The application of the research work of James Clerk Maxwell in electromagnetics to industrial frequency problems

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Faraday’s work inspired the development of electrical motors and generators. Until Maxwell pointed out the significance of Ampere’s Law, there was no rigorous design method for magnetic devices. His interpretation strongly influenced the creation, by others, of the ‘magnetic circuit’ approach, which became the seminal design technique. This, utilizing the concept of reluctance, led to the design method for magnetic machines that is still widely in use today. The direct solution of the Maxwell equations (less the displacement current term) had to await the development of modern continuum methods to yield the field everywhere in, and around, the devices of interest, and this then permitted the application of the Maxwell stress tensor. This final refinement yielded forces and torques, and this resulted in the accurate prediction of electrical machine performance.

Keywords: Maxwell’s equations; magnetic circuits; Ampere’s Law; industrial frequency; Maxwell stresses; electrical machinery

1. Introduction

While Maxwell is well known for his contributions to many areas of physics including optics, planetary motion and electromagnetics, the development of the field equations has often been seen as leading to both the development of Einstein’s work on relativity and the prediction of radio frequency communication (as demonstrated by Hertz). The intention of this paper is to examine the impact that his ideas and theory of electromagnetics have had on the design of low-frequency electrical machinery. The thread that will be followed is largely historical but cannot, in the space available, be considered to be complete and definitive given the wide-ranging explorations of physics that were occurring in the nineteenth and early twentieth centuries. Here we concentrate on Maxwell’s contributions.

What could be easier than to look back and list the applications of equations from Maxwell’s work? All papers on the solution of industrial frequency, 50/60 Hz, field theory problems quote Maxwell’s equations. It is common knowledge that Ampere (1820) produced his law, what became later known as

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One contribution of 20 to a Theme Issue ‘James Clerk Maxwell 150 years on’.
‘Ampere’s Law’, only a short time after Oersted announced his discovery that an electric current produced a magnetic field. Then, Faraday (1832), in a wonderful sequence of inspired experiments, demonstrated that an electric field, and so a voltage, could be produced by a changing magnetic field.

The scene was set; from then on there followed the fast development of electromagnetic energy converters: first motors and then generators, which we call ‘electrical machines’. (Electrostatic devices are part of a quite separate story.) These machines form the backbone of our industrial heritage, supplying electrical power for countless applications, without which modern society could not exist.

From our current position, approximately 170-odd years on, we elide and blur these developments, so that the rolling out of ideas and applications appears to be a smooth, continuously forming interrelated corpus of ideas. These all set firmly in a bedrock established by Ampere, Faraday, Maxwell, Heaviside and all the ingenious inventors of energy converters.

The aim of this paper is to demonstrate that, for the low-frequency applications of interest here, what are known as ‘Maxwell’s equations’ were not employed in the design of real devices until the advent of powerful computers in the late 1960s. The field equations (Laplace and Poisson) could then be solved for ever more complex problems. Then and only then could the little-known Maxwell stress equations be applied to fill in the last part of the jigsaw, the calculation of forces and torques.

There were several development paths, which are as follows. The first was, of course, the ‘electromagnetic propagation’ path set out by Maxwell, and leading to the prediction of the velocity of light and hence the concept of electromagnetic radiation, and also the development, by others, of radio, television, radar, etc. Ideas unarguably placed him firmly in the scientific pantheon alongside Newton and Einstein.

The second, Maxwell-inspired, path, was taking much longer, but was leading to the accurate prediction of forces and torques in electromagnetic power devices, using the Maxwell stress equations, and the melding of industrial frequency problems with propagation effects. The latter requires the use of a considerable amount of computer power. Future developments in computing techniques will lead to the solution of the full set of Maxwell’s equations in the complex structures considered here, but not ubiquitously for some years yet.

The third path was concerned with the electric circuit analysis of commutators. This came and went quickly, but has echoes in modern electric motor drive analysis.

