Recurrence plots for dynamical analysis of non-invasive mechanical ventilation

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Quantifiers were introduced to convert recurrence plots into a statistical analysis of dynamical properties. It is shown that the Shannon entropy, if properly computed, increases as the chaotic regime is developed as expected. Recurrence plots and a new estimator for the Shannon entropy are then used to identify asynchronisms in non-invasive mechanical ventilation. It is thus shown that the phase coherence—easily identified using a Shannon entropy—is relevant in the quality of the mechanical ventilation. In particular, some patients with chronic respiratory diseases or healthy subjects can have a high rate of asynchronisms but a regular breathing rhythm.

Keywords: recurrence plots; Shannon entropy; non-invasive ventilation

1. Introduction

Recurrence plots were introduced by Eckmann et al. (1987) and some quantifiers were later introduced to convert recurrence plots into a statistical analysis (Trulla et al. 1996). Among these quantifiers, a ‘Shannon entropy’ was introduced, but it was decreasing with the development of the chaotic regime. Consequently, it was not a measure of the complexity as the usual Shannon entropy quantifies. Thus, we proposed a new estimator for the Shannon entropy, still based on recurrence plots, which is found to be in agreement with the usual meaning. In particular, the Shannon entropy is found to be correlated with the largest Lyapunov exponent and can thus be used to estimate it.

Recurrence plots analysis, often used in biomedicine (Marwan et al. 2002), is used here to identify asynchronisms in non-invasive mechanical ventilation. One of the most important criteria for a successful assistance by non-invasive ventilation is the comfort. Unfortunately, the comfort is quite subjective to estimate and seems to be strongly related to the presence of asynchronisms. By using phase portraits, first-

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return maps, the rate of non-triggered cycles (a breathing cycle with inspiratory effort unable to trigger the ventilator), recurrence plots and Shannon entropy, the dynamics underlying non-invasive ventilation is investigated.

2. Recurrence plots and Shannon entropy

A recurrence plot $R_{ij}$ is built as follows. Every point of the phase-space trajectory $\{x_i\}_{i=1}^N$ is tested for whether it is close to another point $x_j$ of the trajectory, that is, whether the distance between these two points is less than a specified threshold $\epsilon$. In this case, the point is said to be recurrent and is represented by a black dot. Otherwise, the point is not recurrent and is represented by a white dot. This can be described as an $N \times N$ array,

$$R_{ij} = \theta(\epsilon - ||x_i - x_j||),$$

where $\theta(x)$ is the Heaviside function. Two examples of recurrence plots are given for the logistic map with different $\mu$ parameter values. In the first example, the solution to the logistic map is a period-2 limit cycle and the recurrence plot looks like a chessboard (figure 1a). In the second example, the solution is a chaotic behaviour and the recurrence plots (figure 1b) no longer have the regularity observed in the previous example. Nevertheless, there are still some diagonal black segments which provide a signature of a deterministic dynamics structured around a skeleton of periodic orbits visited time to time. These diagonal recurrent segments arise from the recurrence properties of the dynamics underlying the logistic map.

It has been shown that a recurrence plot analysis is optimal when the trajectory is embedded in a phase space reconstructed with an appropriate dimension $d_E$ (Zbilut & Webber 1992). Such a dimension can be well estimated.

Figure 1. Recurrence plot computed for the logistic map for two different dynamical regimes. (a) Period-2 limit cycle: $\mu = 3.23$ and (b) chaotic behaviour: $\mu = 3.99$. The embedding dimension is equal to 3.
using a false nearest neighbours technique as introduced by Kennel et al. (1992) or improved by Cao (1997). The $d_E$-dimensional phase space is then reconstructed using delay coordinates. The time delay $\tau$ can be estimated using mutual information (Fraser & Swinney 1986) or the first zero of an autocorrelation function (Liebert & Schuster 1989), but most of the time a visual inspection also works well. Basically, the time delay has to be as small as possible and always less than a quarter of the pseudo-period. A parameter specific to the recurrence plot is the threshold $\epsilon$. Many trials lead us to choose (for all the dynamics investigated) a threshold equal to $\sqrt{d_E} \times 10\%$ of the fluctuations of the signal. The threshold is therefore automatically computed from the time series investigated. Thus, only two parameters need to be determined: the embedding dimension, $d_E$, and the time delay, $\tau$. When not extracted from a Poincaré section, the time series used will be sampled at $\tau$. This appears to be a good balance between covering each oscillation and covering the whole attractor.

