We review recent progress in the study of quantum oscillations as a tool for uniquely probing low-energy electronic excitations in high-$T_c$ cuprate superconductors. Quantum oscillations in the underdoped cuprates reveal that a close correspondence with Landau Fermi-liquid behaviour persists in the accessed regions of the phase diagram, where small pockets are observed. Quantum oscillation results are viewed in the context of momentum-resolved probes such as photoemission, and evidence examined from complementary experiments for potential explanations for the transformation from a large Fermi surface into small sections. Indications from quantum oscillation measurements of a low-energy Fermi surface instability at low dopings under the superconducting dome at the metal–insulator transition are reviewed, and potential implications for enhanced superconducting temperatures are discussed.

Keywords: quantum oscillations; high-$T_c$ superconductivity; copper oxide superconductors; quantum critical point; Fermi surface; Fermi liquid

1. Introduction

The normal state of the high-temperature copper oxide superconductors remains a mystery [1,2]. It is thought that an accurate description of the state out of which unconventional d-wave superconductivity arises is likely to hold the key to the origin of the pairing mechanism [3]. The unconventional behaviour of various physical properties such as transport coefficients, photoemission spectra, optical conductivity and heat capacity contribution in the underdoped region has been widely interpreted in the light of the proximity to the Mott insulating parent material as evidence for a breakdown of the fermionic quasi-particle concept [3–12]. With the recent surprising observation of quantum oscillations at low temperatures and high magnetic fields, our understanding of the physics underlying the underdoped cuprates has begun to undergo a re-examination [13–26]. The observation of well-defined quasi-particles with fermionic properties is

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intriguing given other indications of a breakdown of the Landau Fermi-liquid concept. In this review, we focus on the implications of quantum oscillations in the underdoped region of the cuprates for our understanding of normal state physics in this region of phase space.

In §2, evidence is examined to show that magneto-oscillations observed in the underdoped cuprates indeed appear to correspond to orbital quantization in a Landau Fermi liquid.

In §3, we summarize results from quantum oscillation experiments performed at high magnetic fields that reveal Fermi surface sections with surprisingly small area in the underdoped cuprates (see schematic in §2a), in contrast to the overdoped cuprates, in which high-frequency quantum oscillations corresponding to a ‘large’ Fermi surface section obtained from the band structure are observed.

In §4, various scenarios for the observed reduction in Fermi surface area from that expected from band structure are compared with experimental observations.

In §5, a relatively simple possibility is examined: the reconstruction of a large (paramagnetic) Fermi surface by a translational symmetry-breaking order parameter, and the results of angle-resolved quantum oscillations compared with momentum-space-resolved photoemission (angle-resolved photoemission spectroscopy; ARPES) measurements and tight-binding calculations.

In §6, we discuss the surprising findings from doping-dependent and angle-dependent quantum oscillations of a steep mass enhancement at low dopings.

In §7, we discuss the potential interpretation of findings from quantum oscillations in terms of a possible quantum critical point (QCP) in the vicinity of the metal–insulator (MI) transition at low dopings, and possible translational symmetry breaking yielding the observed small Fermi surface sections.

In §8, we treat challenges in reconciling evidence from complementary experiments to discern the electronic structure in the normal state of the underdoped cuprates. In particular, we consider whether an antinodal pocket from quantum oscillations as suggested from the associated negative Hall coefficient can be consistent with the ‘pseudo-gap’ in the antinodal quasi-particle spectrum at the Fermi energy from photoemission and optical conductivity experiments among others.

In §9, we summarize findings from quantum oscillation measurements in the family of underdoped high-$T_c$ cuprate superconductors, and the chief outstanding challenges.

2. Landau quasi-particles and orbital quantization

Magneto-oscillations were first observed in the Hall resistivity of YBa$_2$Cu$_3$O$_{6+x}$ (with $x = 0.5$, i.e. a hole doping $p \approx 0.10$; the correspondence between $x$ and $p$ is illustrated in §6a) [13], as shown in figure 1. A pressing question then concerns whether the low-energy excitations within the underdoped regime of the cuprates can be described as Landau quasi-particles [28,29] governed by Fermi–Dirac statistics

$$ f_{FD}(z) = \frac{1}{1 + e^z} \quad \left( z = \frac{\varepsilon - \varepsilon_F}{k_B T} \right), \quad (2.1) $$

*Phil. Trans. R. Soc. A* (2011)
Figure 1. (a) Oscillatory part of the Hall resistance measured on a single crystal of YBa$_2$Cu$_3$O$_{6.5}$ at temperatures between 1.5 and 4.2 K, as a function of inverse magnetic field; adapted from Doiron-Leyraud et al. [13]. (b) Quantum oscillations in the interlayer resistance measured on a single crystal of YBa$_2$Cu$_3$O$_{6.59}$ at temperatures between 1.5 and 4.2 K, as a function of inverse magnetic field; adapted from Ramshaw et al. [24]. (c) Quantum oscillations in the negative Hall coefficient measured on a single crystal of YBa$_2$Cu$_3$O$_{6.5}$ as a function of magnetic field, showing the evolution with temperature; adapted from LeBoeuf et al. [27]. The dominant $F_\alpha$ frequency is visible to the naked eye in the oscillations shown. (Online version in colour.)

