Uncertainty in quantum mechanics: faith or fantasy?

BY ROGER PENROSE*

Mathematical Institute, 24–29 St Giles’, Oxford OX1 3LB, UK

The word ‘uncertainty’, in the context of quantum mechanics, usually evokes an impression of an essential unknowability of what might actually be going on at the quantum level of activity, as is made explicit in Heisenberg’s uncertainty principle, and in the fact that the theory normally provides only probabilities for the results of quantum measurement. These issues limit our ultimate understanding of the behaviour of things, if we take quantum mechanics to represent an absolute truth. But they do not cause us to put that very ‘truth’ into question. This article addresses the issue of quantum ‘uncertainty’ from a different perspective, raising the question of whether this term might be applied to the theory itself, despite its unrefuted huge success over an enormously diverse range of observed phenomena. There are, indeed, seeming internal contradictions in the theory that lead us to infer that a total faith in it at all levels of scale leads us to almost fantastical implications.

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1. The uncertainty of quantum mechanics

When ‘uncertainty’ is used to describe the actions of physical systems according to fundamental physical laws, this frequently refers to the behaviour of the world according to quantum mechanics. Indeed, ‘quantum uncertainties’ are serious considerations when one refers to small-scale activity. The issue of ‘Heisenberg’s uncertainty principle’ often looms large, as does the fact that quantum measurements usually yield results that are predicted by the theory only with uncertainty—that is, these predictions are given just with probabilities. So we do indeed have these uncertain elements in the theory. All this is true. But I wish to refer to something else about quantum mechanics—a quite different element of uncertainty, namely the uncertainty that should apply to all theories, but that is, surprisingly in my view, not often levelled against quantum mechanics itself. Is the theory always to be trusted? Is it actually ‘true’? How much does our belief in the theory’s implications amount to a matter of faith? And does our faith in it sometimes lead us into the realms of fantasy?

*roger.penrose@maths.ox.ac.uk

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So the question here is whether the word ‘uncertainty’ applies to the theory itself; that is, can there—in view of its uncontradicted success over a huge range—actually be serious doubt that quantum mechanics truly reflects the behaviour of the world? I should explain that there is no question that it is an extremely well-supported theory. Indeed, it is not just well supported; there is no observation that tells against the theory. Its range is absolutely enormous. I think people sometimes say it is the most successful theory that physical science has yet produced, and that its discovery was the greatest of scientific revolutions. I do not myself think it quite compares with Newtonian mechanics, which really was perhaps the greatest revolution in science. There are other somewhat comparable revolutions such as Maxwell’s nineteenth century theory of electromagnetic phenomena, which largely arose from Faraday’s ingenious experiments. And then there is the other twentieth century revolution in physics: Einstein’s general theory of relativity.

There is a nice contrast between the two great twentieth century revolutions because quantum mechanics had absolutely enormous implications, some only just beginning to be revealed in many different areas. For example, the quantum theory provides the explanation of chemical forces, lasers, superconductors, spectral lines, the strengths and colours of materials, the speed and reliability of computers, the reliability of inheritance, etc. As for the comparable theory of general relativity, what is it good for? Well, there is the global positioning system; this is a definite application and it is certainly an important one. Moreover, there are several key uses of general relativity in astronomy and cosmology, such as determining the regions of mass density throughout the universe, by way of the effects of gravitational lensing. However, this hardly compares with the vast range of applications of quantum mechanics, many of which have completely transformed our lives. On the other hand, general relativity is a beautiful, self-consistent whole, whereas quantum mechanics has to live with not only deep puzzles of interpretation but, I would contend, a profound internal inconsistency, which is the reason that one might believe that there is something serious to be attended to about the theory, at some stage. So I want to try to address this issue.

### 2. The basic puzzle of quantum mechanics

I shall need to explain the basics of quantum mechanics because I take it that those reading this article are not all physicists. I should say that there are certain elements of irony and paradox in the previously mentioned issue of ‘quantum uncertainty’. The theory is, in fact, governed by very precise rules: the Schrödinger evolution equation and the Born rule for the probabilities. These are extremely well tested and there is no doubt that the theory works extraordinarily well in practice. So what is quantum mechanics? Let me start by illustrating the two opposing common descriptions that tend to be applied to basic quantum entities: are they particles or are they waves?

One can see these two aspects of quantum behaviour in two very similar idealized experiments. In figure 1a, we have a quantum entity—say a photon—projected at a beam splitter, which we could think of as a half-silvered mirror. This beam splitter transmits half of the light that is shone on it and reflects the other half. Here, we have a laser A that is emitting a single photon and this photon encounters the beam splitter B, angled at $45^\circ$ to the direction in
Figure 1. The laser source at A emits a single photon, which encounters the beam splitter at B. (a) Particle-like behaviour: the detector at C registers reception of the photon or the detector at D does—not neither, not both. (b) Wavelike behaviour (Mach–Zehnder interferometer): with mirrors at C and D, and another beam splitter at E (with equal path lengths), the detector at F always registers reception of the photon, but that at G never does.

which the photon is initially travelling. If the photon is to behave as an ordinary particle, then it would have to be either transmitted or reflected. Thus, if we have detectors at the two marked places, one at C, in the direction in which the laser is aimed and the other, at D, where BD is perpendicular to BC (where we make the ideal assumption that the detection capabilities of the detectors are perfect), then either one or the other detects the photon (each with a 50% probability), not both, not neither. So it is an ‘exclusive or’. That is very much a particle behaviour for a photon.

Suppose now that we arrange things as in figure 1b (Mach–Zhender interferometer), where we replace our two detectors with perfect mirrors, each parallel to the original beam splitter, so that the beams are directed towards a second beam splitter E, again parallel to the original one, and let us suppose that this is arranged so that all four path lengths (BC, BD, CE and DE) are equal (so that the paths between the two beam splitters form the square, BCED). Detectors are placed in the two locations F and G (continuing the paths DE and CE, respectively). One would imagine that the photon, as it leaves the first beam splitter B, should have a 50 per cent chance to go one way (towards C) and 50 per cent to go the other (towards D), and that following reflection at C or D upon arriving at the second beam splitter E, irrespective of the direction it came from, it would have a 50 per cent chance of sending the photon towards F and a 50 per cent chance towards G. But what do we find? We would find, in actuality, for a perfect set-up like this, that on every single occasion, it would be F that detects the photon, and G would never receive it. This is hardly the way that ordinary classical particles behave! As far as the detector G is concerned, it is as though the two possible ways that the photon might have reached it have somehow cancelled each other out!

However, this is just the way that ordinary waves behave. Let us think of the photon as being a little wavelike disturbance that is split at B into two smaller waves, each carrying half the energy—and so \(1\sqrt{2}\) of the amplitude of

Phil. Trans. R. Soc. A (2011)
intensity—of the original one, and these then simultaneously reach the beam splitter E from different directions. At E, each of these smaller waves would again split into two, each now of half the intensity of the original wave. Going each way from the beam splitter E would now be two half-amplitude waves, and these would add up to make a full-amplitude wave if completely in phase, but would cancel each other out if they were completely out of phase. Indeed, we find that (with the equal path lengths that we are envisaging here) the two components aimed at F would be in phase and so would combine to make a full-intensity wave, while those aimed at G would be totally out of phase and the two would cancel completely. This is just what is seen.

Does this mean that we can consistently think of photons simply as such little wavelike disturbances? No we cannot, for the wave explanation would not work for the first experiment depicted in figure 1a. We might try to take the view that the two half-energy components emerging from B would each provide a 50 per cent chance of activating the detector it is aimed at. But this would not do, because with this picture there would be a 25 per cent chance that both detectors would register and a 25 per cent chance that neither would. There would only be a 50 per cent chance of the actual result occurring, namely that just one of the detectors registers, not both, not neither!