The fourth, which is the main area of interest here, was the use of ‘magnetic circuit analysis’ of electrical machines. We argue that Maxwell triggered this off, but further research is necessary to confirm this. Here, we advance this idea as a hypothesis.

In the following, the various paths will be intermingled, as it is convenient to set these in chronological order.

2. The first age: the experimental phase

In April 1820, Oersted demonstrated that a current could move a compass needle. This could be considered as the first electromagnetic actuator. Within a very short time, Ampere (September 1820) formulated his law linking the current
to the magnetic field produced. This was the first major theoretical step. In due course, Maxwell used it as a building block in the development of his electromagnetic theory.

Faraday (1832) in a set of carefully conceived experiments discovered the essential associated effect, whereby a changing magnetic field produced an electric field, a voltage. This idea appears to have been taken up immediately and used by machine designers. The iron structure design had to wait until much later.

These two major discoveries triggered off a wave of experimental developments for motors and generators. Since batteries had been used to supply electrical power, then it was obvious, apparently, to all concerned that direct current was required from a generator. The solution was to use a commutator. As has been pointed out, this set back the development of industrial electrical engineering by some 60 years. Nevertheless, the direct current had to run its course, and so by the 1870s–1880s there were extensive DC electrical schemes producing and using electrical power. The approach to the magnetic design was unscientific, empirical and pragmatic, and not uncommonly quite wrong.

It was not until 1873–1874 that a good working understanding of the link between the magnetic flux (or number of lines) and the field coil currents was achieved.
Manifestly, a great deal of thought was given to the problem, but the thinking was, to say the least, not well founded on any science. During the period 1840–1873, the science upon which electrical engineering would be based had to establish itself by some form of the bootstrap process. Initially, there were no agreed units, no standards, no reliable measuring instruments readily available, nor agreed ways of specifying magnetic materials. Entrepreneurs simply built something that produced a voltage, initially using permanent magnets, and later coils. It was cut and try; there was no agreed procedure. Three examples of such procedures are as follows: first, a large air gap was needed to accommodate the armature coils (figure 1), the opposite of what was really required; second, the poles had to be very long (figure 2); and, third, thought due to Preece, the resistance of the armature was to be increased to equal that of the load in order to increase effectiveness. However, it was a sellers’ market. Provided it did not get too hot, vibrate unduly or make too much noise, someone would buy it.

3. The first age: moving towards a theory of design for magnetic devices

Clearly, there was a major requirement for a significant amount of engineering science, a new theory to enable \textit{ab initio} design of magnetic devices. What

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was required was the application of Ampere’s Law, set out in 1820. This takes the form

\[ \oint H \, dl = IN. \]

This was later referred to and explained by Maxwell (1873, vol. 2, article 499) in his treatise.

Maxwell (1873, second paragraph) states that ‘The line-integral (of the magnetic force) \( 4\pi i \) does not depend on the nature of the medium in which the closed curve is drawn. It is the same whether the closed curve is drawn entirely through air, or passes through a magnet, or soft iron, or any other substance, whether paramagnetic, or diamagnetic’.

That is precisely what was required to assist the machine designers, but the truth of it required some teasing out. There was much interest at that time in the magnetic properties of materials, and in the use of those properties as part of the design process for electric machines (see Stoletov 1872; Rowland 1873, 1874, 1884). Both Stoletov & Rowland were personally well known to Maxwell. Rowland clearly described the idea of ‘magnetic resistance’, the term ‘magnetic reluctance’ came later,

\[ R_{\delta l} = \frac{\delta l}{\mu A}. \]

Then,

\[ IN = \sum \phi R_{\delta l}. \]

Nevertheless, it seems that the significance of this work was not recognized and it was a further decade before Ampere’s Law was generally applied, i.e. after Maxwell’s death in 1879.

