Experimental investigation on the avoidance of self-excited vibrations

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Self-excited vibrations are observed in many technical applications. Frictional contacts are often involved in the mechanism which generates vibrations. Reasons for the excitation mechanisms are decreasing friction characteristics depending on the sliding velocity, fluctuating normal loads or different geometrical effects. First, the mechanisms are explained using simple examples. The practical relevance of self-excited, friction-induced vibrations is exemplified with three technical systems: a system with an axial seal; a tread block of a tyre; and a disc brake.

The knowledge of the excitation mechanism is necessary to introduce successfully design countermeasures. These measures to avoid self-excited vibrations are important to solve practical problems. They are the main focus of this work. Further, additional passive and active subsystems are described and validated experimentally. Therefore, a large range of design, active and passive solutions are given.

Keywords: self-excited vibrations; friction-induced vibrations; vibration control; countermeasures; friction contacts; stick–slip vibrations

1. Introduction

Contacts with friction appear in many technical applications. Friction often induces unwanted self-excited vibrations in these applications. The vibrations can create noise, diminish accuracy or influence the function. From an engineering point of view the friction is distinguished into a static friction and a kinetic friction. The static friction is the resistance against the beginning of relative motion of two contacting bodies. The kinetic friction is a resistance against an existing relative motion in the contact. Stick–slip vibrations are alternating changes between static friction and kinetic friction. It becomes possible if the dynamic properties of the system like elasticity, mass, damping, geometry and friction characteristic allow this kind of a motion.

First, descriptions of the mechanisms to generate friction-induced vibrations are given and some demonstration models are shown. Additionally, experimental and analytical investigations on disc brakes, systems with axial seals and tread blocks of tyres are discussed as practical applications. If the mechanism which generates friction-induced vibrations is understood, it is much easier to find countermeasures.

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Design countermeasures are given, which change the system properties like geometry, damping or friction characteristic. Indirect active solutions influence internal variables of the system like the normal contact force or the relative velocity to suppress the vibrations. They have certain advantages against a direct control of the vibrations if they can be applied in the control loop of existing actuators. Additional passive or semi-active subsystems are introduced to solve the vibration problem without necessary feedback control. All these solutions must be adapted to the self-excited vibrations in a special manner depending on the excitation mechanism.

2. Mechanisms to generate friction-induced vibrations

The friction force can transfer energy from one contact partner to the other. During relative motion the friction force also causes the dissipation of kinetic energy and usually an increasing temperature in the contact. If one contact partner is an oscillator and the other an energy source, e.g. a rotating disc, self-excited oscillations can occur. Necessary for the generation of friction-induced vibrations is a switching mechanism to control the energy flow.

If the energy flow from the energy source to the oscillator is larger than the dissipated energy of the oscillator due to damping during one cycle, the vibration amplitude increases. If it is smaller the amplitude of the oscillator decreases. For a limit cycle the energy input and the dissipated energy during each period are in balance. With these energy considerations it is also possible to check the stability of an equilibrium position or of a limit cycle.

The fundamental structure of a system which shows self-excited vibrations is shown in figure 1a. The feedback of the oscillating system by a switching mechanism results in an energy flow out of the energy source according to the frequency of the oscillator. In particular, the switching mechanism is often difficult to understand. For example, it can have geometrical reasons or can be based on the friction characteristic. Figure 1b shows a woodpecker toy which demonstrates this fundamental structure. The oscillator of the toy is the woodpecker and the energy source is the potential energy. During each period the woodpecker moves a small step down. The control mechanism is the friction contact in the clamp with a well defined clearance. The normal force in the clamp changes due to the oscillations of the woodpecker. This results in sliding, sticking or free falling and controls the input of energy into the oscillator (cf. Leine et al. 2001).

In the following, some fundamental effects which result in friction-induced vibrations are described:

— contacts with decreasing friction characteristic;
— contacts with fluctuating normal forces; and
— geometrical effects and non-conservative restoring forces.

Furthermore, mode coupling (cf. Gasparetto 2001) or sprag-slip (cf. Spurr 1961; Hoffmann & Gaul 2004) can cause self-excited vibrations. In practical applications a mixture of these and other effects can be observed. An exact separation of these effects is often difficult. A detailed overview of excitation mechanisms is given by Ibrahim (1992a,b).
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Contacts with decreasing friction characteristic

The classical textbook example of a friction oscillator is a mass–spring system supported by a moving belt (cf. Magnus & Popp 2002). Figure 2a shows this model and figure 2b a demonstrator, which performs stick–slip vibrations. The system has only one degree of freedom (d.f.) which is described by the coordinate \( x \). The energy source is the belt which moves with a constant velocity \( v_0 \). The oscillator consists of a mass \( m \) and a spring \( c \). The switching mechanism is based on the friction contact. The friction force \( F_R \) often depends on the relative velocity \( v_{\text{rel}} \). Different friction characteristics \( F_R(v_{\text{rel}}) \) are shown in figure 3.

The Striebeck characteristic (A) decreases with increasing relative velocity \( v_{\text{rel}} \) up to a certain velocity. For small oscillations this behaviour can be approximated by a linearly decreasing characteristic (B) for a special equilibrium, e.g. \( v_{\text{rel}} = v_0 \). Decreasing friction characteristics have been observed experimentally in many contacts (e.g. Ibrahim 1992a,b; Lindner et al. 2001). The Coulomb friction characteristic (C) jumps at \( v_{\text{rel}} = 0 \) between a static friction force and a constant kinetic friction force (cf. Coulomb 1785). Characteristic (B) or (C) is frequently used as mathematical approximations of the real contact behaviour.