3. How quantum theory works?

So, how does quantum mechanics deal with this conundrum? We need to have a scheme that accommodates both these potential behaviours at the same time. The key thing here is that if there are two, or perhaps more, possible things that the photon might do—and let us consider the case of just two—we have to envisage that they both can happen at once, which is a strange idea. But that is what we have to do in quantum mechanics. Indeed, we must not just do that, but we have to attach a weighting to each of the alternatives. We might have thought that this weighting could simply be just assigning a probability to each alternative, but things are much weirder than this. Let the alternatives here be \( \Psi \) and \( \Phi \), where in the present situation, these could refer, respectively, to the photon taking the route BC and the route BD, these being the two alternatives open to the photon as it leaves the beam splitter B. We must take the view that they not only occur both at once, but that the weighting attached to each is a complex number—a number of the form

\[
z = x + iy,
\]

where

\[
i = \sqrt{-1}.
\]

This is indeed a very strange idea, especially because a number like ‘i’ is referred to as an ‘imaginary’ number. Quantum mechanics reveals that alternatives such as these do have to be weighted by complex numbers, say \( w \) and \( z \). By allowing the weightings to be complex numbers is what gives us the freedom to incorporate a ‘phase’ into the alternative possibilities for the photon, so that alternatives can sometimes cancel out, such as in the case of the photon being unable to reach the detector G in the second experiment above (figure 1b). To see how this works, we shall first need to appreciate a little more about complex numbers.

Phil. Trans. R. Soc. A (2011)
Figure 2. The complex (Wessel) plane. The complex number \( z = x + iy \) (with \( x, y \) real) is represented as the point with Cartesian coordinates \((x, y)\). Centred at 0, the unit circle represents numbers of the form \( z = e^{i\theta} \), where the real number \( \theta \) is the arc length, measured anticlockwise around the circle, from the point 1.

Indeed, you might well ask why we should be bringing in the \( \sqrt{-1} \) in order to describe the fundamental processes going on at the basis of physical reality. Different people have different attitudes to this appearance of complex numbers at such a fundamental level in theoretical physics. It is certainly mysterious, and some people react negatively to the appearance of such numbers, regarding this as an unpleasant complication to our understanding of Nature. But to me it’s a very beautiful and satisfying thing—but that’s because I’m a mathematician who had been already impressed by the power, elegance and even simplicity of the complex number system before learning about quantum mechanics. In fact, I rather feel that it is the appearance of the real numbers that should be regarded as strange, rather than the complex ones, but that is perhaps a particular mathematician’s prejudice. To many mathematicians, at least, the system of complex numbers does appear as more natural and beautiful than the system of reals. In figure 2, I have drawn the complex plane (or Wessel or Gauss or Argand or Warren plane, your terminology perhaps depending upon whether you are Norwegian, German, French or English—but Wessel was the first; see Crowe [1]), where we plot the real numbers along the horizontal axis, out to the right and the real multiples of \( i = \sqrt{-1} \) vertically, through the origin. All the different complex numbers are represented on a plane, with \( z = x + iy \) represented as the point with Cartesian coordinates \((x, y)\).

Let us again consider a pair of possible quantum alternatives, such as the \( \Psi \) and \( \Phi \) considered earlier. We refer to \( \Psi \) and \( \Phi \) as representing, respectively, the quantum state of each of these possibilities. We now have to consider that we might also have any other state of superposition of these two, such as

\[
 w\Psi + z\Phi,
\]

Phil. Trans. R. Soc. A (2011)
Figure 3. The space of quantum state vectors $\Phi, \Psi$, etc. is a complex vector space of (usually) high dimension—frequently infinite—called a Hilbert space. Sums, such as $\Phi + \Psi$, and products, such as $z\Psi$, can be formed, where $z$ is any complex number.

this combination representing the quantum state of the superposition, where the complex numbers $w$ and $z$ are not to be both zero. There is also the rule that it is really just the ratio of $z$ to $w$ that is important—so the physical nature of the quantum state is unaltered if the whole thing is multiplied by some non-zero complex number. Although the numbers $w$ and $z$, being complex, cannot be simply probabilities, they do have a relation to probabilities via the Born rule. The idea is that if we have some measuring apparatus able to distinguish the state $\Psi$ from the state $\Phi$, and the apparatus is presented with the above state $w\Psi + z\Phi$, then this apparatus will conclude that it has been presented with $\Psi$ or with $\Phi$, where the respective probabilities for each of these two alternatives are

$$|w|^2 \quad \text{and} \quad |z|^2,$$

the modulus $|z|$ of the complex number $z = x + iy$ being $|z| = \sqrt{x^2 + y^2}$, namely the distance out from the origin of the point in the complex plane representing $z$, and correspondingly for $w$. Another way of stating this definition of $|z|^2$ is

$$|z|^2 = z\bar{z} = x^2 + y^2$$

(the Pythagorean theorem), where the complex conjugate $\bar{z}$ of $z$ is

$$\bar{z} = x - iy,$$

the operation of sending a complex number to its complex conjugate being obtained geometrically by reflecting the complex plane of figure 2 in the real axis.

In fact, I have been much too careless, so far, in formulating the Born rule. For it to be true, as stated, we need two conditions to hold for our states $\Psi$ and $\Phi$. The first is that each of these states, and also the superposed state $w\Psi + z\Phi$, be what is called normalized and the second condition is $\Psi$ and $\Phi$ be orthogonal. To understand these notions, we should think of $\Psi$ and $\Phi$ standing for vectors in some high-dimensional space (figure 3), referred to as a Hilbert space, and we refer to $\Psi, \Phi$ and $w\Psi + z\Phi$ as state vectors. The notion of their being ‘orthogonal’ is to be thought of as their being at right angles (or in independent directions), and the notion of a state being ‘normalized’ is to be thought of as its vector
being of unit length (i.e. a unit vector). Both of these notions can be understood more precisely in terms of the mathematical concept of scalar product, which, for vectors in ordinary space, is something that a good many of my readers may well be familiar with. The scalar product between \( \Psi \) and \( \Phi \), which for quantum states is a complex number, would normally be written
\[
\langle \Psi | \Phi \rangle
\]
in the context of quantum mechanics. The rules of a Hilbert space demand that the algebraic relations
\[
\begin{align*}
\langle \Phi | \Psi \rangle &= \langle \Psi | \Phi \rangle, \\
\langle \Psi | w \Phi \rangle &= w \langle \Psi | \Phi \rangle = \langle w \Psi | \Phi \rangle, \\
\langle \Phi | \Psi + \Gamma \rangle &= \langle \Phi | \Psi \rangle + \langle \Phi | \Gamma \rangle, \\
\langle \Phi + \Gamma | \Psi \rangle &= \langle \Phi | \Psi \rangle + \langle \Gamma | \Psi \rangle
\end{align*}
\]
and
\[
\langle \Psi | \Psi \rangle > 0, \quad \text{unless} \quad \Psi = 0,
\]
always hold. In fact, in each of the second and third lines, the second equality follows from the first equality by virtue of the equality in the first line. (For vectors in ordinary space these do all hold, but without the bar.) Two states \( \Psi \) and \( \Phi \) are deemed to be orthogonal if
\[
\langle \Psi | \Phi \rangle = 0,
\]
and a state \( \Psi \) is normalized—i.e. a unit state vector—whenever
\[
\langle \Psi | \Psi \rangle = 1.
\]

From the above, it follows that even for normalized states, there is the freedom of multiplying the state vector by a complex number of unit modulus, such a number lying on the circle of unit radius centred at the origin—the unit circle—in the complex plane (figure 2). Such a number, usually referred to in the context of quantum mechanics as a phase, is normally written as \( e^{i\theta} \), where by a famous formula (due to De Moivre, Cotes and Euler), we have
\[
e^{i\theta} = \cos \theta + i \sin \theta,
\]
the real number \( \theta \) measuring the angle that the line from the origin to the point \( e^{i\theta} \) makes with the real axis, being taken in an anticlockwise sense (figure 2). Any state \( \Psi \) can be converted to a normalized state simply by dividing it by the square root of its norm \( \langle \Psi | \Psi \rangle \),
\[
\Psi \mapsto \langle \Psi | \Psi \rangle^{-\frac{1}{2}} \Psi,
\]
but there is still the freedom of multiplying this by a phase, which does not alter the state’s physical interpretation at a given time. The phase plays an important role in the mathematical expression for how a state evolves with time according to Schrödinger’s equation. We find that for a photon in free flight, moving uniformly

1The scalar product of ordinary vectors \( \mathbf{x} \) and \( \mathbf{y} \) is \( xy \cos \phi \), where \( x \) and \( y \) are the respective lengths of the vectors \( \mathbf{x} \) and \( \mathbf{y} \) and \( \phi \) is the angle between their directions.
in a particular direction (with a definite momentum \( p \)), the state has a time dependence of the form
\[
e^{-2\pi \nu t} \Psi
\]
(taking \( \Psi \) constant in time), where \( \nu \) is the frequency of oscillation of the state and \( t \) is the time. The actual value of the photon’s momentum turns out to be
\[
p = \frac{\hbar \nu}{c},
\]
in quantum mechanics, where \( \hbar \) is Planck’s constant and \( c \) is the speed of light.