where $f_{FD}$ is the equilibrium average occupation number in a one-particle state of energy $\varepsilon$, $T$ is the temperature and $\varepsilon_F$ is the Fermi energy, or whether this description can be seen to fail. Before attempting to answer this question, we will first consider what we know about more conventional layered metals [30,31]. As is the case in conventional three-dimensional metals [32], quantum oscillations in layered systems result from quantization of orbital motion in the presence of a magnetic induction $B$ into a series of energy levels

$$\varepsilon_l = \hbar \omega_c \left( l + \frac{1}{2} \right),$$

(2.2)

where $l$ is an integer, and $\omega_c = eB/m^*$ is the cyclotron frequency defined by $B = |B|$ and the cyclotron effective mass $m^*$. The number density of (electron or hole) particles per unit area $N$ contained within a given pocket of Fermi surface
Figure 2. Schematic cuprate phase diagram. Circles indicate the relative areas of Fermi surface sections corresponding to the observed quantum oscillation frequencies in the different regions of the phase diagram in different materials. Filled circles denote hole carriers, unfilled circles denote carriers of type as yet to be unambiguously identified. Multiple frequencies observed in YBa$_2$Cu$_3$O$_{6-x}$ are labelled $a$, $g$, $b$, where $a$ is the spectrally dominant frequency (detailed discussion in the text). In the phase diagram, ‘SC’ refers to superconductivity while ‘AFM’ refers to large-moment antiferromagnetism. (Online version in colour.)

is expected to remain fixed (or oscillate about a fixed number) so that, in the ground state, only the levels such that $\varepsilon_l \leq \varepsilon_F$ are filled—the occupancy being determined by equation (2.1) together with the degeneracy of each level. An increase in magnetic field increases the momentum-space area associated with each of the Landau levels and the separation of the levels, such that the $N$ particles are contained within fewer levels, and the higher Landau levels exit the Fermi surface. The $1/B$ periodicity at which filled Landau levels exit the Fermi surface yields oscillations in the density of states at the chemical potential, of frequency $F = \pi \hbar N/e = \hbar A/2\pi e$, where $A$ is the extremal area of the Fermi surface pocket in momentum space. The ratio of $F$ to $m^*$ appears in the Fermi energy $\varepsilon_F = e\hbar F/m^*$ in the simplest case.

(a) Orbital quantization

It is instructive first to consider the case of overdoped Tl$_2$Ba$_2$CuO$_{6+\delta}$ [26,33], which resides at hole doping $p \approx 0.30$ (relative to the half-filled antiferromagnetic insulator at $p = 0$) on the far right of the hole-doped region of the phase diagram in figure 2. The high-frequency oscillations with frequency $F \approx 18\ 100\ T$ correspond to a particle number $\approx 1.30 = 1 + p$ holes per layer, consistent with band-structure calculations and other experiments. A few decades of oscillations occur over the accessible range of magnetic fields, demonstrating periodicity in $1/B$. In the case of the underdoped systems, however, only about a decade (or fewer) of oscillations can be observed (figure 2): hole-doped YBa$_2$Cu$_3$O$_{6-x}$ [13,16–22] and YBa$_2$Cu$_4$O$_8$.
Figure 3. Plot of the quantum oscillation indices corresponding to the dominant ($F_a$) frequency (red denotes an oscillation maximum, blue denotes an oscillation minimum) as a function of inverse magnetic induction $1/B$ measured in YBa$_2$Cu$_3$O$_{6.54}$ demonstrating linear behaviour. The integral indices indicate the value of $1/B$ at which the highest occupied Landau level defined by $\varepsilon = \varepsilon_F$ corresponding to $l$ (equation (2.2)) exits the Fermi surface. Data from contactless conductivity quantum oscillations measured in Sebastian et al. [20,34] shown in §3 is used for constructing this plot. (Online version in colour.)

Increasing the experimental sensitivity and accessed magnetic field range in underdoped YBa$_2$Cu$_3$O$_{6+x}$ is crucial for a clear test of $1/B$ quantum oscillation periodicity. Samples of composition $x = 0.54$ have now been measured over a broad range of magnetic fields from 28 to 85 T [20,34]. A plot of the indices of the oscillations versus $1/B$ in figure 3 reveals a constant slope corresponding to a dominant frequency $F \approx 535$ T, thereby unambiguously confirming the periodicity in $1/B$ and enabling the useful application of Fourier methods in $1/B$.