Figure 4 shows the limit cycles of the model shown in figure 2 using the three characteristics. The diagrams show the normalized velocity \( \dot{x}/\omega \) of the oscillator versus the displacement of the oscillator \( x-x_e \) with the natural frequency \( \omega \) of

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If the belt velocity is in the decreasing area of the friction characteristic (A), the equilibrium position $x_e = (F_R(v_0))/c$ is unstable and the oscillation amplitudes increase until they reach the limit cycle (figure 4a). This is similar to the behaviour using the linearly decreasing friction characteristic (B; figure 4b). For characteristic (C) the equilibrium position is not unstable, but there exists also a stable limit cycle (figure 4c). Therefore, the dynamical behaviour of the system depends on the initial configuration, e.g. a run-up of the belt velocity from 0 results immediately in the limit cycle.

To understand the mechanism of the decreasing friction characteristic, time histories of a system with a linear decreasing characteristic (B) are shown over one period in figure 5. An assumed small sinusoidal velocity $\dot{x}(t)$ (figure 5a) results in a sinusoidal fluctuation of the relative velocity $v_{rel}$ around the mean value $v_0$ (figure 5b). Owing to the decreasing characteristic, the friction force $F_R$ is smaller for larger relative velocities $v_{rel}$ and vice versa (figure 5c). The energy flow into the oscillating system is the frictional power $F_R \dot{x}$. It has larger positive than negative values for the decreasing friction characteristic (figure 5d). The total energy input $\Delta E_{in}$ during one period $T$ is

$$\Delta E_{in} = \int_0^T F_R \dot{x} \, dt.$$  \hspace{1cm} (2.1)

If the energy input $\Delta E_{in}$ is positive the vibration amplitude increases until a limit cycle is reached. Then the energy input $\Delta E_{in}$ is equal to 0 during one period.
Contacts with fluctuating normal forces

A positive energy input is also possible for a constant friction coefficient, characteristic (C) with \( \mu_0 = \mu = \text{const.} \), if the normal force fluctuates. Assuming the normal force \( F_N \) as a superposition of a constant part \( F_0 \) and a sinusoidal part

\[
F_N(t) = F_0 + \hat{F}_N \sin(2\pi t / T),
\]

the same diagram sequence of figure 5 is possible. But the reason for the oscillations of the friction force \( F_R \) in figure 5c is different. The oscillations of the friction force are no longer induced by the variation of the relative velocity. Now they are based on the variation of the normal force by \( F_R(t) = \mu F_N(t) \). A maximum energy input results from a fluctuating normal force if the sinusoidal oscillations of velocity \( \dot{x} \) and normal force \( F_N \) are in phase.

Oscillations of the velocity \( \dot{x} \) and the normal force \( F_N \) in antiphase give a negative energy input \( \Delta E_{\text{in}} < 0 \). Therefore, fluctuating normal forces can also be used to damp vibrations. This is described in §4a.

Geometrical effects and non-conservative restoring forces

Geometrical effects and non-conservative restoring forces are based on interactions of different degrees of freedom. Therefore, they can be described by models with at least two degrees of freedom. Figure 6 shows a minimal model of a so-called pin-on-disc application (cf. Spurr 1961). Both the disc and the pin have one translational degree of freedom and are connected to the inertial frame by spring and damper elements. The pin is tilted with an angle \( \gamma \) to the disc. A constant coefficient of friction, characteristic (C) with \( \mu_0 = \mu = \text{const.} \), is assumed in the contact. The stability boundary can be determined by

\[
\mu_{\text{crit}} = \tan \gamma + \frac{2d_2}{\sin(2\gamma)d_1},
\]

with the parameters given in figure 6. It is obvious that the criterion is strongly affected by the geometry, here the angle \( \gamma \). The angular natural frequencies \( \omega_1 = \sqrt{c_1/m_1} \) and \( \omega_2 = \sqrt{c_2/m_2} \) of the bodies do not need to have a specific relation. Additional damping \( d_2 \) of the pin can stabilize the system while the damping \( d_1 \) destabilizes the system. In the case of instability, \( \mu \geq \mu_{\text{crit}} \), the pin sticks on the disc.

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Similar to geometrical effects, non-conservative restoring forces are strongly dependent on the geometry. A simple model based on the model of North with non-conservative restoring forces is given in figure 7 (cf. North 1972). The equation of motion reads

\[
\begin{bmatrix}
    m & 0 \\
    0 & J
\end{bmatrix}
\begin{bmatrix}
    \ddot{x} \\
    \ddot{\phi}
\end{bmatrix}
+ \begin{bmatrix}
    d_1 & 0 \\
    0 & d_0
\end{bmatrix}
\begin{bmatrix}
    \dot{x} \\
    \dot{\phi}
\end{bmatrix}
+ \begin{bmatrix}
    c_1 + c_2 & -c_2s \\
    c_2(\mu h - s) & c_\phi + sc_2(s - \mu h)
\end{bmatrix}
\begin{bmatrix}
    x \\
    \phi
\end{bmatrix}
= \begin{bmatrix}
    0 \\
    0
\end{bmatrix}.
\]

The non-conservative forces can be identified by an asymmetrical stiffness matrix of the system ($K \neq K^T$). The stability boundary of the system according to the Hurwitz criterion reads

\[
\mu_{\text{crit}} = \frac{s}{h} + \frac{c_\phi}{hsc_2} + \frac{c_\phi}{hsc_1}.
\]

The vibration amplitudes grow exponentially in the case of instability, $\mu \geq \mu_{\text{crit}}$. This kinetic instability is called flutter instability.