Otherwise, there is nothing in any of the Maxwell work to indicate that he had any contact with those designing electromagnetic devices. With two exceptions, there appears to be no letters in the three volumes, nearly 3000 pages, of his outgoing correspondence (Harman 1990, 1995, 2002), nor in the two volumes of his papers (Niven 1890, republished 2003a,b), nor in his two-volume treatise (Maxwell 1873), to show the possibility of any interest being shown by ‘heavy electrical engineers’ in Maxwell’s work.

The two possible exceptions to be noted are as follows.

The first is his observations on the work by Weber (1852, article 443, vol. 2) in a paper on nonlinear magnetic material B/H characteristics. This seems to have gone unnoticed at the time. There appears to have been no follow-up correspondence.

The second relates to his attending a Royal Society meeting in February 1867, where Wheatstone and Siemens presented their two papers, separately, on the phenomenon of self-excitation of a generator. Very shortly after, Maxwell submitted a theory he had devised following the meeting (Maxwell 1867). This was couched in electrical circuit element terms and was for an iron-free machine. Heaviside (1881) later referred to the work and extended it.

We note, in passing, that nowhere in all his reported writings does Maxwell ever look at the problem of calculating the magnetic flux in a soft iron ‘magnetic circuit’ with an air gap. Looking back, had he done this, it would have been of

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enormous help to machine designers. Or, as an alternative theory, is it simply that any note or letters he might have made on the subject have not come down to us?

However, and most importantly, he did lay the foundations for the development of magnetic circuit analysis in article 499 of the treatise.

4. The first age: a theory of design for magnetic devices

Until 1879, the design of the iron structure of a machine appears to have been more of a mechanical problem, rather than a magnetic one. It was little more than something to hold the rest of the machine together. However, it was recognized that the presence of iron was helpful magnetically. Lord Elphinstone & Vincent (1879) drew attention to the advantages of a ‘closed’ magnetic circuit.

Crompton (1928, p. 102), in his autobiography, describes ‘putting his best thoughts into the improvement of dynamo design’. That was in 1881–1882. He consulted ‘Sir William Thomson, Fleeming Jenkins and others’. Quoting him again: ‘we arrived at a formula for the variation of electro-motive force, viz. $4\pi \times \text{ampere turns/}(\text{air gap} \times 2)$’, thus ignoring the effect of any iron. Crompton called for an even better method to improve the accuracy. Those named here would have had access to the relevant article 499, in vol. 2 of the treatise.

Again, when reminiscing, Crompton (1922) at the IEE, referring to this same problem, was reported as saying (p. 394): ‘To get this I myself travelled many miles between London, Edinburgh and Glasgow, pointing out to Sir William Thompson and to Rankine that it was imperative to have a formula for our electromotive force’. He then quotes the formula above. He goes on to relate that his chief designer, Kapp, simply refused to accept the idea at that time.

This, we believe, is the first recorded indication that a major manufacturer/designer of electrical machines was adopting a scientifically based method for the design of an air-gap/iron magnetic circuit.

At about the same time, Silvanus P. Thompson published his book on ‘dynamo electric machines’. The first edition appeared in Thompson (1884). However, there was no mention of the use of Ampere’s Law. It was not until the 2nd edn. (Thompson 1886) appeared that Thompson set out the whole recent history very clearly. Bosanquet (1882, 1883) had shown how to add reluctances. Moreover, Kapp (1885, 1886) showed how the mmf/flux/reluctance idea could be used in the design of machines. However, Thompson did note the awkward units used by Kapp. He reserves special mention for the Hopkinsion brothers, John & Edward (1886), who took into account the nonlinear nature of the B/H curves when calculating reluctances.

It is very important to observe that Thompson (1886) also, in that second edition, refers directly to Maxwell’s (1873) article 499, vol. 2. As a side note, it is interesting to examine Thompson’s personal copy of Maxwell’s treatise (1881, 2nd edn.). He heavily annotated article 499, in pencil, underlining keywords in pencil, and marking it as clearly of importance to him.

