<th>$Re_\tau$</th>
<th>name</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>-4.724</td>
<td>0.120</td>
<td>0.315</td>
<td>47.80</td>
<td>1.213</td>
<td>1.504</td>
<td>531</td>
<td>lowC</td>
</tr>
<tr>
<td>60</td>
<td>-15.371</td>
<td>0.390</td>
<td>0.567</td>
<td>26.46</td>
<td>1.213</td>
<td>1.501</td>
<td>960</td>
<td>midC</td>
</tr>
<tr>
<td>60</td>
<td>-43.198</td>
<td>1.097</td>
<td>0.957</td>
<td>16.03</td>
<td>1.199</td>
<td>1.534</td>
<td>1584</td>
<td>highC</td>
</tr>
</tbody>
</table>

Streamwise and wall-normal velocity components were obtained using a TSI 1248A-T1.5 end-flow miniature x-wire probe operated in a constant-temperature mode. The probe consisted of two tungsten wires coated with platinum to provide a fast response. The sensing elements were 1.27 mm long with a diameter of 3.8 µm resulting in an l/d ratio of 400, twice the recommended value of l/d=200 (Ligrani & Bradshaw 1987). Square-wave tests showed a response of at least 25 kHz. It is generally accepted that a wire length of 30–60 viscous wall units is suitable for the capture of small-scale spectra (Fernholz & Finley 1996). The present wire length varied between 32 and 100, depending on the Reynolds number. Thus, there was some averaging of the smaller scales at the highest Reynolds number. However, this effect was minimal on the larger scales of motion.

The x-wires were calibrated both before and after each experiment in a suction-based calibration tunnel using a cooling velocity/yaw-response calibration. A thermistor was used to measure the temperature and compensate for any minor variations in the ambient temperature. Only experimental data that showed minimal variations between the two calibrations were used. Further details about the calibration procedure are available in Balasubramaniam (2005). The probe position with respect to the channel walls was determined optically with accuracy better than one viscous wall unit, even at the highest Reynolds number.

Simultaneous samples of the streamwise and wall-normal velocity components were acquired at 20 kHz. At every wall-normal location, 50 records of $2^{17}$ uniformly spaced samples each were acquired, separated by a fixed time delay of 1 s. The duration of each record was long enough to sample wavelengths of approximately 1250h and 2250h at the lowest and highest Reynolds numbers, respectively. However, only a fraction of this length was needed in the computation of the spectra, to represent the distribution of energy in all wavelengths to good accuracy.

Balasubramaniam (2005)
Zero-pressure-gradient boundary-layer experiments

Two sets of CTA measurements at friction Reynolds numbers of 1476 and 2395 were obtained in the same wind tunnel used by Meinhart (1994) and Adrian et al. (2000), with a documented root-mean-square (r.m.s.) value of the free-stream turbulence less than 0.35% of the mean free-stream velocity, $U_N$. The Reynolds number was varied by moving the location of the measurement station from the trip wire ($x$). The ceiling of the tunnel was adjusted so that the static pressure coefficient was generally less than 0.01, with most of the points falling below 0.005. While the homogeneity in a channel or pipe flow allows the estimation of the friction velocity using a control volume analysis, the inhomogeneity of a ZPGBL in the streamwise direction makes the measurement of $u_t$ non-trivial. In the present work, the friction velocity is obtained by performing a Clauser (1956) fit on the first few points that lie within the log layer. We assume the validity of the log law and the value of $u_t$ to be dependent on an accurate wall-normal location. Considering these factors, the friction velocities reported here are estimated to carry an error of $\pm 2\%$. The experimental parameters for the ZPGBL data are shown in table 2, where $\theta$ is the momentum thickness.

Velocity measurements were obtained using a TSI 1243A-T1.5 boundary-layer probe. The probe had an $l/d$ ratio of 400 and a response of at least 26 kHz. The wire length varied between 32 and 35 viscous wall units, depending on the Reynolds number. A cooling velocity/yaw-response calibration, identical to the case of the channel flow and GHA06, was carried out in the free stream using a rotating fixture and varying the free-stream velocity. The bridge and A/D systems were identical to the ones used in the channel flow experiments. The probe position with respect to the ZPGBL wall was determined within an accuracy better than 0.5$\delta_v$ even at the highest Reynolds number. The duration and sampling rate of the sampled data was identical to the channel flow experiments, and the duration was enough to resolve wavelengths of lengths approximately 945$\delta$ and 545$\delta$ at the mid- and highest Reynolds numbers, respectively.