**3. Practical examples**

In this section three practical applications are given to show the relevance of the effects described above. First, a special seal application is explained, then a tread block of a tyre is discussed and finally self-excited vibrations of a brake system are explained.
An axial seal is used as an example for a system with a Stribeck friction characteristic (A). The seal test rig is shown in figure 8. A bolt moves with a constant velocity \( v_{\text{bolt}} \) through a lubricated seal. For constant bolt velocities the measured axial force in the bolt is equal to the friction force in the contact with the seal. Additionally, the displacement and the velocity of one point of the seal lip are measured by a laser vibrometer.

The measured friction force characteristic of the lubricated seal depending on the bolt velocity \( v_{\text{bolt}} \) is shown in figure 9. The seal has a decreasing friction characteristic for bolt velocities \( v_{\text{bolt}} < 7 \text{ mm s}^{-1} \). The friction force characteristic is approximated by

\[
F_R(v_0) = \frac{9.7}{0.7v + 1} + 1.2\sqrt{v} + 0.01, \tag{3.1}
\]

with \( F_R \) in N and \( v \) in mm s\(^{-1}\). Using the model of figure 2a and adding a linear damper \( d \) in parallel to the spring \( c \), the equation of motion for the seal system is

\[
m\ddot{x} + d\dot{x} + cx = F_R(v_{\text{rel}}); \quad v_{\text{rel}} = v_0 - \dot{x}. \tag{3.2}
\]

A linearization of the friction characteristic at the equilibrium position \( \dot{x} = 0 \), \( v_{\text{rel}} = v_0 \), gives a constant value \( F_R(v_0) \) and the gradient \( \delta_F(v_0) \) of the friction force characteristic

\[
F_R(v_{\text{rel}} = v_0 - \dot{x}) = F_R(v_0) - \delta_F(v_0)\dot{x}; \quad \delta_F(v_0) = \left. \frac{\partial F_R(v_{\text{rel}})}{\partial v_{\text{rel}}} \right|_{v_0}. \tag{3.3}
\]
This leads to a linearized equation of motion with a constant r.h.s.

\[ m\ddot{x} + [d + \delta_F(v_0)]\dot{x} + cx = F_R(v_0). \]  

(3.4)

The equilibrium position \( x_0 = (F_R(v_0))/c \) is asymptotically stable for \( d + \delta_F(v_0) > 0 \), and is unstable for \( d + \delta_F(v_0) < 0 \). A negative gradient \( \delta_F \) describes a decreasing characteristic. Therefore, the gradient \( \delta_F(v_0) \) determines the stability of the equilibrium position.

If the gradient \( \delta_F \) is a function of the relative velocity \( v_{\text{rel}} \), the stability of the equilibrium depends on the velocity \( v_0 \). The stability map for the given friction characteristic is shown in figure 10. For relative velocities \( v_{\text{rel}} \) higher than the critical velocity, the equilibrium is stable. The critical velocity yields from \( d + \delta(v_{\text{rel},\text{crit}}) = 0 \). For lower velocities it is unstable. In this case, the experimentally tested seal shows stick–slip vibrations for bolt velocities \( v_{\text{bolt}} < v_{\text{rel},\text{crit}} = 7 \text{ mm s}^{-1} \), which is in good correlation with the theoretical approach.

(b) Vibrations of a tread block

Another example of a system with friction-induced vibrations is a tread block of a tyre sliding on a surface. Tread block vibrations induce noise during braking or cornering and they can have a negative influence on the friction coefficient and therefore on the safety of the vehicle. The tread block vibrations are investigated experimentally using the tribometer test rig in figure 11. The block has a dimension of 15 mm \( \times \) 15 mm and a thickness of approximately 10 mm. It is fixed on a sample holder of the test rig by adhesive. The material of the tread block is Styrol butadiene rubber, SBR, with 60 phr carbon black and the contact partner is a glass plate. The glass plate instead of a road surface reduces the stochastic excitation. Therefore, dynamic effects of the tread block itself can be seen more clearly.

The dynamical behaviour of the tread block depends strongly on the surface geometry of the block. For rectangular corners a lip is formed at the leading edge (figure 12a). The resulting local contact combined with local wear and temperature effects result in diverse kinds of vibrations, which are not reproducible due to permanent changes in the contact geometry. A radius of approximately 1 mm at the leading and trailing edge results in self-excited vibrations with strong variations of the normal force up to separations of the tread block from the disc (figure 13). The reason is the local contact next to the
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Figure 11. (a) Test rig for a tread block sliding on a disc and (b) detail view of the contact.

Figure 12. Deformation shapes of a tread block: (a) sketch of a rectangular block building a lip, (b) contact of a block with a radius at the edges and (c) contact of a rough block.

Figure 13. Vibrations of a tread block with an edge radius of approximately 1 mm on glass: (a) displacement, (b) velocity, (c) limit cycle, (d) forces (SBR with 60 phr carbon black, pressure 0.13 N mm\(^{-2}\) and disc velocity 50 mm s\(^{-1}\)).