(b) Landau quasi-particles

In order to address the question of Fermi–Dirac statistics governing the lowest-energy excitations in underdoped cuprates, Fourier methods are extended to the energy domain. On performing an inverse Fourier transform of the measured magneto-oscillation amplitude (with respect to the dimensionless parameter $\eta = 2\pi k_B T m^*/\hbar e B$) over a broad range of temperatures $100 \text{ mK} \lesssim T \lesssim 18 \text{ K}$, the extracted form of the probability distribution is found to closely follow that, $|f_{FD}(z)| = 1/2(1 + \cosh z)$, of the Fermi–Dirac distribution as shown in figure 4a,b [19]. The Fermi–Dirac behaviour of the low-energy quasi-particles justifies the use of the model temperature-dependent amplitude

$$a(T) = a_0 \frac{\pi \eta}{\sinh \pi \eta}$$

(2.3)
Figure 4. (a) Renormalized temperature-dependent amplitude $a(T)$ of the dominant ($F_a$) frequency magneto-oscillations (circles) measured in YBa$_2$Cu$_3$O$_{6+x}$ (where $x = 0.56$) using two different oscillatory techniques, plotted versus the dimensionless parameter $\eta = 2\pi k_B T m^*/\hbar e B$ (where $m^* = 1.676 \pm 0.001 m_e$ is the quasi-particle effective mass for this composition). The pink line is a linear interpolation. (b) Numerical Fourier transform of $a(\eta)$ in (a) (thick black line) corresponding to the experimentally determined probability distribution $|f'(z)|$, plotted over a restricted range of positive $z = (\epsilon - \epsilon_F)/k_B T$, with the Fermi–Dirac probability distribution $|f'_{FD}(z)|$ (superimposed thin green line) shown for comparison. Data from Sebastian et al. [19]. (Online version in colour.)

found in the Lifshitz–Kosevich theory to obtain estimates of the quasi-particle effective mass [32]. The close correspondence of the experimentally determined distribution with the Fermi–Dirac distribution in figure 4b implies that, at low temperatures and high fields, the lowest-energy excitations in YBa$_2$Cu$_3$O$_{6+x}$ ($x = 0.56$) behave as well-defined Landau quasi-particles, despite the unconventional high-energy behaviour of underdoped YBa$_2$Cu$_3$O$_{6+x}$ and its positioning well within the strongly correlated regime of phase space.

The observation of Landau Fermi-liquid physics in underdoped YBa$_2$Cu$_3$O$_{6+x}$ may also appear surprising given that the system is in the mixed superconducting state below the thermodynamic upper critical field $H_{c2}$; hysteresis in the magnetization indicates pinned vortices that coexist with a dynamical vortex liquid [17]. The appearance of Fermi–Dirac statistics (figure 4) suggests that the low-energy state of the vortex liquid being accessed in the regime where quantum oscillations are measured in underdoped YBa$_2$Cu$_3$O$_{6+x}$ can be described as a Fermi liquid [35,36]. While additional damping is expected to reduce the quantum oscillation amplitude within the mixed state, any further effect of the superconducting pair potential on the average density of states is not apparent within experimental resolution [37,38]. Quantum oscillations have also previously been reported in conventional type II superconductors in a similar limit as the current measurements on the underdoped cuprates ($\hbar \omega_c \ll \Delta$, where $\Delta$ is the superconducting pairing potential) with the same frequencies and effective masses as the normal state [39,40].
3. Quantum oscillation frequencies from small Fermi surface sections

Fourier transforms in $1/B$ are used to extract oscillation periods in underdoped YBa$_2$Cu$_3$O$_{6+x}$, given the origin of the observed magneto-oscillations as Landau quantization, and the applicability of Fermi–Dirac statistics (justifying the use of Fourier methods). Measured oscillations and Fourier transforms in underdoped YBa$_2$Cu$_3$O$_{6+x}$ over various field ranges between 28 and 85 T using the torque and contactless conductivity techniques are shown in figures 5 and 6. While the strongest amplitude peak ($F_\alpha \approx 530$ T) yields the indices plotted in figure 3, multiple other frequencies ($F_\beta \approx 1600$ T, $F_{\gamma,\text{neck}} \approx 460$ T, $F_{\gamma,\text{belly}} \approx 600$ T) and their harmonics are additionally observed in the Fourier transforms [18,34] (figures 5 and 6), and oscillatory fits yield multiple frequencies [24,34]. A spectrally dominant $F_\alpha$ pocket occupying 1.9 per cent of the Brillouin zone, a spectrally weaker $F_{\gamma,\text{neck}}$ pocket (with components $F_{\gamma,\text{neck}}$ and $F_{\gamma,\text{belly}}$) also occupying 1.9 per cent of the Brillouin zone, and a larger $F_\beta$ frequency approximately three times as large as the $F_\alpha$ frequency [17,20] are identified.