### Table 2. Experimental parameters for the ZPGBL flow.

<table>
<thead>
<tr>
<th>$x$ (mm)</th>
<th>$U_N$ (m s$^{-1}$)</th>
<th>$u_r$ (m s$^{-1}$)</th>
<th>$\delta_v$ (µm)</th>
<th>$\theta$ (mm)</th>
<th>$\delta$ (mm)</th>
<th>$u_t/\nu$</th>
<th>$Re_t$</th>
<th>name</th>
</tr>
</thead>
<tbody>
<tr>
<td>2605</td>
<td>9.32</td>
<td>0.360</td>
<td>43.4</td>
<td>7.9</td>
<td>64</td>
<td>4690</td>
<td>1476</td>
<td>midZ</td>
</tr>
<tr>
<td>5545</td>
<td>9.41</td>
<td>0.331</td>
<td>47.2</td>
<td>14.8</td>
<td>113</td>
<td>8952</td>
<td>2395</td>
<td>highZ</td>
</tr>
</tbody>
</table>

3. Spectral density functions using digital FFT

The distribution of the components of the Reynolds stress tensor among various wavelengths contained in the flow was determined by estimating the one-sided wavenumber co-spectrum using digital FFT. Taylor’s frozen-field hypothesis was used to calculate the two-point spatial correlation functions from CTA time-series. Every CTA record of 6.5536 s duration was divided into blocks containing 8192 and 32 768 points each for the channel and the ZPGBL, respectively.

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Spectral density functions were then computed using a procedure that mirrored GHA06. The ability of the computation to discern the various wavelengths in the flow warrants some discussion. Each block contained enough points to resolve wavelengths of 51 $h$ for the lowC case, 180 $h$ for highC, 120 $d$ for midZ and 63 $d$ for highZ, using Nyquist sampling. To ascertain the adequacy of this resolution, spectra were also calculated by doubling the length of the data records to 16 384 and 65 536 points for the channel and the ZPGBL, respectively. It was found that the variation between the cases was minimal and that the shorter length was enough to represent the larger scales with good accuracy. Approximately 800 blocks were used in the computation of the channel flow spectra, while 200 blocks were used in the computation of the ZPGBL spectra. A nonlinear averaging procedure was used to smooth the computed spectra (Balasubramaniam 2005). Care was taken to ensure that the spectra were not distorted by the averaging process, especially for the longer wavelengths.

Streamwise power spectra are compared in figure 1 for selected wall-normal locations with spectra from Perry & Abell (1975), Perry & Chong (1982), Perry et al. (1986) and Morrison et al. (2002) and GHA06. The present data show good agreement near the ‘−1’ region, where all the other spectra, except GHA06,
collapse. At longer wavelengths, there is a wide spread in the spectra. It is possible that the scatter is due to the different levels of long-term stability in the various experiments. Low-frequency drifts in the bridge, mean velocity, ambient temperature or errors in calibration could all have caused an increase in energy at longer wavelengths. The present data fall between the data of Perry & Abell (1975) and Perry & Chong (1982), but are self-consistent, and at worst, they would only have underestimated the energy content of the LSMs.

4. Spectra of the large eddies

Equal amounts of the streamwise kinetic energy, $\Phi_{uu}$, are carried between equal $k_x$ intervals in a semi-logarithmic plot of the premultiplied spectrum. Premultiplied spectral plots are hence used to illustrate clearly the contributions of various wavelengths to the total spectra, and to locate wavelengths of peak spectral densities. The inset to figure 1 shows a sample of the premultiplied spectra inside the logarithmic layer in the channel and ZPGBL flows. Two distinct peaks are present: one corresponding to a wavelength of $7.5h/5\delta$ and the other to a wavelength of $1h/1\delta$. These values are close to the wavelength reported by Kim & Adrian (1999) and GHA06 for the pipe flow. Kim & Adrian (1999) interpreted their observations based on the organization of the hairpin vortices into large- and VLSMs, with the smaller peak corresponding to a hairpin-vortex packet and the larger peak to a further organization of the packets into VLSMs by alignment. A similar interpretation is equally applicable here, suggesting that the organizational patterns in pipe, channel and ZPGBL flows are quite similar. Following GHA06, we shall take $k_xh=2$ or $k_x\delta=2$ to be the nominal dividing line between LSMs and VLSMs. This corresponds to wavelengths of $\pi h$ or $\pi\delta$.

