Comparing measurements of redshift-space distortions (RSDs) with geometrical observations of the expansion of the Universe offers tremendous potential for testing general relativity on very large scales. The basic linear theory of RSDs in the distant-observer limit has been known for 25 years and the effect has been conclusively observed in numerous galaxy surveys. The next generation of galaxy survey will observe many millions of galaxies over volumes of many tens of Gpc$^3$. They will provide RSD measurements of such exquisite precision that we will have to carefully analyse and correct for many systematic deviations from this simple picture in order to fully exploit the statistical precision obtained. We review RSD theory and show how ubiquitous RSDs actually are, and then consider a number of potential systematic effects, shamelessly highlighting recent work in which we have been involved. This review ends by looking ahead to the future surveys that will make the next generation of RSD measurements.

Keywords: cosmology; cosmology observations; large-scale structure of Universe

1. Linear redshift-space distortions

The rate at which cosmological structure grows provides significant evidence to discriminate between cosmological models. In particular, dark energy models in which general relativity (GR) is unmodified predict formation rates that are different when compared with modified gravity models with the same background expansion [1].

The overdensity field evolves through the motion of material within a co-moving frame, and galaxies act as test particles following this peculiar velocity field. Galaxy redshifts depend on the relative velocity of galaxies with respect to the observer, and so include both the Hubble recession and the peculiar velocity. Consequently, if only the Hubble recession is assumed when converting from redshift to distance, then we recover a distorted field, with radial redshift-space distortions (RSDs) caused by coherent co-moving flows.

If we follow the ‘plane-parallel’ approximation that observed galaxies are sufficiently far away that their separations are small when compared with the distances between them, then assume that the linearized continuity equation

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holds and that we have scale-independent growth as predicted by GR then, to linear order [2,3],
\[ P_{gg}(k, \mu) = P_{mm}(k)(b_5 + b_v f \mu^2)^2, \]  
(1.1)
where \( b_5 \) accounts for a linear deterministic bias between galaxy and matter overdensity fields, \( f \) is the logarithmic derivative of the growth factor by the scale factor \( (f \equiv \frac{d \ln G}{d \ln a}) \) and \( \mu \) is the cosine of the angle to the line-of-sight.

RSDs lead to the \( \mu^2 \)-dependent term in the relationship between the redshift-space galaxy power spectrum \( P_{gg} \) and the real-space matter power spectrum \( P_{mm} \), \( b_v \), which is normally neglected, allows for a linear bias between galaxy and matter velocity distributions: while, locally, galaxies must move with the matter flow, it is not true that the distribution of galaxy velocities in any particular sample has to match that of the matter. In the following, we set \( b_v = 1 \) following this convention, although it should be noted that this is an active area of study.

Equation (1.1) shows that the additive component owing to RSD depends only on cosmological quantities, namely the redshift-dependent growth rate of cosmological structure and the normalization of matter overdensity fluctuations at some epoch. The parameter combination \( f(z) \sigma_8(z) \) has been shown to be a good discriminant between modified gravity models [4]. The first three even Legendre moments of \( P_{gg}(k, \mu) \) contain all of the information in equation (1.1), with both the quadrupole and hexadecapole containing information about fluctuations in the velocity field. Even using just the monopole \( P_0^s \) and quadrupole \( P_2^s \), we can write down a combination that results in a bias-independent function that has the same expected large-scale amplitude as \( P_{qq}^r(k) \) demonstrating how RSD measurements are independent of overdensity bias [5]:
\[ \hat{E}(k) = \frac{7}{48} \left[ 5(7P_0^s + P_2^s) - \sqrt{35}[35(P_0^s)^2 + 10P_0^s P_2^s - 7(P_2^s)^2]^{1/2} \right]. \]  
(1.2)

If we drop the assumptions of scale-independent growth and continuity, then we must rewrite equation (1.1) in terms of the real-space galaxy–galaxy \( P_{gg}^r(k) \), galaxy–velocity divergence cross-power \( P_{g\theta}^r(k) \) and the velocity divergence auto-power spectrum \( P_{\theta\theta}^r(k) \):
\[ P_{gg}^s(k) = P_{gg}^r(k) - 2\mu^2 P_{g\theta}^r(k) + \mu^4 P_{\theta\theta}^r(k), \]  
(1.3)
where \( \theta_g = \nabla \cdot u \) is a divergence of the galaxy velocity field. This form is better able to cope with nonlinear evolution, which breaks the assumption of scale-independent growth [6,7].