The small size of the Fermi surface pockets is in contrast to the expected large Fermi surface from paramagnetic band-structure calculations and seen in Tl$_2$Ba$_2$CuO$_{6+\delta}$ [33,41–43]. The origin of the small size of the Fermi surface pockets in the electronic structure is crucial in understanding the normal state of YBa$_2$Cu$_3$O$_{6+\delta}$.
underdoped $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$, Angle-dependent quantum oscillation measurements have been used to estimate the extent of corrugation, i.e. the interlayer hopping $t_\perp$ for each pocket (figure 7). Indications are for a very weakly corrugated ($t_\perp \lesssim 0.3$ meV) spectrally dominant $F_\alpha$ pocket [20,24].
Figure 7. Quantum oscillation frequencies (and harmonics where observed) measured in YBa$_2$Cu$_3$O$_{6.54}$: dependence on the angle $\theta$ between the orientation of the magnetic induction $\mathbf{B}$ and the crystalline $c$-axis, yielding three distinct sections of Fermi surface. Two of the sections (corresponding to the $F_a$ and $F_\beta$ frequencies) appear to be very close to two-dimensional in form (line simulates cosine dependence), indicating that they reside in a region of the Brillouin zone close to $\mathbf{k}_{\text{nodal}} = (\pi/2, \pi/2)$ where the interlayer dispersion $t_\perp$ is small [34]. The other section (corresponding to the $F_\gamma$ frequency) is proposed to be more corrugated by comparison (line simulates angular dependence of the Yamaji expression [44]). Data from Sebastian et al. [34]. More extensive angle-dependent measurements indicate that $F_\alpha$ and $F_\beta$ could also result from splitting. (Online version in colour.)

4. Possibilities for small Fermi surface pockets

Various possibilities have been proposed to explain the observation of small Fermi surface pockets.

— One possibility is that the observed magneto-oscillations arise not from fermionic quasi-particles as in the Landau Fermi liquid, but rather from novel types of quasi-particles. Among possibilities suggested for an unconventional origin of magneto-oscillations are spinless fermions [45,46], charge $2e$ bosons [47], low-energy Andreev-type bound states [48], oscillations originating from the vortex lattice [49,50,51], or Fermi points accompanied by a shift in chemical potential [52]. These possibilities, however, need to be examined for their agreement with Fermi–Dirac statistics observed by quantum oscillations and measured $1/B$ periodicity over an extended field range (discussed in §2), as well as the doping dependence of the frequency and effective mass (discussed in §3).

— A second possibility is that small Fermi surface pockets arise from within the electronic band structure without further broken symmetry. Small pockets of BaO–CuO character have been suggested due to CuO chains that provide a channel for BaO hybridization [53,54], and small Fermi surface sections arising from the chains themselves have been proposed [25]. The significantly higher disorder in the chains compared with the planes in underdoped YBa$_2$Cu$_3$O$_{6+x}$ [55], however, needs to be taken into account.
while treating the possibility of small pockets involving the chains in the electronic structure—making this possibility appear unlikely. No quantum oscillations have been observed in the isostructural, non-superconducting material PrBa$_2$Cu$_4$O$_8$, containing chains but no superconducting planes [56]. In addition, electronic band-structure calculations thus far yield a single small pocket only in the underdoped cuprate YBa$_2$Cu$_3$O$_{6+x}$.

Small Fermi surface pockets in the double-chain material YBa$_2$Cu$_4$O$_8$ and the chainless electron-doped material Nd$_{2-x}$Ce$_x$CuO$_4$ have also been observed by quantum oscillations. It is not clear that these observations can be explained by modified electronic structure calculations possibly involving shifts in the repulsive potential or a more complete treatment of correlations, which would be required to make this a viable possibility.

— Fermi surface sections of small area could arise as a consequence of strong correlations in proximity to the undoped Mott insulating regime. Two routes have been suggested—either a more conventional one involving translational symmetry breaking, or an unconventional one without any translational symmetry breaking, where Luttinger’s theorem is not respected. In the former case of translational symmetry breaking, a strong coupling order parameter such as a large-moment antiferromagnet or staggered flux state involving a significant reduction in the density of states would result in pockets at the nodal ($k_{\text{nodal}} = (\pi/2, \pi/2)$) locations in the Brillouin zone [57–60] (figure 8a). In the latter case, holes doped into a half-filled Mott insulator aggregate near the nodal ($k_{\text{nodal}} = (\pi/2, \pi/2)$) locations in the Brillouin zone, yielding a nodal density of states in the absence of translational symmetry breaking [61,62]. In this scenario, only one or two sets of pockets at the nodal location are expected. A better understanding is needed for the difference in warping and/or splitting between pockets in this case.

— Multiple Fermi surface pockets of small area with different corrugations can be created by the reconstruction of the large (paramagnetic) Fermi surface by a translational symmetry-breaking order parameter. This possibility is discussed further in §5.