Plots of the streamwise velocity spectra ($\Phi_{uu}$), the wall-normal velocity spectra ($\Phi_{vv}$) and the shear-stress spectra ($\Phi_{uv}$) for the channel and the ZPGBL are presented in the electronic supplementary material and at http://www.efluids.com/databases/BA07PRSL. Each of the spectra resembles their counterpart pipe flow spectra presented by GHA06. Quantitatively, the pipe spectra agree best with the channel spectra, and less well with the boundary-layer spectra, but this may be an issue of finding the correct proportionality between $\delta$, $R$ and $h$ used in the normalization. All wavelengths in $\Phi_{uu}$ behave alike as functions of $y/\delta_o$, contributing to a reduction in the r.m.s. of the streamwise velocity with increasing wall-normal distances. The $\Phi_{vv}$ spectra are an order of magnitude smaller than the streamwise spectra, consistent with the lower values of the wall-normal component relative to the streamwise component. The wall-normal spectra depend upon the position within the log layer and above, and can be summarized as follows. The ‘crossing over’ of the wall-normal spectra that was observed in pipe flows by GHA06 is also observed in the log layer in the channel (except at the lowest Reynolds number) and in the ZPGBL. ‘Cross-over’ refers to the spectral energy density of the low wavenumbers increasing with height in the logarithmic layer, but decreasing with height above it. The spectral densities of the high wavenumbers decrease monotonically with height in and above the logarithmic layer. The cross-over occurs within a range of wavelengths at each Reynolds number, and is generally found between $3 \leq k_xh \leq 20$ for the channel and $1 \leq k_x\delta \leq 23$ for the ZPGBL. The corresponding range for the pipe flow is $1 \leq k_xR \leq 9$.

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A trend similar to that found in the wall-normal power spectra is also found for the velocity co-spectra in the channel and ZPGBL flows. Near the wall, for $y/h \leq 0.3$ in the channel flows and for $y/\delta \leq 0.2$ in pipe flows, the longer wavelengths show increasing power densities with increasing distance from the wall, while the shorter wavelengths show an opposing trend throughout the flow. While the cross-over wavelengths for the channel flow fall within the range $1 \leq k_x h \leq 4$, the corresponding wavelength for ZPGBL falls within $1 \leq k_x h \leq 9$. While the ranges represent the entire range over which the cross-over occurs, the cross-over begins very close to the beginning of the VLSM regime at $k_x h, k_x \delta = 2$.

5. Premultiplied spectra

Premultiplied spectra for streamwise velocity fluctuations are presented in figure 2a–e. Each spectrum shows a bimodal distribution, as in figure 1(inset), in both the channel and ZPGBL flows, at all the investigated Reynolds numbers. In the ZPGBL, at a given Reynolds number and with increasing distance from the wall, the long-wavelength peak reduces in magnitude relative to the shorter wavelength peak. In the channel flow, the ratio of the long-wavelength peak to the short-wavelength peak remains more nearly constant as $y$ increases, i.e. the apparent bimodal shape persists in the channel flow, but not in the boundary layer.

The variations in the wavelengths of the peaks are plotted versus the distance from the wall for various flows and Reynolds numbers in figure 2f. The shorter wavelength peak, which represents the LSM wavelength carrying the highest energy density, behaves in a manner that is completely consistent with the data presented by GHA06. The wavelength increases until $y/\delta_0 \sim 0.5$, and then reverses the trend to decrease towards the channel or pipe centreline or the boundary-layer edge. This behaviour points to the presence of a similar underlying mechanism in the channel, ZPGBL and pipe flows that acts to create the 1–3$\delta_0$ motion. Also, this behaviour is more or less common for the range of Reynolds numbers, $531 \leq Re_\tau \leq 7959$.