To sum up, by the mid-1880s, machine designers had the means to calculate the flux/pole for a given number of ampere-turns, and hence the voltage produced at a given speed. The concept of the magnetic circuit was well established and machines could be designed with a reasonable amount of
confidence, given some allowance for flux leakage, and sensible dispositions of any solid and laminated iron.

What more could a designer wish? All the tools were in place. Apart from fine-tuning to accommodate losses and air-gap flux density distributions, the industry had a fairly robust technique available. Indeed, for many industrial designers, even today, this situation has not changed since that time. There are even computer packages to aid machine design, for sale at the present time, which are based on little more than the theory outlined in S. P. Thompson’s 2nd edn. (Thompson 1886) of dynamo electric machines.

5. The first age: moving towards an effective electromagnetic theory for design

Returning to Maxwell, following the publication of his paper on Faraday’s lines of force (1855) while first at Cambridge, he was developing his theory of electromagnetism, to be published in his two-volume treatise (Maxwell 1873): first at Marischal College, Aberdeen (1856–1860), then at King’s College, London (1860–1865), where he published his paper on the dynamical theory (1865), followed by a period at his home in Glenlair, Scotland (1865); then unfortunately for the subject, at Cambridge as Director of the Cavendish (1871). This was unfortunate because his Cavendish duties greatly limited the time available for him to devote any effort to the further development of his electromagnetic theory before his death in 1879.

Maxwell was the first to come up with a workable and understandable concept for electromagnetic fields. That in itself was a major aid to thinking as it provided a conceptual model of fields existing not only in the obvious magnetic circuit of a device but also in the air surrounding the structure, i.e. the leakage fields. This makes it clear that the form of the iron structures is critical to the device performance.

He developed easily applied equations to calculate forces; these are referred to as the Maxwell stress equations. These are currently of great importance in electromagnetic device design and are given as follows.

The force on a body in free space is given by

\[ F = \int_S \mathbf{T} \cdot \mathbf{n} \, ds, \]

where \( \mathbf{T} \) is known as the ‘Maxwell stress tensor’ and is integrated over the surface surrounding the body (\( n \) is the normal to the surface and \( S \) is the area of the surface). The tensor \( \mathbf{T} \) (in modern notation), from article 642, has the form

\[
\mathbf{T} = \frac{1}{\mu_0 \mu_r} \begin{bmatrix}
B_x^2 - \frac{1}{2} |B|^2 & B_x B_y & B_x B_z \\
B_y B_x & B_y^2 - \frac{1}{2} |B|^2 & B_y B_z \\
B_z B_x & B_z B_y & B_z^2 - \frac{1}{2} |B|^2
\end{bmatrix},
\]

where \( B \) represents the vector of magnetic flux density.
Clearly, to apply this equation in real devices, a detailed field distribution is needed. This only became possible with the advent of powerful computers (see §7).

In addition, of course, we have what are referred to as Maxwell’s equations, but therein lay a problem. Maxwell appears to have chosen to use quaternions (recently developed by Hamilton), to make the theory more compact. Unfortunately, very few people had any knowledge whatsoever of the subject. To quote Mahon (2004, p. 143), ‘the job was completed by Heaviside and Gibbs—20 years later’. They recast them into a more acceptable form by using vector notation. Therein lies a problem, which was not resolved until the development of ‘differential forms’ in the late twentieth century.

The Maxwell equations, in vector form, then became the starting point for every author of electromagnetic industrial frequency research papers/reports/theses. They write down the full set of equations and then remove the $dD/dt$ term, since there is no propagation. What is left is simply the Faraday and Ampere laws, usually recast as the Laplace equation if there are no driving sources, or the Poisson equation if the problem is static or quasi-static, or the diffusion equation if eddy currents are to be accommodated. While the question may well be asked ‘so why bother with Maxwell’s equations at all?’, it is important to recognize that the contribution of the full equation set is that it is possible to identify what is being ignored and how much error is being introduced into the basic solution by ignoring the terms.