Trulla et al. (1996) coupled the recurrence plots with different measures, useful for transforming graphical interpretations into a statistical analysis. Among their different measures, the Shannon entropy was found to be correlated with the inverse of the largest Lyapunov exponent, that is, a property quite opposed to the Pesin conjecture. One of us recently proposed a new estimator for a Shannon entropy which thus increases when the chaotic dynamics is developed (Letellier 2006). The Shannon entropy is usually defined as

$$S_{RP} = - \sum_{n=1}^{H} P_n \log(P_n). \quad (2.2)$$

In the form introduced by Trulla et al. (1996), $H$ is the maximum length of the recurrent (black) diagonal segments observed in the recurrence plot as exemplified in figure 1b. $P_n \neq 0$ is the relative frequency of the recurrent diagonal segments with length $n > 0$. In fact, the Shannon entropy should quantify the complexity of the dynamics and must be correlated with the largest Lyapunov exponent (Pesin 1998). Thus, one of us proposed to replace $P_n$ with the relative frequency of the occurrence of the diagonal segments with length $n$ of non-recurrent points (Letellier 2006). This is simply justified since a white dot is represented by a non-recurrent point, which is nothing else than a signature of the complexity within the data. With this new estimator, the quantifier given by equation (2.2) increases as the bifurcation parameter increases (as shown in the case of the logistic map in figure 2). This estimator removes the departure from the properties commonly presented by the Shannon entropy.

Obviously, the value of the Shannon entropy depends on the threshold $\epsilon$, but the relevant question is to determine whether this dependence is significant or not. It has been observed that the computations depend on the threshold $\epsilon$ in a rather monotonic way as shown in figure 3. The Shannon entropy slightly decreases when the threshold is increased. Such a feature was expected since increasing the threshold can be seen as replacing non-recurrent points by recurrent points; it thus decreases the entropy. The chosen threshold $\sqrt{d_E} \times 10\% \approx 0.17$ avoids the fluctuations observed when the threshold is too low. Quite robust relative comparisons can thus be performed.
3. Experimental device and measurements

Pressure support ventilation is a ventilatory mode where a preset inspiratory positive airway pressure (IPAP) and a preset expiratory positive airway pressure (EPAP) are superposed to spontaneous respiratory cycles. It has proved to efficiently unload the respiratory muscles and, consequently, to decrease the work of breathing in stable patients with obstructive diseases like chronic obstructive pulmonary disease (COPD; Nava et al. 1993) or cystic fibrosis.
Pressure support ventilation is often qualified as a ‘physiological’ ventilatory mode because it allows the patient to keep a control over his respiratory rate, inspiratory time and tidal volume. Therefore, it is not surprising that it has been found to be better tolerated than other ventilatory modes, especially when compared with volume target ventilation (Vitacca et al. 1993; Fauroux et al. 2001). However, it requires an adequately titrated and performing ventilator in order to correctly superpose mechanical breaths to spontaneous respiratory efforts (Brochard & Lellouche 2006), in other words to optimize patient–ventilator synchronization (Richard et al. 2002; Fauroux et al. 2004).

Patient–ventilator asynchrony and, especially, ineffective inspiratory triggering efforts are regularly encountered when performing non-invasive mechanical ventilation. This is the case not only in patients with cystic fibrosis (Fauroux et al. 2004), with COPD having lung hyperinflation and intrinsic positive end-expiratory pressure (PEEPi; Nava et al. 1995), but also in patients with various other pulmonary diseases (Nava et al. 1997). Ineffective inspiratory efforts under pressure support ventilation are more frequent during sleep (Fanfulla et al. 2005; Parthasarathy 2005) or when increasing the level of ventilatory assistance (Leung et al. 1997; Giannouli et al. 1999). Patient–ventilator asynchronisms including ineffective inspiratory efforts may be clearly a cause for non-invasive ventilation intolerance and failure (in critically ill patients, this situation is often described as patients ‘fighting’ against their ventilator). Either in acute or chronic setting, the incidence of non-triggered respiratory cycles and their consequences on non-invasive mechanical ventilation efficacy and comfort is unknown.

The interplay between the patient and the ventilator is complex. Asynchronies may arise at several points during the respiratory cycle. In order to investigate this interplay, an experimental device is built as shown in figure 4. Non-invasive ventilation is delivered to the patient through a well-fitting full face mask. To avoid CO2 rebreathing, an intentional leak is required in the ventilatory circuit and is therefore inserted (figure 4).