The long-wavelength peak, which represents the VLSM wavelength carrying the highest energy density, increases to approximately 20 for the pipe flow in GHA06. In comparison, the values of $A_{\text{max}}$ (wavelength of the spectral peak) are shorter for the channel flow reaching a wavelength of approximately 13–15$h$ near $y/h \sim 0.5$. Beyond $y/h \sim 0.5$, the long-wavelength peak is difficult to discern and the spectra show a unimodal distribution with only the shorter wavelength peak. Generally, the behaviour and trends observed for the pipe flow are also found for the channel flow. However, the ZPGBL spectra make an early transition from a bimodal distribution to a unimodal distribution near $y/\delta \sim 0.2$ beyond the edge of the log layer. The maximum value of the wavelengths also falls short of the values for the pipe and channel flows. This could be an indicator of poorer organization of VLSMs in the boundary layer.

Finally, in the channel flow, as the Reynolds number is increased, more of the kinetic energy is carried by the VLSMs in comparison with the LSMs, and the contribution of the VLSMs to the kinetic energy increases. To investigate this question further, the cumulative energy and shear-stress distributions will be calculated.
Figure 2. Premultiplied power spectrum of streamwise velocity fluctuations, $k_x \Phi_{uu}$, versus streamwise wavenumber, $k_x h$ or $k_x \delta$, for (a) $Re_T = 531$ (channel), (b) $Re_T = 960$ (channel), (c) $Re_T = 1584$ (channel), (d) $Re_T = 1476$ (ZPGBL), and (e) $Re_T = 2395$ (ZPGBL). (f) Wavelengths of the spectral peaks as a function of the wall-normal distance. Hash symbol, $Re_T = 531$ (channel); plus symbol, $Re_T = 960$ (channel); asterisk symbol, $Re_T = 1584$ (channel); less than symbol, $Re_T = 1476$ (ZPGBL); and greater than symbol, $Re_T = 2395$ (ZPGBL). The remaining symbols are as labelled in figure 4 of GHA06.

6. Cumulative distributions of energy and stress

The premultiplied spectra shown in figure 2a–e have been used to determine the wavelengths carrying peak kinetic energy density. However, it is often useful to analyse the relative contributions of the different wavelengths to the total energy (or shear stress). The cumulative contributions to $\bar{u}_i \bar{u}_j$ from all wavenumbers

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between $k_x = 2\pi/A$ and infinity are calculated using discrete summation

$$Y_{ij}(k = 2\pi/A) = 1 - \frac{\Sigma_0^k \Phi_{ij}(\hat{k})\Delta\hat{k}}{\Sigma_{0_{max}}^k \Phi_{ij}(\hat{k})\Delta\hat{k}}. \quad (6.1)$$

Since all of the cumulative energy distributions are rather similar in distribution, those of the streamwise kinetic energy for various wall-normal locations are relegated to figures A6 and A7 in the electronic supplementary material for channel and ZPGBL flows, respectively. At all Reynolds numbers and wall-normal locations, scales larger than $3h$ (VLSMs) carry at least 45% of the streamwise kinetic energy in the channel and ZPGBL flows. Correspondingly, GHA06 finds that scales larger than $3R$ in pipe flows carry more than 65% of the energy. The variation in the fraction of energy carried by the VLSMs as a function of the Reynolds number for the different flows is shown in figure 5a. The variations in the energy carried by the channel, ZPGBL and pipe flows may be due to the differences in the flow geometry (and the attendant differences in the definitions of $\delta_o$) or the Reynolds number effects. This issue will be further explored in §7.
Figures 3 and 4 plot the cumulative Reynolds shear-stress fractions for the channel and ZPGBL flows. In the region $y/\delta_o>0.1$, more than 40% of the stress is carried by scales larger than $3\delta_o$, both for the channel and ZPGBL flows. The corresponding number for pipe flows is approximately 50%, in the same region. The variation in the fraction of the shear stress carried by the VLSMs as a function of the Reynolds number for the different flows is shown in figure 5b.