To sum up, Maxwell gave us the concept of an electromagnetic field, the stress equations and a full set of electromagnetic equations (with some help from Heaviside (1888) and Gibbs), which we whittle down according to the requirements of the device being considered. In addition, what could be regarded as the key starting point for the analysis of machines, i.e. the magnetic circuit.

6. The second age: design calculations increasingly based on field theory

It is at this point that field theory and circuit theory are combined together. The designers of electromagnetic devices tended to make approximations to simplify the equations and these were acceptable for two reasons. First, the ability to control manufacturing processes and material properties to any great degree of accuracy did not exist and thus the errors in constructing the physical device were considerably larger than those introduced by ignoring small terms in the underlying physics model. Second, the overall size to wavelength ratio meant that, for a whole range of devices, the radiation of the electromagnetic field is essentially zero. Such devices are usually referred to as ‘low frequency’, even though, strictly speaking, this is a misnomer.

Although Maxwell had shown designers the equations that described the performance of their devices, they could not be solved for practicable problems, and thus appeared to be of largely academic interest. However, using the magnetic circuit approach, by 1890, designers had a working method that helped them design and produce electromagnetomechanical energy converters of significant industrial size. Efficiency and local overheating had long been of concern. Early on, it was recognized that an alternating magnetic flux in solid iron could produce excessive heating to the point of melting. This could be reduced by using thin iron
laminations, parallel to the flux direction. The iron losses (eddy current and hysteresis) could then be estimated using loss curves based on laboratory tests on ostensibly the same materials. That method is still used for want of something better. The flux densities in the various parts of the device were calculated using simple magnetic circuit models, with corrections applied for leakage fluxes.

Many attempts were made to determine the field patterns in critical areas of the devices. A variety of analogue techniques was developed. As an example, the air-gap region of a machine, over a pole pitch, is one such region. The shape of the flux density distribution over a pole is important for a generator, slightly less so for a motor. It is frequently desirable to achieve as near a sinusoid as possible. This cannot be easily calculated analytically. It was necessary to resort to hand flux plotting techniques to solve the Laplace equation for the field distribution in the air-gap region. An enormous amount of this was done in the USA, when engineering draughtspersons were otherwise idle during the depression. The results for wide ranges of air-gap/pole arc/pitch dimension ratios were plotted and published as air-gap flux coefficients.

Similarly, but using highly sophisticated mathematics, the Carter coefficient was introduced for the air-gap slot/tooth region. This was performed for both singly and doubly dentated regions.

The point here is that the field problems solved were Laplacian. The problems were for specific regions of the machine, not for the whole device. The resulting coefficients could be easily applied by the designer, with no knowledge whatsoever of the field-solving methods employed.

A second set of problems, those with eddy currents, for example losses in solid pole iron and in solid and stranded conductors, required the use of the Poisson and diffusion equations for the current distributions. The loss formulae developed, where solid iron was involved, were often a blend of theory with large amounts of experience and empiricism, guaranteed to give reasonable results, since they were based on measurements. The concept of ‘skin effect’ had been introduced by Lamb (1883). This enabled the designer to choose an appropriate conductor, or lamination thickness, to minimize losses. This thickness was commonly set to less than twice the local skin depth.

In the first half of the twentieth century, as competition to develop more efficient and effective devices increased, better solutions to the equations were needed, and, as has been discussed, this led to the development of analytical solutions to highly idealized parts of devices and the creation of simple hand-based approaches to solving for the fields themselves. Additionally, analogues of the electromagnetic field were created, for example, using fluid flow systems and resistive networks to understand the field structures. These approaches were seriously limited in that if they took longer to create, or were more expensive to construct than the device itself, they were not worth using.

Thus, what has become known as the ‘classical design’ method gradually developed. It was based firmly on magnetic circuit concepts, with ‘bolt on’ correction factors and coefficients derived in a variety of ways, constantly subject to testing against the national and international standards and the requirements of ever more demanding customers.