Figure 4. Experimental device used for investigating the interplay between subject and ventilator (see text for details.)
To adequately assist spontaneous breathing, a ventilator is placed at the other end of the circuit. It detects inspiratory effort through changes in inspiratory flow. The inspiratory phase is triggered by a threshold rate of airflow change. The ventilator thus delivers an IPAP, which is set to several values between 10 and 20 mbar in the present protocol. The pressure delivered by the ventilator increases to the preset IPAP value during a so-called pressure rise time, $T_{pr}$. The expiratory phase is triggered here when the flow decreases below 75% of the maximal value of the peak flow. The pressure delivered by the ventilator then returns to the end positive airway pressure (EPAP) value which was set to 4 mbar in the present study. At the output of the ventilator, an antibacterial filter for breathing system is inserted to prevent bacterial contamination of the ventilator.

During routine measurements of breathing pattern, respiratory flow ($Q_v$) was measured using a pneumotachograph connected to a pressure transducer. The pneumotachograph was inserted between the full face mask and the intentional leak. Airway pressure ($P_{aw}$) was measured with a differential pressure transducer near the pneumotachograph. Two typical excerpts of the datasets recorded during the protocol are shown in figure 5. They correspond to a healthy subject not so well trained to non-invasive mechanical ventilation. When an antibacterial filter is inserted in the ventilatory circuit, this subject has some difficulties to trigger the ventilator and many inefficient inspiratory efforts lead to a high rate of asynchronisms (41%; figure 5a). Basically, a non-triggered cycle can be identified when a small amplitude oscillation of the airflow is associated with a low peak of pressure (about the EPAP value). In this case, it is sufficient to remove the antibacterial filter to greatly reduce the rate of non-triggered cycles to 4.6% (figure 5b).

Twelve subjects (seven females and five males) with various health conditions were studied. All of them were in stable condition, as assessed by clinical examination and arterial blood gases. Among them, four patients had severe COPD, four had obesity hypoventilation syndrome (OHS) and four were healthy subjects. The four COPD patients were smokers and the eight other subjects were not. Basically, COPD is characterized by an increased airway resistance due to an inflammatory process in the bronchial wall and a loss of lung elastic recoil (Rossi et al. 1995). Therefore, airflow limitation leads to an intrinsic (EPAP$_i$) and dynamic hyperinflation (Pepe & Marini 1982). OHS is defined by the coexistence of obesity (body mass index greater than 30; Muir et al. 1998) and daytime hypercapnia (Masa et al. 2001). The mechanism of hypercapnia in OHS results from an alteration of ventilatory control (Krachman & Criner 1998) and/or a decreased thoraco-pulmonary compliance leading to a decreased work of breathing (Sharp et al. 1964).

Three of our patients ($P_1$, $P_3$ and $P_8$) and three healthy subjects ($S_9$, $S_{10}$ and $S_{12}$) were not familiar with non-invasive mechanical ventilation. The other five subjects were treated by non-invasive ventilation for at least 3 years and the healthy subject $S_{11}$ was quite familiar with non-invasive ventilatory techniques. The subjects were ventilated in a quiet seated position. Six working conditions were analysed and compared in this study. The level of IPAP was increased from 10 to 20 mbar by 2 mbar steps. For each value, a 10-min period was recorded once a stable breathing pattern was observed. Since the respiratory frequency $f_R$ is patient dependent and varies between 9.11 and 33.68 cycles per minute, the number of respiratory cycles during each dataset varies between 91 and 337 cycles.
Figure 5. Time series of the airflow and the pressure measured with a healthy subject not so well trained to non-invasive mechanical ventilation: (a) with and (b) without an antibacterial filter. Few types of asynchronism are identified in these two examples. They are labelled as follows: NT, non-triggered cycle; ST, self-triggered cycle; DT, delayed triggered cycle. The beginnings of the respiratory cycle, corresponding to the inspiratory effort of the patient, are designated by circles and the ends of the inspiratory phase (during which the ventilator applies a pressure support at the IPAP value) are marked by black diamonds on the airflow time series. Parameter setting: IPAP = 16 mbar, EPAP = 4 mbar and the inspiratory threshold is set to the most sensitive value, i.e. 0.0167 l s$^{-1}$. 