5. Reconstruction of a ‘large’ Fermi surface

One way to obtain small Fermi surface sections is through Fermi surface reconstruction by an ordering wavevector: coupling between the electronic band $\epsilon_k$ describing the large Fermi surface and the same band translated by $Q$ (i.e. $\epsilon_{k+Q}$) yields two bands

$$\epsilon_{\pm} = \frac{\epsilon_k + \epsilon_{k+Q}}{2} \pm \sqrt{\left(\frac{\epsilon_k - \epsilon_{k+Q}}{2}\right)^2 + V^2},$$

where $V$ is the matrix element for scattering from state $k$ to $k + Q$. In the limit of large coupling $V$ and for a small number of doped carriers (i.e. $|p| \ll 1$), an indirect gap between $\epsilon_+$ and $\epsilon_-$ leads to a single type of pocket for $Q = (\pi, \pi)$. These pockets are of hole character for a hole-doped system, and occur at four
symmetry-related pockets near \( \mathbf{k}_{\text{nodal}} = (\pi/2, \pi/2) \), akin to the strong correlation translational symmetry-breaking scenario outlined earlier [45,52,57–61,65] (figure 8a). For smaller \( V \), both electron and hole pockets can be obtained (a schematic is shown in figure 8b). These are only two of many possible examples of pockets of different sizes, carrier types and locations in momentum space obtained depending on the size of the coupling \( V \) and the ordering wavevector \( \mathbf{Q} \) [36,63,64,66–75].

We consider quantum oscillation findings alongside photoemission results that are able to identify the spectral density in momentum space, and tight-binding calculations for guidance in locating the pockets in momentum space. The d-wave symmetry of interlayer hopping anticipated from band-structure calculations [20,41] suggests one possibility that the dominant \( F_a \) pocket corresponds to the nodal section, while the weaker-amplitude \( F_y \) pocket corresponds to the antinodal section of the Fermi surface [36]. On comparison with photoemission results, the dominant coherent density of states from photoemission is located at the nodes, consistent with the spectrally dominant quantum oscillation \( F_a \) frequency corresponding to the nodal locations. However, it is not yet clear that the suggested antinodal pockets can be reconciled with the gap at the Fermi energy observed by photoemission—alternatives are discussed in the penultimate section of this review.

Fermi surface reconstruction by a translational symmetry-breaking order parameter with weak enough coupling could result in both electron and hole pockets. Owing to symmetry reasons, a pocket at the antinodes \( \mathbf{k}_{\text{antinodal}} = (\pi, 0) \) would be associated with the electron pocket in such a translationally symmetry-broken scenario, with a pocket at the nodes \( \mathbf{k}_{\text{nodal}} = (\pi/2, \pi/2) \) corresponding to hole carriers (figure 8b). Were the dominant \( F_a \) pocket to be associated with
holes and the weaker-amplitude $F_\gamma$ pocket to be associated with electrons, the additional hole doping required to satisfy Luttinger’s theorem could be provided by associating the $F_\beta$ frequency [17,34] with a hole section of Fermi surface. This is an example of one of several possible interpretations, leaving open other possibilities.

The observation of particle–hole asymmetry from photoemission experiments could be consistent with the reconstruction of a large Fermi surface by a translational symmetry-breaking order parameter, only the nodal pockets of which have been observed [76,77]. Translational symmetry breaking has been detected by real-space-resolved techniques such as scanning tunnelling microscopy [78]. Forms of symmetry breaking are also indicated by polarized neutron diffraction measurements and anisotropic Nernst effect measurements [79,80]. A dominant contribution to the Hall resistivity from the electron pocket in this scenario could also explain the reported quantum oscillations in the negative Hall resistivity [27].

6. Further unexpected findings from quantum oscillations

Thus far, the observed quantum oscillations in the underdoped cuprates have revealed surprisingly small sections of Fermi surface. On careful investigation, more unconventional aspects of the measured quantum oscillations emerge.

(a) Steep mass enhancement at low dopings

It has been suggested that the presence of a universal zero-temperature continuous phase transition located at optimal doping could be a potential origin of the pseudo-gap phase and elevated superconducting temperatures. The observation of a dramatic change in Fermi surface topology from the large unreconstructed Fermi surface measured to the right of optimal doping ($\text{Tl}_2\text{Ba}_2\text{CuO}_{6+\delta}$) to small sections of reconstructed Fermi surface measured to the left of optimal doping ($\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ and $\text{YBa}_2\text{Cu}_4\text{O}_8$) consistent with a translational symmetry-broken phase suggests a continuous instability separating the two regimes (figure 2). In this case, a divergence in quasi-particle effective mass may be expected at optimal doping, characteristic of a divergent susceptibility. Alternatively, the physics of strong correlations has been suggested to be chiefly responsible for unconventional superconductivity. The spotlight on low-energy coherent quasi-particles provided by quantum oscillations is useful in further exploring these questions.