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leading edge. Owing to the incompressibility of the rubber material a tangential deformation caused by the friction forces results in a deformation in the normal direction (figure 12b). This deformation influences the normal contact force and its distribution. This coupling of normal and tangential direction is typical for non-conservative restoring forces, which are here the mechanism of the self-excited vibration (cf. §2c).

If the rubber surface is rough (figure 12c)—for example, after preparation with grinding paper—the normal force remains constant because a tangential deformation of a rubber asperity causes no change in the normal direction. The observed stick–slip vibration in figure 14 is based now on a decreasing friction characteristic.

Furthermore, the tread block shows stick–slip vibrations with a relatively constant normal force if it slides on a plate with grinding paper (Corundum K400) or concrete road surface (cf. Kröger et al. 2004).

This example demonstrates that different mechanisms to generate friction-induced vibrations can occur for slightly different system properties. This makes it very difficult to identify the excitation mechanism in this application.

(c) Vibrations of a brake system

The friction contact of brake systems can lead to self-excited vibrations of the system. Noise showing tonal frequencies above 1 kHz is referred to as brake squeal. In the case of a squealing brake, there is an energy transfer from the

Figure 14. Vibrations of a tread block with an edge radius of approximately 1 mm and rough surface on glass: (a) displacement, (b) velocity, (c) limit cycle, (d) forces (SBR with 60 phr carbon black, pressure 0.25 N mm⁻² and disc velocity 50 mm s⁻¹).
rotating disc to the brake system. Like in other self-excited vibrations, this energy transfer is controlled by the vibration itself. There are numerous theories which describe the phenomenon of brake squeal. An overview of these theories is given by Kinkaid et al. (2003). Most of them are based on a decreasing friction characteristic with increasing relative velocity, or on the effect of non-conservative restoring forces.

In the first case, the energy input is proportional to the gradient of the friction characteristic and the normal force, similar to the calculations performed for the 1 d.f. oscillator in §2a.

The effect of non-conservative restoring forces is analysed here with a multibody system of a disc brake (figure 15). It consists of six rigid bodies; each of them has 3 d.f. if a planar motion is considered. The coefficient of friction \( \mu \) between the disc and the pad is assumed to be constant, but it can be varied. The non-conservative restoring forces can be identified by the asymmetry of the stiffness matrix of the system. In the linearized equations, the skew-symmetric part of the stiffness matrix is proportional to the coefficient of friction \( \mu \).

The parameter \( \mu \) is increased stepwise starting from \( \mu = 0 \) to illustrate the effect of non-conservative forces. For each step the corresponding system matrix and its eigenvalues have been calculated. Figure 16 shows the eigenvalues in the complex plane. The real part of two eigenvalues is affected by the variation, one real part is decreased and the other increased. The latter eigenvalue crosses the imaginary axis for a critical value \( \mu_{\text{crit}} = 0.64 \). For \( \mu > \mu_{\text{crit}} \) the system is unstable. The imaginary part of that eigenvalue gives a squeal frequency (cf. Rudolph & Popp 2000).

This brake model is experimentally verified with a test sequence of 225 brakings. Each braking lasted for 12 s with a constant disc speed of 40 rpm. Different pressures in the range from 2 to 30 bar and different disc temperatures between 30 and 310°C are adjusted. The appearing sounds are recorded with a microphone and analysed by FFT. Every braking with a sound pressure level

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higher than 60 dB is classified as a squealing event. These brakings are marked by crosses in figure 17. The measurements indicate the onset of a squealing of 2350 Hz for pressures above 12 bar. This result is in good accordance with the theoretical prediction of a squealing of 2140 Hz starting above 15 bar brake pressure.

4. Avoidance of self-excited vibrations

In this section, measures to avoid self-excited vibrations are discussed. From a practical point of view, the understanding of the excitation mechanism is only the first step and does not solve the problem immediately. Therefore, practical
measures to avoid the self-excited friction-induced vibrations are needed to solve the problem. After summarizing different design possibilities, some measures using additional subsystems are investigated experimentally and theoretically in more detail.

Measures to avoid vibrations for systems with a decreasing friction characteristic are

— an increase in the belt velocity for friction characteristic (A);
— an increase in the damping by additional external dampers or by materials with higher damping;
— a change of the friction characteristic by an optimization of the surface properties, the materials in contact, the geometry of contact or the lubrication; and
— a smaller normal load or a larger stiffness of the spring, which can reduce the amplitude of the limit cycle.

Measures to reduce vibrations for systems with fluctuating normal loads are

— an increase in the damping;
— a reduction in the oscillating part of the normal load or a change in the phase angle of the normal load with respect to the velocity \( \dot{x} \) of the oscillator;
— a change in the frequency of the normal load oscillations out of the resonance of the stick–slip oscillator or a shift of the eigenfrequency by changes in the mass \( m \) or the spring \( c \);
— disturbance in the fluctuating normal load; and
— a fluctuating normal load controlled with a switching mechanism by the velocity \( \dot{x} \); a change in this coupling mechanism can successfully reduce the vibrations.

Measures to avoid vibrations for systems with geometrical effects resulting in non-conservative restoring forces are

— a reduction in the non-conservative restoring forces, e.g. this is possible by a reduction in the coefficient of friction (cf. equation (2.4));
— a change in the contact geometry, for which the important parameters can be found by a stability analysis (cf. equation (2.5)) or by a sensitivity analysis (cf. Rudolph & Popp 2000); and
— an increase in damping in the system, which can have a positive or a negative influence on the stability (cf. Kirillov & Seyranian 2005).