So long as \( \Psi \) and \( \Phi \) are both normalized and orthogonal to each other, then the condition for \( w\Psi + z\Phi \) to be normalized as well is
\[
1 = \langle w\Psi + z\Phi | w\Psi + z\Phi \rangle
\]
\[
= \bar{w}w \langle \Psi | \Psi \rangle + \bar{z}z \langle \Phi | \Phi \rangle + \bar{w}z \langle \Psi | \Phi \rangle + \bar{z}w \langle \Phi | \Psi \rangle
\]
\[
= \bar{w}w + \bar{z}z = |w|^2 + |z|^2,
\]
so that \(|w|^2\) and \(|z|^2\) sum to 1, as they indeed should if, as the Born rule asserts, the quantities \(|w|^2\) and \(|z|^2\) are to be interpreted as the respective probabilities for our measurement to result in the state \( \Psi \) or the state \( \Phi \), these exhausting the possibilities, in the present situation. A similar result holds in situations for which there are more than two alternative outcomes in a quantum measurement.

Let us now see how this applies in the two idealized experimental situations considered in §2. Since the arm lengths BC, BD, CE and DE are all equal, we can ignore the oscillatory phase factor \( e^{-2\pi \nu t} \), this being the same for both routes that the photon might take in each situation under consideration (the photon frequency remaining the same throughout). The only additional thing we really need is a rule that upon reflection at a mirror or beam splitter, the state picks up a factor of \( i \). This plays no significant role in the first experiment, where we may consider that the total state emerging from the beam splitter \( B \) (ignoring the oscillatory overall phase factor) is the superposition
\[
\frac{1}{\sqrt{2}} (\Psi + i\Phi)
\]
(the factor \( 1/\sqrt{2} \) being inserted to normalize the state). In the particular situation considered in figure 1a, our measurement would be to determine whether the photon is in the beam directed at \( C \) from the beam splitter \( B \) (state \( \Psi \)) or in the alternative beam directed at \( D \) (state \( \Phi \)). The Born rule now gives the probability of \(|1/\sqrt{2}|^2 = 1/2\) for the photon to reach \( C \) and the probability of \(|1/\sqrt{2}|^2 = 1/2\) for the photon to reach \( D \), which is indeed the correct answer.

Now let us consider the experiment of figure 1b. The mirror at \( C \) converts the rightward propagating state \( \Psi \) into an upward state \( i\Phi \) and the upward state \( \Phi \) coming from \( B \) is converted to a rightward state \( i\Psi' \) by the mirror at \( D \),
\[
\Psi \to i\Phi \quad \text{and} \quad \Phi \to i\Psi',
\]
whence our total state evolves
\[
\frac{1}{\sqrt{2}} (\Psi + i\Phi) \equiv \frac{1}{\sqrt{2}} (i\Phi' + i \times i\Psi') = \frac{1}{\sqrt{2}} (i\Phi' - \Psi').
\]
When the rightward part of the state $-\psi'$ encounters the beam splitter at E, it emerges as

$$-\psi' \sim -\frac{1}{\sqrt{2}}(\psi' + i\phi'),$$

in exact analogy with what happened originally when $\psi'$ encountered the beam splitter at B. Similarly, when the upward state $i\phi'$ encounters the beam splitter at E, it emerges as

$$i\phi' \sim \frac{1}{\sqrt{2}}(i\phi' - \psi').$$

Combining these two, we find that the entire state $(1/\sqrt{2})(i\phi' - \psi')$ emerges from the beam splitter E as

$$\frac{1}{\sqrt{2}}(i\phi' - \psi') \sim \frac{1}{\sqrt{2}} \left\{ \frac{1}{\sqrt{2}}(i\phi' - \psi') - \frac{1}{\sqrt{2}}(\psi' + i\phi') \right\} = -\psi',$$

the part of the state $i\phi'$ that is aimed at the detector G cancelling out completely. The minus sign, being merely an overall phase, has no consequence, and we see that it is indeed the detector F that always receives the photon.

### 4. Spin: a quantum paradigm

To a certain extent, the previously mentioned descriptions give a misleading impression of quantum superpositions, suggesting that there are actual ‘alternative realities’ that, while coexisting in some sense, are nevertheless simply different possible happenings that the theory tells us we must consider as somehow existing simultaneously. But quantum theory is much more even-handed and subtle than such descriptions would seem to imply. This is illustrated particularly clearly with the spin of a particle or atom, especially in the case where the spin has the smallest non-zero value allowed by quantum mechanics, namely $\bar{h}/2$, where $\bar{h}$ is Dirac’s ‘reduced’ version of Planck’s constant $h$, given by

$$\bar{h} = \frac{h}{2\pi}.$$

The most basic particle constituents of ordinary matter, electrons, protons and neutrons are such particles of ‘spin 1/2’, as are the more primitive entities that constitute the latter two, namely the quarks.

It is usual to represent the spin states of such a particle as being superpositions of two basic quantum spin states that I shall write simply as $\uparrow$ and $\downarrow$, where the first describes the particle’s spin as being right-handed about the upward direction, and the second, as being right-handed about the downward direction (i.e. left-handed about the upward direction). The states $\uparrow$ and $\downarrow$ are orthogonal to one another (in the quantum-mechanical sense that is, where we note the fact that the up and down directions are at 180° to each other, not the 90° that would be the geometrical meaning of ‘orthogonal’). We take each of $\uparrow$ and $\downarrow$ to be normalized, and consider the general quantum superposition of the...
Uncertainty in quantum mechanics

Two $w \uparrow + z \downarrow$. What we find, rather remarkably, is that (assuming $w$ and $z$ are not both zero) this always represents a state of right-handed spin about some direction, say $\uparrow$,

$$w \uparrow + z \downarrow = \uparrow.$$

The spin-axis direction $\uparrow$ is determined by the ratio

$$u = \frac{z}{w}.$$

In figure 4, the geometry of this situation is displayed explicitly (where a specific choice of phase for each of $\uparrow$ and $\downarrow$ has been made, so that $\uparrow + \downarrow$ points off to the right). The sphere $S$, depicted, is to be viewed as the unit sphere whose equator coincides with the unit circle of the horizontal plane $C$ in the picture, which is taken to represent the complex plane of the complex number $u$. This sphere $S$, which is referred to as the Riemann sphere of $u$, is related to $C$ by stereographic projection, from its south pole $S$. That is to say, the point $U$ on $C$, representing the complex number $u$, is in a straight line with $S$ and the point $U'$ on $S$, which is the representation of the same complex number $u$, but now on the Riemann sphere $S$. We note that $u = 0$, which is represented on $C$ by the central point $O$ corresponds to the north pole $N$ of the sphere $S$, but the south pole $S$ itself would represent infinity ($u = \infty$) on the complex plane. Indeed, the entire Riemann sphere can be regarded as a ‘compactification’ of the complex plane, where an additional ‘number’ $\infty$ is adjoined, enabling the complex plane to be ‘folded up’ into a sphere. This is just what is needed here because the ratio $u = z/w$ indeed becomes infinite when $w = 0$, which occurs when the superposed spin state is that corresponding to the downward direction $\downarrow$. 

Phil. Trans. R. Soc. A (2011)
What we now find is that there is a direct correspondence between the possible directions in space about which our particle can spin (right-handed), and the points on the Riemann sphere $\mathbb{S}$. Whatever value the ratio $u = z/w$ takes (including $\infty$), the direction out from the origin $O$ to the point $U'$ on $\mathbb{S}$ that represents $u$ will in fact be the direction of spin of our particle of spin $1/2$. I have always found it to be a very striking and beautiful fact that the geometry of ordinary three-dimensional space can be related so graphically to the seemingly mysterious complex numbers that feature in such a fundamental way at the basis of quantum mechanics. As one would have anticipated, there is nothing special about the particular directions ‘up’ and ‘down’ for the spin of a particle. ‘Left’ and ‘right’ should have done just as well, as, indeed, would ‘forward’ and ‘backward’, or any other pair of diametrically opposite spatial directions. This symmetry is reflected exactly in the quantum-mechanical formalism, where any pair of orthogonal spin-states constitute a basis$^2$ for this two-dimensional Hilbert space describing the states of spin for a particle of spin $1/2$.