Quantum oscillations have been measured at multiple dopings to the left of optimal doping [20,21]. Unexpectedly, quantum oscillation measurements reveal a steeply enhanced effective mass under the superconducting dome at a low doping ($x \approx 0.46$) significantly less than optimal doping of $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$—shown in figure 9 [20]. The location of this steep mass enhancement is coincident with an earlier reported MI transition [4,85]. The threefold increase in effective mass over a very narrow doping range of the dominant $F_\alpha$ pocket and probably all parts of the Fermi surface suggests a putative MI QCP at a finite doping, which is unexpected given its anticipated occurrence at half-filling within the Mott insulating scenario [86]. Intriguingly, the location of this steep
Figure 9. Plot of the Fermi velocity ($v_F = \sqrt{2e\bar{h}F/m^*}$) averaged over the $F_a$ orbit on the left-hand axis as a function of hole doping ($p$) in underdoped YBa$_2$Cu$_3$O$_{6+x}$ shown in pink and blue symbols (pink measured in an 85T magnet [20]; blue measured in a 55T magnet [34]; and dark blue measured in an 85T magnet [21]). The corresponding inverse effective mass $m_e/m^*$ is shown on the right-hand axis. The top axis shows the corresponding oxygen concentration. In green is the temperature corresponding to the resistive upturn (from [4,81]), the red line denotes the superconducting transition temperature [82] (arbitrary units). Inset: plot of the nodal Fermi velocity as a function of hole doping in Bi$_{2201}$ (cyan from [83], purple from [84]) showing a collapse in $v_F$ at low dopings subsequently found in ARPES studies. (Online version in colour.)

enhancement in mass is under a local maximum (plateau) in the YBa$_2$Cu$_3$O$_{6+x}$ superconducting dome. An understanding of the origin of the observed effective mass enhancement may shed light on possible mechanisms contributing to the enhancement of superconducting temperatures; possible explanations are discussed in §7.

(b) Zeeman splitting at the Fermi surface

In the case of a conventional paramagnetic metal, the Landau levels are expected to experience the same Zeeman splitting $\pm \gamma \mu_B B$ as the unreconstructed bands $e_k$ [22], where $\gamma \approx 1$ is the effective moment (equivalent to the Wilson ratio in the absence of ordering). While this splitting does not depend significantly on the orientation of $B$, the cyclotron energy $\hbar\omega_c = \hbar eB/jm^*_0 \cos \theta$ depends on the component of magnetic induction perpendicular to the layers (here $\theta$ is the inclination of $B$ to the crystalline $c$-axis, $j$ is the harmonic index ($=1$ for the fundamental, 2 for the $2F$ harmonic)). For a finite value of $\gamma$, therefore, the change in the ratio of these energies with $\theta$ causes the relative phase between spin-up and spin-down Landau levels to rotate. The location of nodes in the experimental quantum oscillation amplitude occurring at ‘spin-zero’ angles $\theta_{j,\text{spin}}$ is given by

$$\cos \theta_{j,\text{spin}} = \frac{2\gamma j m^*_0}{2n + 1} \frac{m_e}{m^*}$$ (6.1)
where $n$ is an integer and $m_e$ is the free-electron mass [32]. On either side of such ‘spin-zero’) angles, the phase of quantum oscillations is expected to change by $\pi$, resulting in their inversion. The value of $\gamma$ may therefore be constrained by an experimental determination of the location of such spin-zero angles. We note that, from equation (6.1), it can be seen that the experimental location of a single spin-zero angle could correspond to an infinite series of integer values of $n$, while the identification of the lowest spin-zero angle would restrict these possibilities to a subset. However, in order to uniquely identify a single value of $n$ (and consequently of $\gamma$), the experimental location of at least two spin-zero angles is required (equation (6.1)).

The presence of such spin-zero angles would be revealed in scans of dopings and angles performed at high experimental resolution to discern the individual frequencies—the phase of the observed quantum oscillations would be expected to invert at the spin-zero angles in the measured quantum oscillations.

Results of a search for spin-zero angles is reported in work by Ramshaw et al. [24] and Sebastian et al. [22]. Measurements in Ramshaw et al. access fundamental quantum oscillations at discrete angles in the $c$-axis transport and report a phase inversion between $51^\circ$ and $57^\circ$ in the measured oscillations, from which the argument is made by the authors for free spins (i.e. no suppression of the spin moment and $\gamma \approx 1$). Measurements in Sebastian et al. [22,34] access quantum oscillations at discrete and continuous angles in contactless conductivity, and report an absence of phase inversion up to accessed angles (continuous rotation up to $52^\circ$, shown in figure 10), from which the authors infer a suppression of the spin moment (i.e. $\gamma < 1$). Since only a few oscillations are observed at high angles of $50^\circ$ or more, further experiments will be useful in shedding light on this issue. These findings have been reconciled in a recent rotation study (Sebastian et al. 2011, unpublished data), where a phase inversion observed in the measured quantum oscillations at $\approx 56^\circ$ in low magnetic fields is found to shift down to $\approx 52^\circ$ in high magnetic fields. Unexpectedly, however, despite this observed phase inversion in the fundamental oscillations, the phase of the measured $2F$ harmonic oscillations (figure 10) remains uninverted up to $\approx 50^\circ$ [34]. These findings are treated in a more recent study (Sebastian et al. 2011, unpublished data).

7. Implications of steep effective mass enhancement and possible translational symmetry breaking

(a) Origin of low-energy Fermi surface instability at low dopings

Potential explanations for the steep enhancement of quasi-particle effective mass at low dopings include those that either invoke or do not invoke a QCP. Explanations which have been suggested include a topological Lifshitz transition [87]; a superlattice instability [20] perhaps associated with an excitonic insulator transition [34]; and the collapse of a coherence temperature (M. Norman 2010, private communication).