In the following, the transfer of these design countermeasures to the described applications is given. In the seal application the decrease in the normal load by geometrical changes is a first step to solve the problem, but this is limited by leakage problems. Additionally, the materials of the rubber can be changed to get larger material damping, which may have a negative impact on the friction characteristic, especially in the critical mixed friction regime. More successful are investigations done to change the friction characteristic. To reduce the gradient of the friction characteristic for small sliding velocities, additives in the lubricant can be used. For hydrodynamic lubrication, the influence of the contact
geometry, the pressure distribution and the pressure gradients are very important. Another possibility is to produce special surfaces by surface activation, surface coating or surface structuring (cf. Etsion et al. 1999). An optimization of a seal system, especially to avoid self-excited vibrations, has to consider all these influences (cf. Lindner et al. 2004).

The tread block design influences a lot of different tyre properties like grip on dry or wet roads and on ice or snow, the aquaplaning and the wear. Therefore, it is very complicated to reduce the stick–slip vibrations during braking or cornering without worsening the other properties. An optimization of these contacts can be done by changing the surface structure and the material properties of the road as well as by changing the material and geometry of the tyre (cf. Sandberg & Ejsmont 2003).

The design measures to avoid brake squeal are based often on an increase of damping in the brake systems (cf. Fosberry & Holubecki 1961). Practical examples are vibration shims between the backing pads and calliper or special grease in the piston to backing plate contact. Geometrical changes, like chamfering of the pad, or surface treatments, like sanding of the brake disc, can be useful. These countermeasures are summarized by Kinkaid et al. (2003).

If the design measures are not successful or unwanted, additional active or passive subsystems can be used, such as

— an active normal force control;
— an active belt velocity control;
— an additional passive-tuned mass damper system; and
— a passive or semi-active vibration compensation, e.g. by shunted piezos.

These measures are described in the following sections. Furthermore, active vibration control can suppress the vibrations, additional excitation can reduce the amplitudes of the stick–slip vibrations (cf. Popp et al. 1995) or high-frequency excitation of the contact, the so-called dither control, can be used to destabilize the stick–slip limit cycle (cf. Littmann et al. 2001; Curefare & Graf 2002; Michaux et al. 2005).

(a) Active normal force control

The concept to control the normal force is based on the idea of a negative energy input during one period

$$\Delta E_{in} = \int_0^T F_R \dot{x} \, dt < 0.$$  (4.1)

A suitable phase shift between the friction force $F_R$ and the oscillator velocity $\dot{x}$ is needed to reach negative values $\Delta E_{in} < 0$. Small oscillations of the mass in the vicinity of the equilibrium position with a harmonic motion

$$\dot{x}(t) = \dot{x} \sin(\omega t); \quad \omega = \frac{2\pi}{T}$$  (4.2)

are assumed. The controlled normal force should oscillate with the same frequency and a phase shift $\varphi_x$ in relation to the velocity $\dot{x}$. The friction is described by the characteristic (B) with

$$\mu(v_{rel}) = \mu_0 - \delta \mu v_{rel}; \quad \mu_0 = \mu(v = 0)$$  (4.3)
and approximated by \( \mu(v_{\text{rel}}) \approx \mu_0 - \delta_\mu v_0 \) for small changes of the relative motion \((\hat{x} \ll v_0)\). Here, \( \delta_\mu \) is a positive value representing the decreasing friction characteristic \( \delta_\mu = -\left(\delta F/F_N\right) > 0 \) (cf. equation (3.3)). Then the energy input can be calculated from equation (4.1) as

\[
\Delta E_{\text{in}} = \int_0^T (\mu_0 - \delta_\mu v_0)(F_0 + \hat{F} \sin(\omega t - \varphi_\hat{x})\hat{x} \sin(\omega t)) dt \\
= \frac{T\hat{x}}{2} [(\mu_0 - \delta_\mu v_0)\hat{F} \cos \varphi_\hat{x} + \delta_\mu \hat{x} F_0] < 0. \tag{4.4}
\]

A minimum of the energy input is reached for a phase angle \( \varphi_\hat{x} = \pi \) and maximum normal force amplitudes \( \hat{F} = F_0 \). Then the normal force is in antiphase with the velocity \( \hat{x} \). For larger normal force amplitudes \( \hat{F} > F_0 \), the mass lifts off the contact.

With \( \varphi_\hat{x} = \pi \) and \( \hat{F} = F_0 \) the energy input \( \Delta E_{\text{in}} \) is negative if

\[-\mu_0 + \delta_\mu v_0 + \delta_\mu \hat{x} < 0 \iff \mu_0 > \delta_\mu (v_0 + \hat{x}) \tag{4.5}\]

holds. The phase shift can be provided by directly using the velocity signal \( \hat{x} \) in the feedback control

\[F_N(t) = F_0 + \hat{F} \sin(\omega t - \pi) = F_0 - \hat{F} \sin(\omega t) = F_0 - \frac{\hat{F}}{\hat{x}} \hat{x}(t). \tag{4.6}\]

For a displacement or an acceleration feedback a time shift of \(-T/4\) has to be added, which is difficult in a practical application. In analogy to figure 5 the diagram of figure 18 shows the reason for the negative energy input \( \Delta E_{\text{in}} < 0 \) of the Coulomb friction characteristic (C) with \( \mu_0 = \mu = \text{const} \).