There is, nevertheless, still something strange about the behaviour of quantum-mechanical spin. Although the spin state is determined (up to phase and normalization) by some spin direction in ordinary Euclidean space, there is no actual measurement that can determine which direction this happens to be. The best we can do is to select some direction in space, say $\text{↖}$, and then take the quantum spin states $\text{↖}$ and $\text{↘}$, corresponding to the direction $\text{↖}$ and the diametrically opposite direction $\text{↘}$, and perform the measurement that decides between the states $\text{↖}$ and $\text{↘}$. What we find is that there are respective probabilities, given by the Born rule, of finding the spin to be $\text{↖}$ or $\text{↘}$, but when the measurement has been performed, we have lost the chance of determining what might have been the direction (say $\text{↙}$) of the original spin state.

Some physicists would question whether the particle’s spin state before measurement (namely $\text{↗}$) should be assigned any actual physical ‘reality’. Yet, there are some situations where it would appear to be somewhat churlish not to assign a direction to a particle’s spin. Such would arise in those cases where the particle emerges from a system where its spin direction has just been measured and found to be (say) $\text{↗}$, and where it has undergone no further interactions that could affect its spin, so that a subsequent measurement on the particle to determine whether its spin is $\text{↗}$ or $\text{↘}$ would necessarily find it to be $\text{↗}$. No other spin state would have this property, since all other spin directions would give merely a probability of less than certainty to find the result $\text{↗}$.

Yet, there are other situations where it does not appear to make sense to assign such a reality to the direction of an individual particle’s spin. Such could occur when the particle’s spin state is what is called ‘entangled’ with the spin state of another particle. A classic such example arises when our particle of spin $1/2$ might have been produced in a decay process in which an unstable particle whose spin is 0 splits into two particles of spin $1/2$. What we find, according to a famous theorem due to John S. Bell, is that although the spin state of the pair of particles as a whole can be considered to be ‘objectively real’, individual states of spin cannot be assigned to each particle separately.

$^2$A basis refers to a set of vectors, none of which can be expressed as a linear combination of the others, and in terms of which, any vector in the space can be expressed as such a combination of elements of the basis. These ‘linear combinations’ are, in the case of quantum mechanics, the linear superpositions introduced in §3.
which would have been in accordance with the viewpoint referred to as local realism. Bell exhibited inequalities that would have to be satisfied by the joint probabilities of measurement of two systems that act independently in accordance with local realism, but he also showed that quantum systems, such as the one under consideration here, can violate such ‘Bell inequalities’; so local realism is in fact inconsistent with the observable consequences of quantum mechanics.

To be more explicit about what happens in this situation, suppose we find that if we measure the spin of one of the particles in any direction, say →, and find that this particle indeed has the state →, then the other particle would necessarily have the opposite spin state ←, in the sense that a measurement of the spin of that particle, to decide whether it is ← or →, would find ← with probability 1. The corresponding situation would occur if any other pair of opposite directions in space had been chosen, say ↖ and ↙, where if one particle is measured to decide between these two, and its spin state is found to be ↖, then the other’s spin state is immediately established as being ↙. When this fact is combined with the Born rule, the two particles being widely separated, and where directions completely independent of one another are chosen to make spin measurements on the two particles, then it is found that the correlations between the probabilities that the Born rule provides for the two particles violate Bell’s inequalities and so cannot be satisfied by any ‘local-realistic’ model in which the particles are treated as completely independent entities, unable to communicate with one another after they have separated. Experiments of this nature have been performed many times, with increasing sophistication (though usually with pairs of widely separated entangled photons, being subjected to independent polarization measurements), and the results are invariably found to be in accordance with the predictions of quantum mechanics and inconsistent with the expectations of local realism.

5. Unitary evolution and state reduction

In any quantum-level process, such as with the separating particles of spin 1/2 in the above experiment of §4, or when the photon in the experiments in §2 encounters mirrors or beam splitters and when it travels on its course between them, the state of the quantum system is considered to evolve in a deterministic way—and this includes the ‘splitting into two’ that the photon indulges in when it meets a beam splitter. This deterministic evolution takes place according to the ubiquitous Schrödinger equation, a particular instance of which is the evolution \( \Psi \rightarrow e^{-\frac{2\pi}{\hbar}it}\Psi \) for a free photon considered in §3. Another name for this deterministic and continuous evolution of the quantum state is unitary evolution, and I shall use the letter \( U \) to describe it. This, however, is just one of the two ways in which a quantum state is considered to evolve. The other is what happens to a quantum state when it is subjected to a measurement. According to the quantum measurement process, there would be a number of possible outcomes that could result when the measurement is performed, and it would be the quantum state itself that (via the Born rule) determines probability values for each of these alternative outcomes. The spontaneous evolution of the state to a particular one of these alternative outcomes is sometimes referred to as ‘the collapse of the wave function’ or the reduction of the quantum state, and I shall use the letter \( R \) to denote this process. It may be noted that \( R \) contrasts blatantly
with the deterministic continuous $U$, since $R$ is both non-deterministic and discontinuous! Despite this contrast—and, indeed, mutual inconsistency between the two behaviours—there is a sense in which the two procedures dovetail with each other in a remarkable way. Indeed, the fact that the probabilities arising (according to $R$’s Born rule) do make consistent sense is dependent upon the particular mathematical nature of $U$, which we have witnessed a little of above, and unitary evolution is sometimes referred to as ‘conservation of probabilities’.

Yet, there is a deep paradox lurking here, and this also arises from an important mathematical feature of $U$, namely what is called its linearity (which has been implicitly used in the considerations of §3). The linear nature of $U$ can be characterized in the following way. Let us express the unitary evolution of a state $\Psi$ from a time $t$ to a later time $s$ as

$$\Psi_t \rightarrow \Psi_s,$$

and the corresponding evolution, over the same time period of another state $\Phi$ as

$$\Phi_t \rightarrow \Phi_s,$$

then any quantum superposition of these two states, say $w\Psi + z\Phi$, will evolve similarly

$$w\Psi_t + z\Phi_t \rightarrow w\Psi_s + z\Phi_s,$$

the complex numbers $w$ and $z$ remaining constant. Thus, the evolution of each part of a superposition simply carries on independently of the presence of the other parts.

Why is linearity important? Well, it certainly helps a great deal when solving the equations. It is also important because of the things discussed in other articles in this volume about uncertainties in various systems, such as weather systems, and so on (the Lorenz equation and all that sort of the thing), where the chaotic evolution that such systems often indulge in can become effectively unpredictable. Now, these systems are chaotic only because they are nonlinear, whereas the $U$-evolution of quantum mechanics is fundamentally linear. Yet the term ‘quantum chaos’ is often applied to quantum systems, even though they do not have this chaotic character. That phrase has always puzzled me, as it seems to involve a contradiction in terms. I take it that the term refers to a chaotic (and thereby necessarily nonlinear) classical theory that has been brought under the umbrella of standard (linear) quantum mechanics by the process of ‘quantization’. The resulting quantum theory would then, technically, not be ‘chaotic’, that term now referring to the classical unquantized theory.3

I think that the usage of the term ‘uncertainty’, when applied in quantum theory, frequently refers to the $R$-evolution, since $R$ is probabilistic, in contrast with the deterministic $U$. In addition there are ‘uncertainties’ in quantum mechanics of a different kind, such as Heisenberg’s uncertainty principle, which

3The behaviour of the probabilities of quantum mechanics, calculated according to Born’s rule, is closely in accordance with the equation known as the Liouville equation of classical physics that similarly governs the flow of probabilities in the classical theory, where chaotic behaviour frequently occurs. This suggests [2] that a deeper theory underlying the probabilistic aspects of present-day quantum mechanics (perhaps in accordance with the suggestions of §§8 and 9) might tie in with a chaotic nature of such a theory, and possibly also with proposals for a necessary non-computable behaviour that has been argued for [3].
arise when one attempts to assign different classical notions simultaneously to a quantum system in a precise way (such as the position and momentum of a particle) in situations where this is inappropriate in quantum theory. This notwithstanding, we do not find any uncertainties emerging in the $U$-evolution of a quantum state, provided that we use only variables appropriate to the precise quantum description of the state.