Interestingly, the location of the steep mass enhancement observed in underdoped YBa$_2$Cu$_3$O$_{6+x}$ coincides with a local maximum in superconducting temperature, perhaps mirroring the putative QCP at optimal doping. Experimental evidence has been lacking thus far for a divergent susceptibility
characteristic of a proposed QCP under the superconducting dome [88–94], in part due to the robust superconductivity that obscures any such signatures [95]. While an array of thermodynamic signatures from experiments such as heat capacity, muon spin resonance, neutron scattering and others show a discontinuity above the superconducting dome [96–99], only a few hints of such a second-order phase transition under the dome have been indicated by the polar Kerr effect in YBa$_2$Cu$_3$O$_{6+x}$ in the vicinity of optimal doping [100].

A possible scenario is a secondary symmetry-breaking transition located at the MI QCP that destroys the Fermi surface [20,34], subsequent to a symmetry-breaking transition at optimal doping that reconstructs the Fermi surface. The second symmetry-breaking transition potentially drives the insulating transition, while coinciding with a local enhancement of superconducting transition temperature (schematic shown in figure 11). Quantum oscillation measurements could potentially detect a divergent effective mass characteristic of a low-energy continuous phase transition on using high magnetic fields to suppress superconductivity.
Figure 11. Schematic of the superconducting dome, possibly comprising two subdomes. Each of the subdomes is potentially centred on a continuous Fermi surface instability; the transition from large Fermi surface to small Fermi surface sections is labelled as QCP₁, and the transition from small Fermi surface sections to the absence of a Fermi surface is labelled as QCP₂. (Online version in colour.)

The observed mass enhancement of low-energy quasi-particles serves as a further meeting point for two complementary techniques: quantum oscillations and ARPES. Recently, a similar mass enhancement (i.e. collapse in the Fermi velocity \(v_F\)) at the nodal region for a similar doping level has also been observed by photoemission in Bi2201 to occur solely at low energies in the vicinity of \(\varepsilon_F\)—shown in figure 9 inset [83,84]. The previously discussed identification of the Fermi surface pockets in momentum space (§3) is further supported by this observation that the \(F_a\) (nodal) pocket observed by quantum oscillations and the nodal spectral weight observed by ARPES display similar effective mass enhancements (i.e. collapse in \(v_F\)) in the vicinity of \(p \approx 0.09\) hole doping.

(b) Possibilities for translational symmetry breaking

An incommensurate triplet order parameter is suggested at low dopings in YBa₂Cu₃Oₓ₆₊ₓ \((p \leq 0.085)\) from neutron scattering experiments that measure spin correlations at wavevectors \(Q_s \approx (\pi \pm \frac{\pi}{4}, \pi)\) (and by symmetry \(Q_s \approx (\pi, \pi \pm \frac{\pi}{4})\)) [101–106]. This could imply the existence of charge correlations corresponding to four wavevectors \(Q_c = 2Q_s\), which translated into the first Brillouin zone yield \((\pm \frac{\pi}{2}, 0)\) (and by symmetry \((0, \pm \frac{\pi}{2})\)).

However, density-wave order driving translational symmetry breaking appears to be strangely elusive in the region where quantum oscillations are observed in underdoped YBa₂Cu₃O₆₊ₓ \((0.09 \leq p \leq 0.12)\) [23]. Neutron scattering or X-ray diffraction evidence for broken translational symmetry in samples of the same composition where quantum oscillations are observed is lacking, except when doped with a non-magnetic impurity [105]. One possibility suggested is that antiferromagnetism appears only where superconductivity is suppressed in the vortex cores [107,108], becoming recognizable as a traditional antiferromagnetic phase only when superconductivity is suppressed by strong magnetic fields (causing the increased ‘halos’ of extended states around the vortex cores).
to overlap [109]) or non-magnetic impurities [61,88]. A related possibility is the growth in antiferromagnetic correlation length with field, with a low-field glassy phase evolving to a high-field broken-symmetry phase [65]. In these scenarios, it could be argued that neutron scattering and X-ray diffraction experiments have thus far been performed in magnetic fields too weak to suppress superconductivity significantly, although the non-magnetic impurity Zn is more effective at suppressing superconductivity. Magnetic field-dependent neutron scattering experiments find evidence of field dependence in samples YBa2Cu3O6+x of composition x = 0.45, where a low-magnetic-field nematic phase (or glassy phase exhibiting twofold symmetry) is found to evolve progressively into a phase closely resembling an incommensurate spin-density wave in fields of about 15T [101,104], and static magnetic order has been found in YBa2Cu3O6.6 doped with 2 per cent Zn [105]. Other possibilities suggested for translational symmetry breaking include dynamic forms of magnetism [110–112], a d-density wave [63–67] and a charge-density wave, among others.