The theoretical solution is proved by a test rig with a rotational pendulum (figure 19). The normal force in contact is controlled by an electromagnetic actuator using the control law of equation (4.6). The control input is the pendulum velocity measured with a laser vibrometer. Figure 20 shows the time histories and their changes after switching on the control loop for different gain...
Figure 19. Pendulum test rig with an electromagnetic actuator for the normal force control and alternative with a tuned mass damper.

(a) controller

\[ F_N = F_0 - K \dot{x}(t) \]

(b) Velocity \( \dot{x} \) (mms\(^{-1}\)) and normal force \( F_N \) (N)

(c) For different gain factors \( K \):

- \( K = 0 \) (N s mm\(^{-1}\))
- \( K = 0.1 \) (N s mm\(^{-1}\))
- \( K = 0.2 \) (N s mm\(^{-1}\))
- \( K = 0.3 \) (N s mm\(^{-1}\))
- \( K = 0.4 \) (N s mm\(^{-1}\))

Figure 20. Experimental time histories of \( \dot{x} \) and \( F_N \) for the normal force control with different gain factors \( K \): (a) system and control loop, (b) velocity \( \dot{x} \) and normal force \( F_N \), (c) system behaviour for different gain factors \( K \).

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factors $K = \hat{F}/\hat{x}$. In a more detailed view, the phase shift $\varphi_x \approx \pi$ between the normal force $F_N$ and the velocity $\dot{x}$ can be noted. For gain factors $K \geq 0.2$ the limit cycle breaks down after switching on the control loop. A practical application of this countermeasure is shown in figure 21. It shows the declining oscillations of a squealing disc brake after turning on the control loop. A piezoceramic is placed between the brake pad and the piston in order to influence the normal force. The in-plane oscillation of the pad is measured with a laser vibrometer (cf. Neubauer et al. 2005). The influence of an active or passive displacement or velocity-dependent normal load on the vibration behaviour of a single degree of freedom system has been studied by Anderson & Ferri (1990).

\[ (b) \text{ Active control of the belt velocity} \]

An alternative to a normal force control is a control of the belt velocity $v_0$ to again reach a negative energy input into the oscillator. A linear feedback control with

\[ v_0(t) = \bar{v}_0 + \frac{\hat{v}_0}{\hat{x}} \dot{x}(t) = \bar{v}_0 + K \dot{x}(t) \quad (4.7) \]

is used. The control law directly influences the relative velocity

\[ v_{\text{rel}}(t) = v_0(t) - \dot{x}(t) = \bar{v}_0 + (K - 1) \dot{x}(t). \quad (4.8) \]

If a gain factor $K = 1$ is chosen, the feedback control compensates the influence of the oscillator velocity $\dot{x}$ on the relative velocity $v_{\text{rel}}$, which is then constant $v_{\text{rel}} = \bar{v}_0$. This prevents the energy flow into the oscillator, $\Delta E_{\text{in}} = 0$.

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For a linear decreasing friction characteristic (B), assuming $v_{rel} > 0$, the combination of equations (4.1)–(4.3) and (4.8) yields

$$\Delta E_{in} = \left\{ \begin{align*}
\int_0^T F_R \dot{x}(t) dt \\
= F_N \int_0^T [\mu_0 - \delta_{\mu} \ddot{v}_0 - \delta_{\mu} (K - 1) \dot{x}(t)] \dot{x}(t) dt
\end{align*} \right\}$$

and

$$\Delta E_{in} = -\frac{1}{2} F_N \delta_{\mu} (K - 1) \dot{x}^2 T.$$

It can be seen that the energy input is negative ($\Delta E_{in} < 0$) for gain factors $K > 1$. $\Delta E_{in} = 0$ follows for $K = 1$ and the energy input is positive ($\Delta E_{in} > 0$) for smaller gain factors with $K < 1$. Therefore, gain factors $K \geq 1$ suppress self-excited vibrations.

This control concept is tested again experimentally at the rotational pendulum (figure 19). The time histories for different gain factors $K$ are shown in figure 22. In a detailed view it can be seen that the base velocity $v_0(t)$ is in phase with the oscillator velocity $\dot{x}$.

The measurements show the stabilization of the system. They demonstrate the possibility to break down stick–slip vibrations by a belt velocity control. The analytical calculations are only exact for harmonic vibrations, whereas the

Figure 22. Experimental time histories of $\dot{x}$ and $v_0$ for the belt velocity control with different gain factors $K$: (a) system and control loop, (b) velocities $\dot{x}$ and $v_0$, (c) system behaviour for different gain factors $K$. 

For a linear decreasing friction characteristic (B), assuming $v_{rel} > 0$, the combination of equations (4.1)–(4.3) and (4.8) yields
experiments show that a belt velocity control also works well for non-harmonic oscillations, like stick–slip vibrations.

(c) Passive-tuned mass damper

In this section a passive countermeasure is discussed. A tuned mass damper is added to the above described oscillator tilted by an angle $\alpha$ (figure 23a). This tuned mass damper, here named absorber, increases the damping of the whole system and also generates fluctuating normal forces. The fluctuating normal forces reduce the vibrations for positive tilting angles in this application.