Yet there remains a profound conundrum in the contradiction between the continuous deterministic $U$-evolution and the discontinuous probabilistic $R$-evolution, as we shall be seeing in §6. In any case, a probabilistic and discontinuous outcome ($R$) certainly could not be the result of a continuous deterministic process ($U$), unless some approximation procedure is involved. Most physicists appear to take refuge in the puzzling issues involved in finding the right ‘physical interpretation’ of the quantum state and its modes of evolution. For my own part, I would regard it as likely that the linear quantum mechanics that we now use is merely an approximation to some more refined nonlinear evolution that we shall someday discover, and according to which both the $U$- and $R$-evolutions would arise as excellent approximations in their respective contexts. If this proves to be the case, then the present-day inconsistency between the $U$ and $R$ procedures could be removed.

6. Quantum linearity at the classical level? Schrödinger’s cat

Let us try to understand the essential contradiction between $U$ and $R$, sometimes called the ‘measurement problem’. I think that this terminology is under-rating the issue; it is better referred to as the ‘measurement paradox’. The issue is that it would seem that a measuring device itself ought to be treated as a quantum object since, after all, the measuring device is made up out of electrons, quarks, photons, gluons and so on, all of which are quantum entities that are supposed to satisfy the rules of quantum mechanics. So why does not the entire system, consisting of the quantum system under consideration, the measuring apparatus and their common environment, all together, behave as a continuously and deterministically evolving system obeying the strict rules of $U$’s Schrödinger equation, rather than jumping probabilistically in accordance with $R$’s Born rule?

Schrödinger himself put this in the dramatic form of what has become known as ‘Schrödinger’s cat’, where by a simple quantum experiment of a cat could be put into a quantum superposition of being alive and dead (figure 5). It should be emphasized that Schrödinger put this forward as an entirely hypothetical experiment. Being a humane person, he would certainly have had no intention that such an experiment be performed in practice—and in any case, there would be no point in actually doing such an experiment, as the result would be obviously not be in accordance with what would be directly expected from pure $U$-evolution.

In my description, I shall depart slightly from Schrödinger’s original version, which employed a radioactive atom to supply the quantum input. To be more in keeping with the type of the ideas described in §2, a photon encountering a beam splitter will be employed instead for the quantum input.

Thus, let us consider a laser, again emitting a single photon aimed at a beam splitter. After encountering the beam splitter, the photon finds itself in a quantum superposition of two routes, one reflected and one transmitted. Let us suppose
that the reflected route leads to the immediate death of our poor (hypothetical) cat—say (in accordance with Schrödinger’s original prescription) by its triggering a device that smashes a beaker of cyanide within the container where the cat has been constrained (figure 5a)—whereas the transmitted one takes the photon harmlessly in a different direction, leaving the beaker intact and the cat alive (figure 5b). If we accept that the linearity of $U$-evolution extends to the level of a cat then, in accordance with the $U$-linearity described in §5, we must find that the cat herself must indeed also be in a superposition of life and death (figure 5c), as Schrödinger had asserted would follow from his own equation.

Clearly, Schrödinger was providing this example simply to illustrate the absurdity of using his equation to describe physical reality at the level of a cat, where common experience would lead us to be convinced that we would not actually find the cat to be in some such superposition of life and death, but that we would inevitably find the cat to be either alive or dead, and not in a strange superposition of the two. How is this conundrum to be resolved? My own position, as alluded to above (and which I imagine would have been essentially Schrödinger’s also), would be that strict $U$-evolution cannot be the whole story when the level of a cat is reached. Yet, most physicists seem to be much more reluctant than I am to abandon unitary evolution, even for a cat. Such people might say that my description of the cat experiment is all very well, except that I have left out the environment. Or that I have left out an observer who might look at the cat, or that I have not considered such an observer’s conscious state, and so on and so forth. Well, to cut a long story short, in figure 6, I have tried to put all these things together. In figure 6a, we have the case where the photon is reflected and the cat is killed, whereas in figure 6b, we have the case where the photon goes straight through, and the cat survives. In each case, I have tried to indicate the environment, where the dots represent molecules in the air, etc.

An issue arises, concerning the observer’s conscious experience. Should that also be included in the quantum state representing the observer? In figure 6a, b, I have tried to illustrate the observer’s state of mind by suggesting a mental...
image of the dead cat when the photon is deflected and of a live one when it is transmitted, but I have also indicated this by the expression on his face, which is frowning when the cat is killed (figure 6a) and smiling when she survives (figure 6b). I see no reason of principle why facial expressions should not be subject to the same laws as inanimate objects; so if quantum superpositions of inanimate objects are allowed at that level, they should also be allowed for facial expressions. In figure 6c, we have a quantum superposition of the two. Some people consider that a solution to the problem arises as soon as conscious experience is involved, for it is considered that experience cannot be aware of a superposition. Personally, I do not see why conscious experience should be restricted in this way. If such macroscopic superpositions had been commonly presented to us as part of the normal behaviour of the physical world, then I see no reason why they should not have been accepted by us as part of normal experience. The world of our experience simply does not include such macroscopic superpositions—and it is that unsuperposed macroscopic world, the one that we actually perceive, which needs a good description in terms of our physical theories.

Nevertheless, it appears to be the case that a good many physicists and philosophers of science regard it as plausible that conscious experience somehow ‘splits’, as soon as it becomes aware of a quantum superposition, so that there would be a conscious experience of one macroscopic component of the superposition, and a separate consciousness experiencing each other macroscopic component. In the case of Schrödinger’s cat, as described here, the observer’s conscious awareness would have to have two ‘parallel’ sets of experiences, according to this point of view, where one instance of his awareness would experience the dead cat and the other, a live cat. This kind of viewpoint is referred to as the ‘many-worlds’ or Everett, interpretation of quantum mechanics, after Hugh Everett III [4], who first enunciated a serious proposal of this kind. Harvey Brown referred to this point of view in his talk at this meeting [5].

Personally, I regard this picture as highly unsatisfactory. It seems to lay the ‘blame’ for the R procedure entirely on a phenomenon, namely that of conscious experience, of which we have very little understanding in physical terms. Moreover, it is not even clear to me how a ‘stream of consciousness’ can split in this way, and be consistent with our normal experience. And does this ‘splitting’ occur in past directions as well as into the future? Or if not, why not? I have to
confess that the picture makes little sense to me, and I fear that this strict faith in the linear U-rules of standard unitary evolution assumed to hold at all levels of scale is leading us into a world of fantasy.

Some proponents of a ‘many-worlds’ point of view would argue that conscious experience is not the essential factor in determining when a quantum superposition becomes a probability mixture of classical alternatives. Instead, they would claim that the criterion for deciding whether a superposition appears to become such a mixture is when it becomes a practical impossibility to detect the tell-tale interference effects of a quantum superposition such as that which occurred at the detector G in the second experiment of §2. When such a stage has been reached, where observing quantum interference becomes a practical impossibility, it is said that ‘decoherence’ has occurred, and it is deemed that the superposition has become effectively simply a probability mixture of alternatives. Indeed, most supporters of this ‘decoherence’ point of view would not go to the lengths of supposing that there are coexisting alternatives of conscious experience, but would take the view that the R process has in effect taken place as soon as this decoherence has occurred. Again, I must express my dissatisfaction with such a viewpoint, since I believe that one needs a consistent ontology for a physical theory. To argue that ‘reality’ can somehow change from a quantum superposition to a probability mixture of alternatives, merely because it has become a technical impossibility to measure the interference effects that would clearly distinguish the superposition from a probability mixture, strikes me as introducing an unsatisfactory measure of obscurity into the physical interpretation of the theory itself.

Moreover, as we have seen in §4, the general formalism of quantum theory does not favour particular states over others. The states of spin $\uparrow$ and $\downarrow$ are on an equal footing with those corresponding to any other pair of opposite directions, say $\nearrow$ and $\searrow$. Indeed, any (normalized) linear combination (i.e. quantum superposition) of $\uparrow$ and $\downarrow$ is on an equal footing with any other, since these simply refer to the different directions in space. Something of this applies to cat states also. Although these alternatives are now not simply spatial rotations of one another, there is nothing in the general framework of quantum mechanics that regards the pair of states dead and alive as being more ‘real’ than any complex-number-weighted superposition of these two, and this would apply also to superpositions that include the environment or an observer’s body. Although it would be easy to distinguish particular macroscopic states such as the cat states dead and alive, one from the other, it is hard to imagine any kind of practical experiment that could distinguish between, say, the orthogonal pair $(\text{dead} + \text{live})/\sqrt{2}$ and $(\text{dead} - \text{live})/\sqrt{2}$. This is in striking contrast with the orthogonal pair $\rightarrow = (\uparrow + \downarrow)/\sqrt{2}$ and $\leftarrow = (\uparrow - \downarrow)/\sqrt{2}$, which would be no harder to distinguish from one another than the spin states $\uparrow$ and $\downarrow$. Yet, as far as the general framework of quantum mechanics is concerned, the distinction between these two situations is merely one of practicality and not of principle.