Figures 4. Cumulative Reynolds stress fraction, $Y_{uv}$, associated with structures having wavelengths less than $A$ for (a, b) $Re_{\tau}=1476$ (ZPGBL) and (c, d) $Re_{\tau}=2395$ (ZPGBL).

Figure 5. Variation in the fraction of (a) kinetic energy and (b) shear stress carried by the VLSMs as a function of the Reynolds number for channel (open symbols), ZPGBL (grey filled symbols) and pipe flows (black filled symbols) for open squares, $y/\delta_o\sim0.1$; open up-triangles, $y/\delta_o\sim0.2$; open down-triangles, $y/\delta_o\sim0.3$; open diamonds, $y/\delta_o\sim0.5$; and open circles $y/\delta_o\sim0.7$.

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7. Derivative of the Reynolds shear stress

The integral of the wall-normal derivative of the co-spectra occurs in the mean momentum equation as the net fictitious force exerted by the shear stress on the mean

\[
\frac{\bar{D}U}{Dt} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{\partial (\bar{w} \bar{v})}{\partial y} + \nu \frac{\partial^2 U}{\partial y^2} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \int_0^\infty \frac{\partial (-\Phi_{uv})}{\partial y} \, dk_x + \nu \frac{\partial^2 U}{\partial y^2}. \quad (7.1)
\]

Thus, a negative (positive) contribution to the \(y\)-derivative of \(-\bar{w}\bar{v}\) from a given range of wavenumbers translates into mean flow retardation (acceleration) caused by those wavenumbers. Study of the net force allows an analysis of the roles played by the different motions in the flow in accelerating or retarding the mean. For this purpose, the net force is more appropriate than the Reynolds stress.

Figure 6. Vertical derivative of premultiplied co-spectra, \(\Phi_{uv}\), for (a) \(Re_t=531\) (channel), (b) \(Re_t=960\) (channel) and (c) \(Re_t=1584\) (channel). A positive peak exists for all but the lowest Reynolds number.
Figures 6 and 7 plot the premultiplied finite-difference derivative of the co-spectra as a function of $k_x \delta_o$. Away from the wall for $y/\delta_o > 0.2$, the spectra generally remain negative for both the channel and ZPGBL flows. This implies that the component scales of the Reynolds shear stress all act to retard the mean flow outside the log layer. However, even in this region, a large fraction of the force is created by the VLSMs. As the wall is approached, the contributions of the VLSMs to the retardation reduce, and for $y/h \sim 0.17$ (channel flows) and $y/h \sim 0.10$ (ZPGBL flows), the VLSMs show a tendency to accelerate the flow, as is evident from the positive peak in the net force spectra. This peak is not detected at the lowest Reynolds number of the channel flow, which is probably an artefact of the larger error in the shear stress in comparison with the other cases. Most importantly, the observation of the positive peak in the channel and ZPGBL flow substantiates a similar tendency of the VLSMs to accelerate pipe flows in the near-wall region, as observed by GHA06. As the shear stress peak, $y_p^+ \sim 2Re^{1/2}$ (Sahay & Sreenivasan 1999), is approached, a substantial portion of the mean flow acceleration, and hence the fictitious force, is attributable to the VLSMs. Thus, the importance of the VLSMs away from the shear-stress peak is quite clear, but the roles they play at, and on, the wall side of $y_p^+$ remain as open questions.

8. Conclusions

Large-scale eddies are similar in pipes, ZPGBLs and channels over the range of Reynolds numbers, $3815 \leq Re_r \leq 7959$ (pipe), $1476 \leq Re_r \leq 2395$ (ZPGBL) and $531 \leq Re_r \leq 1584$ (channel), with qualitatively similar spectral shapes, similar

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distributions of the cumulative kinetic energy and the Reynolds shear stress over the component wavelengths. A substantial portion of the kinetic energy (40–65%) and the Reynolds shear stress (30–50%) is carried by VLSMs in pipe, channel and ZPGBL flows. Thus, the dividing boundaries between VLSMs and LSMs (taken to be $\pi \delta_o$) and LSMs and the main turbulent motions (taken to be $0.1 \pi \delta_o$) are nominal, but happen to be useful rules of thumb for all of these flows. Large eddies are, therefore, universal phenomena in canonical wall flows with smooth walls, fully developed flows or in equilibrium flows like the ZPGBL with a low free-stream turbulent intensity. These flows allow ample time for long structures to develop. It is not known if deviations from these conditions could break up the very large scales. For example, roughness on the wall is known to reduce the streamwise integral length-scale (Krogstad et al. 1992; Tomkins 2001), suggesting that it disrupts the formation of long structures (although recent results for channels by Flores & Jimenez (2004) and Krogstad et al. (2005) do not show shortening).