The 2-d.f. system in figure 23a is described by the coordinates $x$ and $y$ measured from the corresponding equilibrium positions. The force $N_0$ is used to consider the weights of both the bodies, $N_0 = (m_1 + m_2)g$. The equations of motion are written

$$ (m_1 + m_2)\ddot{x} + m_2 \sin \alpha \ddot{y} + d_1 \dot{x} + c_1 x = F_R = \mu(v_{rel})F_N $$

and

$$ m_2 \sin \alpha \ddot{x} + m_2 \ddot{y} + d_2 \dot{y} + c_2 y = 0 $$

with the parameters given in figure 23a. The normal force $F_N$ is a reaction force, which must be calculated to solve the equations above

$$ F_N = N_0 + d_2 \cos \alpha \dot{y} + c_2 \cos \alpha y + m_2 \sin \alpha \cos \alpha \dot{x}. $$

To show the properties of the system, first, the simple friction characteristic (C) with $\mu_0 = \mu = \text{const.}$ is assumed and the dampers are neglected, $d_1 = d_2 = 0$. Then the following matrix equation describes the system dynamics:

$$
\begin{bmatrix}
    m_1 + m_2 - \mu m_2 \sin \alpha \cos \alpha & m_2 \sin \alpha \\
    m_2 \sin \alpha & m_2 
\end{bmatrix}
\begin{bmatrix}
    \ddot{x} \\
    \ddot{y}
\end{bmatrix}
+
\begin{bmatrix}
    c_1 & -\mu c_2 \cos \alpha \\
    0 & c_2
\end{bmatrix}
\begin{bmatrix}
    \dot{x} \\
    \dot{y}
\end{bmatrix}
=
\begin{bmatrix}
    0 \\
    0
\end{bmatrix}.
$$

(4.13)
The stiffness matrix in equation (4.13) is asymmetric for $\mu c_2 \cos \alpha \neq 0$. Therefore, the non-conservative restoring forces influence the energy flow in the system. If, for example, $\cos \alpha = 0$ ($\alpha = \pm \pi/2$), the stiffness matrix becomes symmetric and the system is conservative. Conservative means that the total mechanical energy in the system does not change. For $\sin \alpha = 0$ ($\alpha = 0$), the motion of the absorber is uncoupled from the motion of the oscillator (cf. equation (4.11)), but the motion of the oscillator is still dependent on the absorber motion.

If the system is calculated with the more important friction characteristic (B) this results in nonlinear equations, which should not be deduced here (cf. Popp 2005). The stability boundaries are shifted due to the effect of fluctuating normal forces. The stable area is larger for positive tilting angles $\alpha > 0$ and smaller for negative tilting angles $\alpha < 0$ (figure 23b). The asymmetry bases on the fluctuating normal forces which can be an excitation mechanism as well as a countermeasure of self-excited vibration depending on the phase angle (cf. §§2b and 4a).

The absorber is tested experimentally with the rotational pendulum (figure 19). Two different damping ratios $D_2 = d_2/(2\sqrt{c_2 m_0})$ of the absorber have been realized. Tests with decaying amplitudes are marked with plus symbols and limit cycles with open circles (figure 23b). Considering the complexity of the real friction contact, the measurements show a good agreement with the simulations. Especially the asymmetry of the stability boundary is shown by the measurements and is demonstrated by two time histories for identical tilting angles $|\alpha|$ with different signs. These experiments and simulations show the potential of additional tuned mass dampers, which are well established for external excitation, also for self-excitation.

(d) **Passive and semi-active piezoelectric shunt technique**

Shunt damping with piezoelements is a technique in which the electrodes of a piezoceramic are connected to a shunt in order to achieve damping of the vibrations. Typically, passive-resonant shunts are connected to the electrodes. These LR shunts consist of inductive and resistive elements. They can be adapted to the excitation frequency in a similar manner as a tuned mass damper. The performance of such shunts is often not sufficient, so that active elements have to be added to the shunt. A negative capacitance element has been proven to increase the performance significantly (cf. Tang & Wang 2001; Neubauer et al. 2006). However, the implementation requires an operational amplifier and thus electrical power supply. Therefore, the subsystem is semi-active.

The model of the piezoelectric transducer is given in figure 24. The piezoceramic is deformed by a harmonic oscillation $x(t)$. This induces a voltage on the electrodes. As a result of the interaction with the attached network, the piezoceramic generates a force. With the common linear description of the piezoceramic, the force $F$ can be obtained in Laplace domain $s=j\omega$ by

$$F(s) = -\left(c_{33} d_{33}\right)^2 \frac{Z(s)}{C_{ps} \left(Z(s) + \frac{1}{sC_{ps}}\right)} \Delta X(s) + c_{33} \Delta X(s), \quad (4.14)$$

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where $c_{33}$ is the mechanical stiffness; $d_{33}$ is the sensitivity; and $C_{ps}$ is the capacitance of the piezoceramic. It is reasonable to split the force into a mechanical part $F_{\text{mech}}$, which is the conservative force due to the mechanical elasticity, and an electrical part $F_{\text{elec}}$, which is dependent on the attached shunt. The shunt is represented by its complex impedance

$$Z(s) = R + sL + \frac{1}{sC}.$$  \hspace{1cm} (4.15)
In the following, the external capacitance $C$ will be normalized to the piezoceramic capacitance $C_{ps}$:

$$\delta = \frac{C_{ps}}{C},$$

(4.16)

with $\delta = 0$ for the passive $LR$ shunt and $\delta < 0$ for the $LRC$ shunt with negative capacitance. Figure 25 shows the frequency response functions of the generated piezoceramic force for a harmonic excitation of the piezoceramic

$$x(t) = \hat{x} \sin(\Omega t).$$

(4.17)