To illustrate this point, it would not be difficult to envisage a modification of the previously mentioned version of the Schrödinger cat experiment to one in which the two routes that the photon might take are entangled with the spin states $\uparrow$ and $\downarrow$ of an external particle of spin 1/2 so that, as with the two-particle experiment considered in §4, the $\uparrow$ state would be accompanied by
the photon being deflected and the $\downarrow$ state accompanied by the photon being transmitted. Thus, a measurement on the particle by an external experimenter, intended to distinguish $\uparrow$ from $\downarrow$, would find $\uparrow$ if the photon is deflected and the cat killed, and $\downarrow$ if the photon is transmitted and the cat survives. However, if a spin measurement was performed on the particle in some other direction, then the photon would emerge from the beam splitter in some superposition of being reflected and transmitted, so that if $U$-linearity is considered to persist all the way up to the level of the cat, we would have to consider that the cat’s state is a superposition of life and death. It should be emphasized, however, that although the external experimenter’s choice of spin direction for the particle is fully under the control of the experimenter, the result of this measurement is not. As a consequence, the external experimenter would not be able to influence the probability that the cat is found to be alive when finally an observation is made in order to ascertain whether she is indeed alive or dead. This is despite the fact that prior to such a final observation, the external experimenter’s choice of measurement seems to have a decisive effect on whether or not the cat’s quantum state is actually a superposition!

7. Quantum ontology

The essential puzzle, as raised in §6, would appear to be the issue of how the quantum state of a system relates to what is actually happening in the world of physical reality. Quantum theory is very vague about the whole issue of the relation between the formalism of the theory and physical reality, that is, with the ontology provided by the theory. This is particularly strange for a theory that is supposed to be providing for us a more precise and far-reaching perspective on our physical universe than the classical physics that preceded it. Often physicists take what they might regard as a pragmatic or operational view of what the theory tells us, namely that it makes superb predictions concerning the probabilities of the different outcomes of an experiment, but it does not otherwise provide us with a picture of physical reality.

Personally, I find this very unsatisfying. This is partly because there is a sense in which, according to the general framework of quantum mechanics, any quantum state can be assigned an ‘element of reality’ when a principle put forward by Einstein et al. [6] is adopted. This was illustrated in §4 with a spin state of a particle, or atom, of spin 1/2. There, it was noted that such a state, say $\uparrow$, is characterized by the fact that no other state of spin 1/2 would be guaranteed ‘with certainty’ to yield the result $\uparrow$ in a measurement that decides between $\uparrow$ and the quantum-orthogonal state $\downarrow$. Here, I am considering an unentangled situation, where we have an individual particle in some definite state of spin. Since this guarantee applies to no other spin direction, it would assign such an element of reality to the actual spin state $\uparrow$. According to the general quantum formalism, this is not a property that is peculiar to spin states; for it applies to any quantum state $\Psi$ whatever, of any quantum system. For there would in principle be a measurement 5 that can be performed on the system, which is

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4I am here considering (non-zero) complex multiples of a state all to count as the ‘same’.

5For those familiar with the quantum formalism, such a measurement would be that provided by the Hermitian operator $|\Psi\rangle\langle\Psi|$, in the Dirac notation.
guaranteed to yield the result $\Psi$ with certainty, only if the state happens to be $\Psi$; so again, this assigns an Einsteinian element of reality to the state $\Psi$. This would apply even to a superposed cat state such as $(\text{dead} + \text{live})/\sqrt{2}$; so this seems to provide an element of reality to this strangely superposed state that we find in the Schrödinger cat experiment. But in such a situation, the experiment would be utterly impractical. We appear to be left in the ontologically highly unsatisfactory situation, if we do not regard such a superposed cat state as being ‘objectively real’, that physical reality depends upon the present state of technology, since it is our state of technology that determines which experiments can be performed in practice and which cannot! A modified quantum formalism that tells us that certain types of experiment are in principle impossible to perform might resolve this particular ontological conundrum, but that is not the case as quantum mechanics stands today.

In addition, if we are inclined to assign no ‘objective reality’ to the quantum state of a system, we encounter a difficulty of assigning reality to anything in the theory at all. If we assign reality only to the ‘results of measurement’, we need to ask what processes actually qualify as measurements. Quantum theory does not really tell us which kinds of collections of material particles actually constitute ‘measuring devices’ and which kinds simply give us ‘quantum systems’ to which perhaps no physical ‘reality’ need be assigned. Ultimately, as was the case with Niels Bohr’s (and Werner Heisenberg’s) ‘Copenhagen interpretation’, measuring devices are taken to behave as classical objects, as regards their observationally distinct alternative outcomes, which is a good ‘pragmatic’ attitude to dealing with quantum mechanics, but it hardly provides us with a philosophically satisfying overall ontology for the theory.

8. Conflict with principles from Einstein’s general relativity

My own position is not to put the blame on the vagueness of quantum ontology, and to accept, provisionally, the ‘reality’ of the quantum state (in some appropriate sense) at least at a small-scale level of quantum phenomena. Instead, I would take the mathematical framework provided by quantum theory as not providing us with the whole story, and that we shall eventually find that the behaviour of the physical world actually deviates significantly, at some larger scale, from what is implied by the U-quantum framework. Accordingly, when a physical system becomes ‘large’ (such as Schrödinger’s cat) in some appropriate sense, its behaviour will begin to depart from what present-day quantum theory predicts, and we shall find that the larger the system becomes, the more closely its behaviour accords with classical, rather than quantum, physics. Accordingly, for any such large system, its description would not even be well approximated by the present-day notion of a ‘quantum state’ (or wave function).

It must be made clear, however, that this measure of ‘largeness’ would not refer simply to physical size. There are experiments involving quantum-entangled photons that reach huge separations from each other while maintaining their entanglements over such distances. Although I am not aware of the present distance record, there are certainly measurements where quantum entanglements are confirmed for which the distance between the entangled photons is more than 10 km (see Tittel et al. [7]). Thus, it is not simply distance that determines
the ‘size’ at which quantum effects might begin to fade and classical behaviour takes over. What I am arguing for is a criterion of ‘size’ that refers more to a measure of mass displacement between quantum alternatives. In the case of the entangled photons, the mass displacement is utterly negligible, since it is only in the photons’ polarization states that the entanglements are manifested.

My reason for considering that the amount of displacement of mass between quantum superposed states is the key issue in determining where classical physics begins to take over from quantum physics is that it is mass distributions that provide the source of the gravitational field and, according to Einstein’s general theory of relativity, the gravitational field provides a curvature for space–time. This space–time curvature provides an awkwardness for the very formalism of quantum mechanics, in various different ways. If a change is to be sought for its foundations, then it seems not unreasonable for this change to enter at a point where the formalism already encounters some difficulties in its formulation.

Normally, the problem for theoretical physicists with formulating the appropriate union of quantum mechanics with Einstein’s theory has been phrased in terms of finding the appropriate theory of quantum gravity. This term is usually taken to mean a theory resulting from a correct application of the standard rules of quantum mechanics to Einstein’s general relativity, or perhaps to some other theory of gravity if that were to turn out to be a classical theory of gravity that is more accommodating, with regard to the principles of quantum mechanics. The common view (see [8]) would be that space–time structure itself might well have to be drastically modified, when space–time curvatures get extremely large—say at the Big-Bang or the space–time singularities that classical theory tells us would arise at the cores of black holes—where radii of classical space–time curvature might begin to approach the tiny Planck scale of approximately $10^{-35}$ m. I shall return to these issues in §9.

My own position is different, in many essential respects [9–12]. As the arguments from §6 and §7 above strongly imply, I take the view that quantum theory itself is in need of some serious modification, when applied to systems that are large in some appropriate sense. I believe that there are several reasons for taking the view that it is when quantum phenomena start to involve the gravitational field in a serious way that we shall see that changes in quantum mechanics begin to manifest themselves. Often, people point to the extreme weakness of the gravitational field, as compared with the other forces of Nature, and would argue that gravitational forces are so incredibly weak at the level of an ordinary table-top quantum-mechanical experiment that they could hardly be responsible for the ubiquitous R phenomenon. However, the issue is not the strengths of the ‘forces’ involved in an experiment, but a clash of the fundamental principles lying at the foundations of quantum mechanics and general relativity.