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4. Dynamical analysis

Ideally, during mechanical ventilation, the triggering of the ventilator should result from inspiratory muscle activity. The most frequent subject–ventilator asynchrony is when the subject’s inspiratory effort does not trigger the ventilator. For instance, the healthy subject $S_{12}$ had 41% of ineffective triggerings with an antibacterial filter, but less than 4.6% when this filter was removed. Indeed, the filter can be viewed as a 64% increase of the threshold value for triggering the ventilator (Achour et al. 2007). Phase portraits were reconstructed from the airflow in subject $S_{12}$ with IPAP = 16 mbar. The time delay $\tau$ is equal to $(1/10) T$, where $T$ is the time duration of the respiratory cycles averaged over the whole dataset.

Figure 6. Effect of an antibacterial filter placed in the ventilatory circuit on the occurrence of ineffective triggerings: (a) with (41% of asynchronisms) and (b) without (4.6% of asynchronisms) a filter. Phase portraits reconstructed from the airflow in subject $S_{12}$ with IPAP = 16 mbar. The time delay $\tau$ is equal to $(1/10) T$, where $T$ is the time duration of the respiratory cycles averaged over the whole dataset.

One of the advantages of the recurrence plots is that the succession of the events versus time can be followed. Since the recurrence plot analysis computed in a Poincaré section helps to obtain a more reliable characterization (Letellier 2006), the maximum of the airway pressure $P_{\text{max}}$ during a respiratory cycle (figure 7) is used to build a ‘discrete’ time series.

As an example, the recurrence plots are computed from the maxima of the airway pressure for subject $S_{12}$ in working conditions, similar to those of figure 6. The threshold $\epsilon$ is set to $\sqrt{d_E} \times 0.1 \times \text{IPAP}$. Compared to the phase portraits, we now have a representation of the succession of the cycles, black domains corresponding to recurrences of the dynamics, that is, succession of similar cycles (figure 8). Since the rate of ineffective efforts is always less than 50%, the recurrent points are quite rare with the antibacterial filter (figure 8a).
This means that consecutive cycles are very different from the pressure point of view, that is, a triggered cycle is very often followed (or preceded) by a non-triggered cycle and vice versa. As soon as the filter is removed (figure 8), the non-triggered cycles decrease to 4.6% and the recurrence plot is almost black everywhere. Only small sets of cycles reveal large fluctuations of the airway pressure, that is, asynchronisms. The recurrence plots reveal that the asynchronisms mainly occur in clusters. This feature is often encountered when the patient has some tendencies to provide inspiratory efforts near the threshold value required to trigger the ventilator and when the patient (not so well trained in the present case) tries to react to the non-triggered respiratory cycle. By such a reaction, the patient deeply modifies the dynamics of his breathing and, consequently, sends to the ventilator dynamical features that are no longer well identified by the ventilator. Typically, such a fight against the

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**Figure 7.** Time series made of the successive maxima of the airway pressure $P_{max}$ during respiratory cycles. Basically, a non-triggered cycle is associated with a value about the preset EPAP value.

**Figure 8.** Recurrence plots computed from the maxima of the pressure airway during respiratory cycles recorded for subject $S_{12}$ (a) with (41% of asynchronisms) and (b) without (46% of asynchronisms) an antibacterial filter. The embedding dimension is equal to 3.
ventilator is observed for few consecutive cycles before returning to ‘normal’ respiratory cycles. The recurrence plots thus confirm that the mechanical ventilation is much more efficient when the antibacterial filter is removed.

The Shannon entropy is computed from these recurrence plots according to our new estimator. When computed from the maxima of the airway pressure, the Shannon entropy will be denoted $S_P$. It is equal to 2.3 without the filter and equal to 0.4 with the filter. It has been found that the Shannon entropy $S_P$ is strongly correlated to the rate of non-triggered cycles. In a previous study (Achour et al. 2007), we found that an asynchronism frequency below 10% was not relevant for ventilatory comfort. Such a rate corresponds to a Shannon entropy slightly less than 1. Thus, a Shannon entropy $S_P$ less than 1 corresponds to a situation where inefficient efforts are not clinically relevant on the subjects’ comfort.

Another dynamical characteristic relevant for the quality of the assisted mechanical ventilation is the rate of fluctuations of the total duration of the respiratory cycle. Since the subject is in a quiet seated position, the breathing rhythms should be regular. In particular, the patients very familiar with mechanical ventilation should be able to manage their ventilator for breathing in a regular way. On the other hand, we assume that the more regular the dynamics is, the better the comfort is. Moreover, the fluctuations over the total duration of the respiratory cycle, $T_{tot}$, are not necessarily correlated with the occurrence of asynchronisms.