The force is normalized to the maximum appearing amplitude in the frequency spectrum for $\delta = 0$. The generated force shows a strong amplification at the resonant frequency of the shunt

$$\Omega_{res,shunt} = \frac{\sqrt{1 + \delta}}{\sqrt{C_{ps}L}}.$$

(4.18)

To maintain a constant resonant frequency, the inductance has to be reduced to $(1 + \delta)$ times the value for the $LR$ shunt. For the passive $LR$ shunt, $\delta = 0$, the amplification is limited to a very narrow frequency range around the resonant frequency. With a negative capacitance, this range can be broadened. Assuming a harmonic oscillation and a harmonic response of the system with the phase delay $\phi$,

$$x(t) = \hat{x} \sin(\Omega t), \quad \dot{x}(t) = \Omega \hat{x} \cos(\Omega t)$$

and

$$F(t) = \hat{F}(\Omega) \sin(\Omega t - \phi(\Omega)),$$

(4.19)

Figure 26. Energy dissipation per vibration period of a passive $LR$ shunt ($\delta = 0$) and semi-active $LRC$ shunts, $R = 100 \, \Omega$. 

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the energy dissipation $\Delta E_{\text{diss per vibration period}} = \frac{2\pi}{\Omega}$ can be obtained using equation (2.1),

$$\Delta E_{\text{diss}}(\Omega) = \int_{0}^{T} \dot{x}F(t)\,dt = \int_{0}^{2\pi/\Omega} \Omega\dot{x}\cos(\Omega t)\dot{F}(\Omega)\sin(\Omega t - \phi(\Omega))\,dt$$

$$= -\pi\dot{x}\dot{F}(\Omega)\sin \phi(\Omega).$$

(4.20)

The result is depicted in figure 26. The energy takeout per period is given for the passive $LR$ shunt, $\delta = 0$, and $LRC$ shunts with various capacitance ratios. While the maximum energy takeout at $\Omega_{\text{res,shunt}}$ is constant, the $LRC$ shunts comprise a larger area around it. The total energy takeout is significantly increased for excitations $\Omega \neq \Omega_{\text{res,shunt}}$.

(e) Suppression of brake squeal with shunted piezoceramics

A disc brake is an important application of measures to avoid friction-induced vibration (cf. §3c). Therefore, the method of shunted piezoceramics for squeal suppression has been performed analytically and experimentally.

The brake test rig in figure 27 has been modified in order to incorporate piezoceramics between one brake pad and the calliper. Tests are conducted with passive $LR$ shunts and semi-active $LRC$ shunts with a negative capacitance. In addition to the experiments the brake with the connected shunt is modelled as an electromechanical multibody system. The model consists of rigid bodies and a flexible brake disc that is modelled according to the Kirchhoff theory (cf. Neubauer et al. 2006).

The stability of the brake model is calculated by a complex eigenvalue analysis. The system is unstable if the maximum real part, termed $\text{Re}(\lambda_{\text{max}})$, of at least one eigenvalue is positive. Related to the brake this will be interpreted as a strong tendency to squeal. Figure 28 shows the maximum real part for the brake depending on the parameters $L$, $R$ and $C$ of the shunt. The inductance $L$ is normalized to the value $L_{\text{LRC}}$, for which the resonant frequency of the shunt is equal to the squealing frequency of the brake. It is evident that the brake can be
stabilized if the shunt is tuned to the squealing frequency \( \frac{L}{L_{LRC}} \). The bandwidth of the effect is maximized for a resistance value of approximately \( R = 50 \, \Omega \). For values of \( R > 150 \, \Omega \), the force amplification of the piezoceramic is too low and the system cannot be stabilized.

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A variation of the capacitance ratio $\delta$ shows that a passive $LR$ shunt with $\delta = 0$ is capable of suppressing the squealing, but the effect is limited to a very narrow range of inductance values. Consequently, this system is very sensitive for changes in the inductance or the squealing frequency. Therefore, this method cannot be practically used to suppress brake squeal. A negative capacitance significantly increases the bandwidth, thus making this method applicable for a more robust suppression.

Figure 29 shows the experimental verification of the proposed technique. The sound pressure of the brake has been recorded with a microphone at a distance of 0.5 m to the friction contact. In the experiments, the electrodes of the piezoceramic have been connected and disconnected in a sequence of approximately 10 s with the $LRC$ shunt. The measurements clearly show the strong sound pressure when the shunt is disconnected. The squealing frequency is in the range of $f = 3400$ Hz with a small variation due to the disc inhomogeneities. The disc rotates at 23 rpm resulting in a period time of $T = 2.6$ s. After connecting the optimal tuned shunt, the squealing stops immediately until the shunt is disconnected again. The remaining sound pressure originates from the motor that drives the brake disc at the test rig and other environmental sounds.

5. Conclusions

Different mechanisms generating self-excited vibrations are discussed analytically and experimentally. As practical applications an axial seal, a tread block and a disc brake are investigated showing the relevance of this research. The design countermeasures depend on the excitation mechanism. Therefore, it is essential to understand the mechanism before optimizing the system systematically. Additionally, passive or active solutions can be used to avoid the unwanted self-excited friction-induced vibrations. These active or passive subsystems are explained analytically and their function to avoid vibrations is validated experimentally. A large range of countermeasures is demonstrated using simple models as well as the real applications.

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