I think that the most powerful case for the existence of such a clash comes from the Galilei–Einstein principle of equivalence, in relation to the quantum principle of superposition. The principle of equivalence, which is the main foundation stone of Einstein’s general relativity, asserts that a gravitational field (at least for a field whose spatial variation can be ignored) is physically equivalent to an acceleration of the frame of reference. The first issue to be considered is how this relates to the way in which the Earth’s gravitational field is to be treated in our table-top quantum experiment. Suppose we wish to incorporate its effect into our Schrödinger equation describing the quantum system under consideration.
There are two ways that we might try to do this. The most direct would be to adopt a standard procedure that physicists would normally employ, namely to incorporate the gravitational potential as ‘a term in the Hamiltonian’. The experts will know what this means, but it is not helpful for me to try to explain this explicitly here. It amounts to treating the gravitational force in the same ‘Newtonian’ way as any other force. Solving this Schrödinger equation, we would obtain a wave function (or state vector, see §3) $\Psi_N$. The alternative ‘Einsteinian’ procedure would be to do our calculations in a freely falling frame of reference, in which the gravitational field has now disappeared (according to the Galilei–Einstein equivalence principle)—so our Hamiltonian does not include the term that the Newtonian procedure gives us—and then transform back to the original frame that we used before. Solving the Schrödinger equation that we obtain in this way, we find a slightly different wave function $\Psi_E$.

It turns out [11,12] that the wave functions that we obtain using these different procedures differ by only a phase

$$\Psi_E = \phi \Psi_N$$

(so that $\phi$ is a complex number of unit modulus). From the discussion given in §3, we might think that the phase $\phi$ would be of no consequence. Indeed, a classic experiment was performed by Colella et al. [13], confirming agreement between these two ways of looking at the gravitational field, showing that the principle of equivalence is actually compatible with quantum mechanics at this level. However, a subtlety emerges if we look at the form of $\phi$ explicitly. What we find is that $\phi$ actually depends on the time $t$, involving a term in $t^3$. This results in $\Psi_E$ and $\Psi_N$ actually belonging to what quantum field theorists refer to as ‘different vacua’ [11,12]. Again, it is not helpful for me to try to explain what this means, but the essential point is that the rules of quantum field theory actually forbid us from forming superpositions of states from different vacua. In the present situation, this would not be a matter of concern because whichever description we use, we would get a consistent mathematical scheme in which quantum superpositions can be performed, either in one vacuum or in the other, and the standard quantum procedures hold.

However, for a more complete description, we need to take into consideration the gravitational fields of the massive objects that constitute the actual experiment under consideration. Were we to use the Newtonian viewpoint, then this could be carried out consistently for every massive constituent, and no conflict with the general formalism of quantum mechanics would arise. But I am arguing that it is the Einsteinian viewpoint that is, in a deep sense, the more physically appropriate one. If the massive constituents of the system are in superposed states of different locations, then their gravitational fields would also be superposed. If we try to adopt the Einsteinian perspective consistently, we encounter a problem because we now have different vacua for the different

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6This is the $c \to \infty$ limiting case of what is known as the Unruh effect ([14]; see also [15,16]), according to which an observer with constant acceleration $a$ experiences a ‘thermal vacuum’ of temperature $\hbar a/2\pi ck$ (with $k$ being Boltzmann’s constant). This temperature goes to zero in the limit $c \to \infty$, but the accelerating observer’s vacuum remains different from that of an unaccelerated one in just the way stated in the text (B. S. Kay (2002), personal communication).
locations of these massive constituents, and we are, technically, not allowed to perform the quantum superpositions that are needed in order to describe the overall state of the system.

In order to proceed further, according to my own view, we require the unknown extension of quantum theory to which the U and R procedures would both be limiting approximations. Without knowing this extended theory, we cannot continue the evolution of the state. However, it is possible to estimate the level at which the R part of the theory ought to take over as a real physical phenomenon. The estimate takes advantage of that form of Heisenberg’s uncertainty principle\(^7\) that relates time to energy and that is used to relate the lifetime \(\tau\) of an unstable atomic nucleus to an uncertainty \(\Delta E\) in its rest energy by the formula

\[
\tau \approx \frac{\hbar}{\Delta E},
\]

where the symbol ‘\(\approx\)’ stands for ‘roughly equals’.

The idea is that a superposition of two stationary quantum states would be similarly unstable, and the illegal superpositions of different vacua arising in the Einsteinian viewpoint can be subsumed into an effective ‘energy uncertainty’ \(\Delta E\) that can be estimated from an examination of the coefficient of \(t^3\) in the phase factor arising from the differing gravitational fields in the two mass distributions. We find that this \(\Delta E\) can be calculated as the gravitational self-energy \(E_G\) of the mass distribution obtained by subtracting the mass distribution in one component of the superposition from that of the other. The Earth’s field then cancels out, and makes no contribution to \(E_G\). In the case when one mass distribution is simply a spatial translation of the other, \(E_G\) can be more simply described as the gravitational interaction energy of the two mass distributions. This is the energy that it would cost to separate two instances of the given mass distribution, where we start them both in one of the locations of the superposition and then displace one of them to the other location of the superposition, considering only the gravitational attraction between the two mass distributions. Identifying \(\Delta E\) in the above Heisenberg formula with \(E_G\), we obtain

\[
\tau \approx \frac{\hbar}{E_G}
\]

for the expected length of time \(\tau\) that the superposition could last before spontaneously becoming either one mass distribution or the other,\(^8\) in accordance with R.

It might be thought, since these gravitational effects would be ridiculously tiny, for any reasonable table-top experiment, that the time scale \(\tau\) for R to take place would be as good as infinite for a table-top experiment. However, \(\hbar\) is also extremely tiny, and one needs to look carefully at any particular situation in order to get a good estimate for the lifetime of a quantum superposition, according to

\(\text{Phil. Trans. R. Soc. A} \ (2011)\)

\(^7\)Although the arguments being presented here are aimed in a direction that might ultimately shed a different light on the kind of ‘uncertainty’ that is involved in Heisenberg’s principle, this principle still has its role to play in demonstrating the limitations of the use of classical physical notions.

\(^8\)Diósi [17] first suggested this expression for a rough average lifetime of a quantum superposition of two stationary quantum states; for subsequent ideas, differently motivating this expression from principles of general relativity, see [9,11,12,18].
this scheme. There does not appear to be any conflict between this proposal and any experiment carried out to date, but various experiments are underway which might begin to approach the level that could seriously test this scheme. In one of these [19], it is proposed to put a tiny mirror—a cube of about 1 μm in dimension (around one tenth of the thickness of a human hair)—into a quantum superposition of two locations that differ by about the width of an atomic nucleus. This would be achieved through a million-fold repeated impact by one component of a beam-split photon. In calculating the time scale $t$ for the lifetime of such a superposition according to this scheme, it is essential that one takes into account the fact that the mass is almost entirely concentrated in the atomic nuclei. The superposition might have to be held for several minutes to test the theory, which is clearly a huge experimental challenge.

9. Cosmology and the second law of thermodynamics

There are also other reasons to expect a conflict between the principles of quantum mechanics and those of classical general relativity, one of the most important being in the role that quantum gravity might be expected to have with regard to the space–time singularities of the classical theory. This conflict arises from the intimate relation between the structure of these singularities and their relation to the second law of thermodynamics (henceforth: second law)—that fundamental law of Nature which was referred to in the discussion given by Lord May [20].

The Big-Bang singularity, and also the singularities in black holes, have been referred to already in §8. It is normally assumed that a physical understanding of what actually happens at these classically singular places would require the appropriate application of quantum field theory to Einstein’s general relativity. However, one of the most striking features of these singular places is the gross time asymmetry that is exhibited, whereas both quantum field theory (as normally understood) and general relativity are completely time symmetrical. Accordingly, the normal expectation would be that the combination of the two would be time symmetrical also.