8. Quantum oscillations and complementary photoemission and optical conductivity measurements

Given the relatively recent discovery of quantum oscillations in the cuprates against a backdrop of two decades of research involving a battery of other experiments, it is crucial to relate the findings from quantum oscillations to those from other techniques. We focus on an aspect of complementary experiments that has been thought to be particularly challenging to reconcile with results of quantum oscillation experiments, and examine to what extent these can be explained within a common framework. An antinodal energy gap (known as a pseudo-gap) is indicated directly by ARPES experiments that do not observe coherent antinodal quasi-particle states at the Fermi energy [6,113–115], and less directly by optical conductivity experiments [11,116].

It has been suggested from the observation of quantum oscillations in a state characterized by a negative Hall coefficient that the associated Fermi surface pocket is an electron pocket, located at the antinode for symmetry reasons [27]. Furthermore, the observation of multiple frequencies has been interpreted as corresponding to different sections of the Fermi surface [17], and the association of different warpings with each of the Fermi surface sections interpreted as pockets of different carriers located in different regions of momentum space [19]. While it is important to consider options for the observation of antinodal pockets in quantum oscillation measurements and not in photoemission experiments, an explanation other than antinodal pockets to explain these observations cannot be ruled out.

With regard to comparison with optical conductivity experiments, we compare the measured interlayer hopping of the Fp pocket proposed to be more corrugated, to the scattering rate (1/τ) [20] from measured quantum oscillations. A value of scattering $\hbar/\tau \approx 2.3 \text{meV}$, similar to the value of the interlayer hopping $t\perp \approx 2.4 \text{meV}$, implies that interlayer tunnelling of quasi-particles along the corrugated section of the Fermi surface is on the border of being coherent. Given this marginally coherent behaviour of antinodal quasi-particles, it may not be too surprising that a Drude peak is unobserved in c-axis optical conductivity in underdoped cuprates [116].
We consider the possibility that antinodal pockets suggested by quantum oscillation measurements in contrast to an antinodal gap observed by photoemission experiments could be related to the different regimes in which ARPES ($B = 0, T \gtrapprox T_c$) and quantum oscillations ($B \gg T_c$) experiments are performed. The large antinodal energy gap (approx. 50 meV) observed by ARPES is unlikely to be closed by the modest applied magnetic fields ($\gtrapprox 25$ T) in which quantum oscillation experiments are performed. However, it is possible that coherent spectral weight (i.e. low-energy excitations) in the vicinity of the antinodes may be resurrected at low temperatures and applied magnetic fields despite the existence of this gap. Outlined below are two such proposed scenarios for the destruction of coherent antinodal quasi-particles in ARPES; and their recovery under conditions at which quantum oscillation experiments are performed (schematic shown in figure 12).

— In the first scenario, the absence of coherent antinodal spectral weight is attributed to a zero-momentum instability involving incoherent superconducting pair formation preceding the d-wave superconducting gap. In this scenario, in the regime of applied magnetic field and low temperatures below the coherence temperature in which quantum oscillation experiments are performed, the evolution of low-energy vortex core states into Landau levels is argued to resurrect the spectral weight within the antinodal gap present in zero field [35,117].

— In the second scenario, the antinodal spectral weight is destroyed by a finite-moment density-wave order parameter as suggested in [34,118,77]. In this scenario, it has been proposed [119] that strengthening the
spin-density wave order parameter responsible for the pseudo-gap enhances the low-energy coherent excitations, such that the antinodal region evolves into pockets observed by quantum oscillations in an applied magnetic field.

While other proposals involve novel quasi-particles that constitute the pockets observed in quantum oscillations, yet yield arcs in ARPES \cite{48,120}, these need to be viewed in the light of the $1/B$ periodicity and Fermi–Dirac statistics displayed by quantum oscillations, indicating orbitally quantized Landau quasi-particles.

9. Summary

The recent quantum oscillation experiments demonstrate the orbital quantization of low-energy Landau quasi-particles in underdoped cuprates, bringing the familiar Fermi-liquid paradigm back into relevance in our understanding of these ultra-strongly correlated systems. While high-frequency quantum oscillations reveal a single large Fermi surface in the overdoped cuprates, small Fermi surface sections are revealed in the underdoped cuprates. The transition in electronic structure indicated between these two regimes is followed by a subsequent transition at lower dopings at which quantum oscillations and hence the Fermi surface appear to suddenly vanish (schematic shown in figure 11). With the familiar comes the unfamiliar—quantum oscillations reveal a steep enhancement in the quasi-particle effective mass at the second of the Fermi surface transitions (QCP$_2$) at which the Fermi surface is destroyed, indicating a continuous instability; yet comparatively little is known about the evolution from large to small Fermi surface at the first (QCP$_1$ at higher doping) transition. Although a relatively simple explanation of the small Fermi surface sections is Fermi surface reconstruction by translational symmetry breaking, evidence for long-range order has not yet been observed in the regime where quantum oscillation measurements are performed. A more radical possibility would be the creation of small Fermi surface sections without translational symmetry breaking, potentially involving unconventional quasi-particles. A pressing question raised by quantum oscillation experiments in the underdoped cuprates then remains: whither translational symmetry breaking?


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