From 1960 onwards, there were three major developments that changed the whole picture.
First, a resurgence in the use of algebraic analysis techniques. A leading proponent here was Hammond (1959), who developed methods for solving for the field distribution, albeit in somewhat idealized electrical machines. This seminal set of papers led to great insights into the fundamentals of machine theory.

Second, the advent of powerful computers to solve the equations from continuum theory for real magnetic devices. At last, it was possible to deal with real, complicated, geometries, with realistic representation of materials. This was accompanied by a seminal paper by Carpenter (1960) on the application of the long neglected Maxwell stresses (Maxwell 1873, ch. XI), as powerful tools for the calculation of forces and torques, leading into the third age.

Third, forcing the use of the new computer-based methods, from the 1970s, the development of a plethora of new devices for which the classical design method was simply not applicable. These included the following: linear induction motors; axial flux machines; claw pole generators; internal permanent magnet (IPM) machines; brushless DC motors; superconducting machines; and novel forms of limited motion actuator. In addition, quite commonly, high levels of magnetic saturation were employed. Hence, classical design methods could no longer be used with any degree of confidence.

7. The third age: computational electromagnetics from 1940 onwards

From approximately 1940 onwards, there occurred a convergence of electrical engineering technologies (several themselves fuelled by the understanding provided by Maxwell) and mathematical developments of the 1930s, leading to the birth of the modern era of digital computing. Of course, some of the background had been in place since before the time of Maxwell with the development of the ‘difference engine’ by Babbage (1823). Lord Kelvin (William Thompson 1876) produced an analogue computer for tidal predictions, but this system was purely mechanical. The first analogue computer involving any electrical systems capable of solving simple equations appeared in approximately 1927 (Vannevar Bush; Bush & Hazen 1931) and the first electronic analogue computer appeared in approximately 1941. Turing (1937) described a ‘universal machine’ that can be programmed to duplicate the function of any other existing machine. In a sense, this statement sets the scene not only for solving the electromagnetic field equations of Maxwell but also for all other phenomena. If the electromagnetic field can be considered to be a deterministic ‘machine’, then Turing showed that it can be simulated.

The development of ENIAC (1946) is generally considered the starting point of the modern era of computing—largely because it resulted in the first publicly published patent in the area. This machine could complete 5000 adds and 300 multiplies per second, and it had a 200 byte memory and was programmable. While lacking in power compared with modern processors, it was a major advance in its day. It was immediately put to use in solving pressing physics problems. Now, all the pieces were in place to look at an effective computer-based simulation of an electromagnetic field based on Maxwell’s equations.

By 1955, IBM had moved the stakes to 4000 adds and 800 multiplies per second and had a storage system of 10 kB. In addition, the system could retrieve 15 kB s\(^{-1}\) from a magnetic tape. Although the computer power was increasing.
and the basic algorithms for solving partial differential equations had been demonstrated, there was still no major move towards using this capability to compute electromagnetic fields. In fact, the first application of a digital computer to the problem of electrical machine design seems to have been that by Veinott (1956) and this was an implementation of, essentially, classical, equivalent circuit-based design methods rather than an attempt to determine the fields. If it is tempting to ask why, when all the pieces were finally coming together to obtain a solution to the field equations of Maxwell for a particular structure, the step was not taken, the answer is probably that there was no pressing need in an industrial sense and thus the solution would have still been only of academic interest. As always, a tool is only developed or used when either a need is recognized or it is seen that the tool can provide information either faster or with more accuracy than existing methods. There is no doubt that the equivalent circuit-based models coupled with tabular data and simplified analytical solutions always require less in the way of computing power than a full-field solution, and thus the real thrust for an effective solution method for Maxwell’s equations occurs only when the commercial pressures require improved accuracy or an ability to understand the phenomena which cannot be modelled any other way in order to build more effective devices.