Recurrence plots are computed from $T_{tot}$ using a threshold $\epsilon$ set to $\sqrt{d_E} \times 0.1 \times \overline{T}$, where $\overline{T}$ is the ‘ideal’ time duration cycle corresponding to a respiratory frequency equal to 12 breaths per minute. For the two cases investigated in figures 6 and 8, the recurrence plots from $T_{tot}$ also reveal obvious departures, but not in the same way as when computed from $P_{max}$. With the filter, both recurrence plots, from $P_{max}$ (figure 8a) and $T_{tot}$ (figure 9a), are quite similar as revealed by the Shannon entropy, $S_P=2.3$, and those computed from $T_{tot}$, $S_T=2.5$. The fact that $S_T$ is slightly greater than $S_P$ means that the lack of recurrences over $T_{tot}$ affects a slightly larger number of cycles than those over $P_{max}$.

Figure 9. Recurrence plots computed from the total duration of the cycle, $T_{tot}$, recorded for subject $S_{12}$ (a) with (41% of asynchronisms) and (b) without (46% of asynchronisms) an antibacterial filter in the ventilatory circuit.
The regularity of the breathing rhythm is thus not correlated with the occurrence of asynchronisms alone. For instance, this was obviously confirmed when the filter was removed (figure 9b) because $S_T > 1.9 \gg S_P = 0.4$.

These two Shannon entropies thus characterize two different dynamical properties that are important for investigating the dynamics underlying patient–ventilator interactions (Rabarimanantssoa et al. 2006). The Shannon entropy $S_T$ was computed for the 69 datasets recorded and plotted versus the Shannon entropy $S_P$ (figure 10). These estimations of the Shannon entropies are computed over the whole 10-min dataset. Depending on the respiratory frequency, the number of data points is therefore between 91 and 337 as previously explained. Working at fixed number of data points is not natural in such a study. Typically, when more than 100 points are considered, estimations are not significantly dependent on the length of the dataset. The dependence on the number of points is therefore not significant. Basically, four different regions are distinguished in this figure. First, the square defined by $S_T < 1$ and $S_P < 1$ corresponds to subjects who have fluctuations over neither $P_{max}$ nor $T_{tot}$. There is no ambiguity for these subjects since they have almost no asynchronism and their breathing rhythms are very regular. It was found that the ability to manage the ventilator was not related to the type of pulmonary pathology.

Second, there is the rectangle defined by $S_T > 1$ and $S_P < 1$. These datasets correspond to subjects with less than 10% of non-triggered cycles but quite significant fluctuations over the total duration of the ventilatory cycle. They correspond to two subjects, $S_1$ and $S_9$, not familiar with non-invasive mechanical ventilation. There are no OHS patients under these conditions. This could suggest that OHS patients would not display significant fluctuations over the...
total duration of the ventilatory cycle, $T_{tot}$. Indeed, obesity tends to reduce lung volumes and there is no longer possibility for varying the inspiratory volume and/or the respiratory time. Third, the rectangle defined by $S_T < 1$ and $S_P > 1$ corresponds to cases where there are many ineffective efforts although the breathing rhythm is regular. The fourth part of the graph shown in figure 10 is associated with a sector such as $S_T > 1$ and $S_P > 1$. Most of the points located in the fourth sector correspond to subjects not familiar with mechanical ventilation (only patients $S_2$ and $S_6$ are familiar). For all of the subjects being not familiar with mechanical ventilation, there is an obvious correlation between $S_P$ and $S_T$. For these patients, the fluctuations over the time duration $T_{tot}$ result from asynchronisms.

5. Conclusion

Recurrence plots can be used to investigate the properties of complex dynamics as widely used, particularly in biomedicine. A new estimator for the Shannon entropy has been proposed to obtain an entropy, which increases as the chaotic dynamics is more developed as expected for a Shannon entropy. It is thus strongly correlated to the largest Lyapunov exponent, according to Pesin’s conjecture.

The recurrence plots and the associated Shannon entropy were used to investigate the dynamics underlying patient–ventilator interactions. We showed that two statistical quantities, the Shannon entropies computed from the maxima of the airway pressure and from the total duration of the ventilatory cycle, are useful to characterize the quality of the non-invasive mechanical ventilation. The first entropy estimates the rate of ineffective triggerings and the second entropy quantifies the fluctuations over the total duration of ventilatory cycles. These two measures are not necessarily correlated and strongly depend on whether the subjects are familiar with mechanical ventilation or not. Basically, when these two entropies are less than 1, it can be assumed that, from a mechanical point of view, the parameter setting of the ventilator is satisfactory.

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References