The importance of large and very large scales in carrying kinetic energy and shear stress has been demonstrated without doubt. The form that they take is less certain. The discussion by GHA06 develops a picture in which the LSMs are hairpin packets, whose sizes range up to the size of turbulent bulges. When averaged over the scales of the hairpin-vortex cores, the smoothed fields of the packets and the bulges look like quasi-streamwise, counter-rotating vortices, with a low-speed streak between them and weaker, high-speed sweeps on the sides. If, as suggested by Kim & Adrian (1999), the VLSMs consist of a concatenation of bulges that do not align perfectly in the spanwise direction, then the smoothed VLSM motion would produce a long, meandering pair of quasi-streamwise vortices. This is consistent with the observations made here and in other spectra analyses, including an atmospheric work by Drobinski & Foster (2003). It is also consistent with flow visualizations by PIV and a spanwise rake of hot-wire probes in the very recent work by Hutchins & Marusic (submitted). The smoothed field of a concatenation of misaligned hairpin packets would look like a pair of meandering counter-rotating streamwise vortices, whose diameters would be of the order of the height and width of the packets.

It is perhaps not unreasonable to conclude by indulging in some conjectures concerning the mechanism(s) that create the LSMs and VLSMs. One idea is that the bulges are the end result of autogeneration, the process by which a short three-dimensional disturbance grows into a hairpin, which then creates new hairpins up- and downstreams (Zhou et al. 1996, 1999). The process repeats for the new upstream hairpin, which then autogenerates another, and so on, until a long packet of hairpins is created from the short disturbance. Autogeneration provides a fundamental mechanism for scale growth in the streamwise direction. The packets grow vertically as well as streamwise until they reach the height imposed on the flow by its geometry ($\delta_o$). In the spanwise direction, growth may also occur by vortex pairing (Adrian et al. 2001; Tomkins & Adrian 2003). The largest packets are the bulges. Another scenario is that long disturbances near the wall are amplified by instability. At low Reynolds numbers, Waleffe (2001) and Wedin & Kerswell (2004) predict large outer flow solutions, whose scale is of the order of $\delta_o$. Until further evidence accumulates, we must allow for the possibility that variations of motion like these could occur at higher Reynolds numbers. Packets near the wall could be the stimulus for growth of these motions.
leading to LSMs. Del Alamo & Jimenez (2006) predict the growth of structures as long as 40h, and this might be the mechanism for the formation of VLSMs. But a complete model based on the amplification of very long modes requires an explanation of the origin of the long streamwise initial disturbance needed to stimulate the growth mode. One possibility is that the long outer modes are stimulated by wall packets formed by autogeneration, and the wall packets are organized into very long groups by the large-scale outer modes. Some evidence for outer modes organizing wall vortices can be found in the work of Toh & Itano (2005) in low Reynolds number DNS of periodic, short channels, and of Iwamoto et al. (2005) in larger Reynolds number DNS of longer channels. Toh & Itano (2005) show that ‘the large-scale structures are generated by the collective behaviour of near-wall structures and that the generation of the latter is in turn enhanced by the large-scale structures. Hence, near-wall and large-scale structures interact in a co-supporting cycle’. Iwamoto et al. (2005) show that large scales in the form of quasi-streamwise vortices spanning a channel interact with small scales near the wall by sweeping the small scales sideways into the region of the stagnation point between the larger flows.

These pictures do not speak directly about the formation of very long structures, but they do suggest that large-/small-scale interaction may be a means of supporting the long-wavelength amplification model of del Alamo & Jimenez (2006) or a similar mechanism. This could provide a mechanism for the apparent concatenation of bulges into VLSMs.

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