Thus, two questions drive research and development of methods for the solutions of the field equations. First, ‘can we solve the problem and obtain sufficiently accurate answers?’ Second ‘can the problem be solved more efficiently?’. The answer to the first requires that the mathematical background is in place and that the computational machinery is powerful enough to provide meaningful answers in a reasonable time frame. The answer to the second drives the search for new lower order complexity algorithms and effective approximations to reduce problem size.

The goal for the designers of the analysis tools is, as it has always been, to try to produce a sufficiently accurate simulation of reality, and thus increases in computer capability have been applied to the solution of Maxwell’s equations to produce a virtual world which will allow designers to perform virtual experiments and analyse the performance of a device by making measurements that are not feasible in the real world. For example, in the virtual world, it is possible to measure the field distribution within the iron core of a device, and thus identify points that may be working at levels which will cause increased local losses potentially leading to device failure (extremely expensive) or areas where the material is, in effect, doing nothing magnetically and might be removed.

In the late 1970s, the arrival, and subsequent development, of the microprocessor has meant that the computational power needed to provide a solution to the field equations can be placed on a design engineer’s desktop. The tool is finally ubiquitously available and can begin to provide answers faster and more accurately than any other approach. Simple systems capable of the solution of the two-dimensional magnetostatic approximation of Maxwell’s equations in the presence of nonlinear materials and complex geometries (consisting of structures having teeth and slots) were routine by the early 1980s and, for the next quarter century, each increase in processor power has meant that more of the approximations made to the field equations could be removed. By approximately 2004, it was possible to solve Maxwell’s equations, fully coupled
to the equations of dynamics, to solve the transient problem of an electromechanical device in motion. This latter feat, at present, owes more to Maxwell’s work on electromagnetism than just the equations. The links between the electromagnetic fields and the mechanical effects of those fields are determined through the application of the Maxwell stress formulae. However, without full-field solutions, the electromechanical coupling, which is the basis of the operation of all motors and actuators and some sensors, would not be easily computable.

8. Conclusions

This paper has been focused on Maxwell’s contributions to the design of low-frequency electromagnetic devices, i.e. those in which the $dD/dt$ term can be neglected. Attention has been drawn to the fact that, until very recently, the custom has been to quote the ‘full set’ of Maxwell’s equations and then cut back to those representing the Ampere and Faraday laws. It might be clearer to all concerned if the term ‘equations from Maxwell’ were adopted, but with the addition of the indispensable Maxwell stress equations. The latter is all important in any analysis of electromagnetomechanical devices.

It has also been pointed out that article 499 in the treatise clearly points the way ahead for magnetic circuit analysis. An appreciation earlier of the import of that article for machine designers would have been of considerable value.

As a result of finally being able to obtain accurate solutions for the fields in a device under a range of operating conditions, modern design tools can deliver more efficient and more effective solutions to a vast range of electromechanical and sensor applications. The cost of an electromechanical device is now very low that it can be found throughout our homes, offices and transportation systems; generating stations can achieve approximately 98% efficiency in mechanical to electrical power conversion; microelectromechanical systems are found in everything from automobiles to video projectors.

Furthermore, the pace of development has been so rapid of late that we are now at the point where the full solution of Maxwell’s equations can be embedded in advanced design tools, thus enabling engineers to further improve the quality of life for people around the world and to make their lives easier. For the most though, it is the pre-Maxwell set of equations that is still used by the low-frequency community.

We are indebted to many colleagues, past and present, for their informative and helpful discussions. In particular, we wish to thank the ever helpful staff of the IET (formerly IEE) Library Archives, London. Their encyclopaedic knowledge and unfailing ability to find rare material never fail to impress.

We have benefited greatly from numerous discussions with Prof. C.W. Trowbridge, whose all embracing knowledge of the history of electromagnetics has thrown much light on this study.

Also, we have drawn heavily on the book by Mahon (2004) (an essential starting point for anyone interested in this topic) and the monumental, titanic, set of tomes by Harman, as well as the almost contemporary work of Campbell & Garnett (1882). These are essential key texts to anyone wishing to study the work of James Clerk Maxwell.
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