In order to understand what is involved here, let us first examine the fundamental role of the time asymmetry that appears to be actually exhibited by the singularities. Not only is this asymmetry intimately related to the second law, but the very fact that there is a ubiquitous second law at all in our actual universe can be attributed to the Big-Bang singularity having been of an extraordinarily special type, with an overwhelmingly tiny entropy, whereas the singularities in black holes are quite the opposite, having an overwhelmingly large entropy. We must recall what the term ‘entropy’ actually means. In rough terms, entropy is a measure of the degree of ‘randomness’ exhibited by a system. Somewhat more precisely (but still a little vaguely), if we consider a classical system composed of a vast number of constituent ingredients (say elementary particles), then the complete description of the system would require knowledge of all the parameters determining the positions and momenta of all the particles. However, we would normally try to describe the system in terms of a much smaller collection of ‘macroscopic parameters’, such as density, temperature, pressure, chemical composition, direction of flow, momentum and so on. There would
be many arrangements of the submicroscopic individual constituents that give rise to the same macroscopic parameters, and the (Boltzmann) entropy $S$ of the system is basically given as the logarithm of the number\(^9\) of such submicroscopic arrangements for the given macroscopic state.

Although the precise measure of a black hole's entropy, in the above terms, is somewhat elusive and, indeed, controversial (compare \([21,22]\)), it is clear that this entropy, in such terms, must nevertheless be enormous because a black hole itself requires only a very small number of parameters (only its total mass, angular momentum, position and velocity), considering the huge number of degrees of freedom that would have been involved in its creation. Moreover, the precise Bekenstein–Hawking value $S_{\text{BH}}$ for its entropy, namely

$$S_{\text{BH}} = \frac{k c^3}{4 G \hbar} A,$$

where $A$ is the surface area of its horizon ($k$ and $G$ being Boltzmann’s and Newton’s constants, respectively), is generally accepted. In view of this formula, the present entropy of the universe is completely dominated by the huge black holes that seem to reside within essentially all galaxies, including our own Milky Way, which appears to have a black hole whose mass is some four million times that of the Sun.

The high-entropy singularities that reside within black holes are anticipated to have a wildly oscillatory, diverging and highly irregular space–time geometry \([23]\). These would have a completely different character from the Big-Bang singularity, which would necessarily be very uniform and of very low entropy in order that there be a second law at all. The very fact that there is indeed a ubiquitous second law in our actual universe can be attributed to the fact that the Big-Bang was a singularity of an extraordinarily special type, whereas the singularities in black holes are unrestricted. By relating the mass content of the observable universe to the Bekenstein–Hawking entropy of the black hole that would result if all this matter were to form a black hole, and combining this with Boltzmann’s entropy formula described above, we can estimate the probability that the Big-Bang’s type of singularity could have come about in this particular form purely by chance to be roughly\(^{10}\)

$$1 \text{ in } 10^{10^{124}}.$$  

The Big-Bang’s low-entropy characterization is that the vast numbers of available degrees of freedom in the free gravitational field were, for some reason, completely inactivated. No standard theory of quantum gravity can explain this huge difference, since all those singularities that can arise in gravitational collapse to black holes should also be available in time-reverse form (‘white-hole’ singularities) and, according to standard quantum-field-theoretic ideas, ought to have been potentially present in the initial quantum state. Their absence (perhaps complete absence) is completely unexplained by quantum gravity theory if that

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\(^9\)Strictly, this only comes out as a ‘number’ if we take quantum mechanics into account. Classically, it would be a phase-space volume.

\(^{10}\)See \([11]\); the uppermost index ‘124’ (rather than the frequently used ‘123’) is to include a contribution from dark matter.

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theory is to be the time-symmetric kind of scheme that would seem to be the
normally expected implication of the application of the principles of quantum
(field) theory to a time-symmetrical classical theory such as general relativity.

If we ignore $\mathbf{R}$ and regard quantum field theory to be a formalism that operates
entirely within the framework of $\mathbf{U}$, as is normally performed, then we are indeed
presented with a conundrum. For $\mathbf{U}$ is completely symmetrical under the reversal
of the direction of time (where we replace the complex number $i$ with $-i$). However, if we are looking towards some kind of scheme that reaches beyond $\mathbf{U}$, and that incorporates something resembling $\mathbf{R}$ as an actual physical process, there are reasons to expect that some measure of time asymmetry should be incorporated. For $\mathbf{R}$’s Born rule, as it is normally employed, definitely does violate time symmetry, in the sense that it gives completely wrong answers if simply used as it stands, but in the opposite direction in time.

For example, in the first experiment of §2, illustrated in figure 1a, the Born
rule gives the correct probability 1/2 for the probability that given that a high-
energy photon is emitted by the laser at A in the direction of the beam splitter
B, it will reach the detector C, and there would be the similar conclusion that
it would have probability 1/2 of reaching the detector D. But if we ask the
corresponding question in the reverse time direction, we get an absurdity. The
time-reversed Born rule would tell us that if the detector C receives a high-
energy photon, coming at it from the direction of the beam splitter B, then
the probability that it came from the laser A is likewise 1/2, with a similar
conclusion, again with probability 1/2 that it came from a point H on the floor
of the laboratory in the line DB extended downwards. This is clearly the wrong
answer, the correct answer being more like a probability 1 that it came from A and
0 that it came from H. This very much suggests that there should be some kind
of inbuilt time asymmetry in the sought-for extension of quantum (field) theory
(see Penrose [11], §30.3) which would be intended to include some operation of
the nature of $\mathbf{R}$.

Nevertheless, I do not regard the above argument, in its present form, to be
getting very close to the heart of the time asymmetry in the singularities arising
in Einstein’s general relativity. Furthermore, I believe that the whole issue of
the role of quantum gravity in determining the specific nature of the Big-Bang
singularity needs to be re-examined in the light of some recent ideas about the
origin of its low entropy [24]. Yet, I think that the above considerations indeed
have a role to play, and elsewhere, I have tried to relate this time asymmetry
to the issue of black-hole information loss (see [11], §§30.3, 30.9 and references
therein). Some recent analysis by Palmer [2] moves beyond the arguments that
I had previously put forward and appears to shed some interesting new light on
this type of consideration.

The matter of information loss in black holes also has strong relevance to the
overall entropy balance of the universe [24]. Although Hawking, in his original
article on the temperature and entropy of a black hole, made a powerful case
for information loss in black holes [25], which would result in $\mathbf{U}$-violation in the
ultimate black-hole evaporation, in more recent years, he has changed his mind
and argued against this information loss. Indeed, there must be no information
loss if $\mathbf{U}$ is to hold at all levels. But I believe that Hawking’s original arguments
remain powerful, and are not overturned by these more recent considerations.
The consequent violation of $\mathbf{U}$ in these circumstances, where quantum effects are
tightly locked with those of general relativity, provide, for me, further powerful reasons to believe that quantum mechanics will not survive in its present form at levels where gravitational effects become important and, as suggested earlier, must be superseded by a more far-reaching theory in which both $U$ and $R$ arise only as limiting approximations.

What kind of a theory might this be? I would certainly expect it to agree with the $U$ part of present-day quantum mechanics in the limit when the mass displacements between states in linear superposition are small, but for large mass displacements, some form of nonlinear instability would begin to show itself, leading to an objective reduction process that behaves like $R$, with its Born rule, in the large time limit, where classical behaviour, consistent with Einstein’s general relativity, would begin to appear. Nevertheless, despite its necessary agreement with standard quantum mechanics when mass displacements are small, I feel that the formalism of this putative theory would not resemble that of standard quantum mechanics very closely, as presently formulated. By this, I mean that, in my expectations, the new theory would not arise as ‘tinkering’ with present-day formalism, such as the adding of some nonlinear term into the Schrödinger equation. I would expect, instead, some radically different formulation of the theory, but which nevertheless agrees with our present-day picture in the low-mass-displacement limit.

The sort of thing that I have in mind would be some kind of revolution resembling the way that Einstein’s gravitational theory radically overturned the very basis of Newton’s gravitational theory. Newton’s theory is nevertheless a limit of Einstein’s when gravitational potentials are small and velocities are small compared with that of light, and most predictions of the two theories are, in practice, identical. Yet the mathematical frameworks of the two theories are utterly different, Newton’s is dependent on gravitation being a force acting in a linear way within a fixed background of Euclidean three-dimensional geometry, and Einstein’s crucially requires the full framework of four-dimensional non-Euclidean differential geometry, within which gravity is not actually treated as a ‘force’ in the ordinary sense of that word. I would expect that, as with Einstein’s theory of gravity, the new quantum formalism would differ fundamentally from that of the present-day viewpoint, but for which most predictions of the two theories would, in practice, be identical, and would agree for all the presently known phenomena that are so marvellously well explained by present-day quantum mechanics.

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