\section{2. Fingers-of-God}
On small scales, the dominant nonlinear contribution to the RSD signal is due to the virialized motion of galaxies within dark matter halos. These velocities can reach amplitudes such that, when misinterpreted as being owing to the Hubble flow, the clusters are stretched along cosmologically important distance along the line-of-sight in apparent ‘Fingers-of-God’ (FOG). This effect can be approximated by including an extra term in equation (1.1) that reduces power on small scales [8],

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and popular functions for this damping term are
\[ F_{\text{exponential}}(k, \mu^2) = \frac{1 + (k\sigma_v \mu)^2}{1} \] (2.1)
and
\[ F_{\text{Gaussian}}(k, \mu^2) = \exp[-(k\sigma_v \mu)^2]. \] (2.2)
Alternatively, one can use a more complicated expression based on higher order computations in perturbation theory [6,9]. Unfortunately, the phenomenological damping terms used to describe the FOG effects are not accurate [6,7] and the results of perturbation theory are not easy to implement in a computationally fast and efficient way.

3. Degeneracy between redshift-space distortions and Alcock–Pacyznski

Redshift surveys measure the angular positions of galaxies and their redshifts, and these need to be translated to co-moving coordinates before we can measure clustering and compare directly with equation (1.1). If the wrong cosmological model is used to make this transformation then we can induce anisotropic distortions in the observed map [10], which are similar to the RSD signal [11].

Suppose that the angular and radial distances in our assumed model are different from real distances by the factors \( a_\parallel = R_r / \hat{R}_r \) and \( a_\perp = R_A / \hat{R}_A \), where quantities without a hat are computed in an assumed model and a hat denotes true value. In this situation, equation (1.1) becomes
\[ P_{gg}(k, \mu) = \frac{1}{a_\parallel a_\perp^2} P_{mm}(k, \mu) \left( \frac{k}{\alpha_\perp} \sqrt{1 + \mu^2(A^{-2} - 1)} \right) \left( b + \frac{\mu^2 f}{A^2 + \mu^2(1 - A^2)} \right)^2, \] (3.1)
where \( \mu = k_\parallel / k \) and \( A = a_\parallel / a_\perp \) (for details see [11,12]). The galaxy power spectrum is scaled by the additional factor \( \Delta V = a_\perp^{-2} a_\parallel^{-1} \) because the reference cosmology under(over)estimates the survey volume by a factor of \( \Delta V \).

In fact the degeneracy is not severe, and even the weak assumption that the Universe follows a Friedman–Robertson–Walker metric couples \( R_A \) and \( R_r \) sufficiently to break much of the degeneracy [13].

4. Effect of redshift-space distortions on angular clustering

Although RSDs affect only inferred distances and not angles, they still distort the projected angular clustering of galaxy samples selected using redshift-dependent quantities. An edge to a window function (or a contour of constant galaxy density) that is straight in redshift-space is systematically distorted in real-space. In general, clusters of galaxies within a sample will tend to ‘pull in’ galaxies as their velocities tend to be aligned towards the sample. Similarly, voids tend to ‘push out’ galaxies, so we see that both projected positive and negative overdensities have their amplitudes increased.

The gain in clustering strength gives an increase in the number of galaxy pairs when compared with that expected, which is how the correlation function is usually measured [14]. Total pair conservation shows that RSDs move pairs from the cross-correlations into the auto-correlations. We can therefore imagine...
creating an estimator for the angular clustering signal that is independent of RSD by re-binning. Nock et al. [15] showed using symmetry arguments that binning based on the centres of galaxy pairs, rather than on redshift-space galaxy positions, is sufficient for this.

Ross et al. [16] took this argument one stage further, showing that surveys such as the Dark Energy Survey (DES; see §8) will be able to make interesting measurements of the RSD signal using only angular clustering measurements of samples selected based on photometric redshifts.

5. Complementarity with integrated Sachs–Wolfe effect

The integrated Sachs–Wolfe (ISW) effect [17] measures changes in the gravitational potential by observing fluctuations in the energy of cosmic microwave background (CMB) photons owing to the large-scale structure through which they have passed. If $\dot{\Phi} - \dot{\Psi} < 0$ as a consequence of dark energy, then the photons gain (lose) energy depending on whether they pass through an overdensity (underdensity), and this signal can be detected by cross-correlating large-scale structure with the CMB [18]. RSD will play a part in these cross-correlation measurements [19] as, for narrow redshift-selected bins, they act as an effective bias boosting the apparent angular clustering of the large-scale structure (see §4). This is not true for wide bins in redshift, where RSD can reduce the ISW signal [20].

Under GR, the ISW effect and observations of RSD probe similar information: the ISW effect depends on a projection of $G(z)[1 - f(z)]$. For models with anisotropic stress, observations of RSD are complementary to ISW observations as they only depend on time-like metric fluctuations $\Psi$. In GR, RSD and ISW observations covering the same large-scale structure volume are not correlated to a significant degree because the RSD predominantly depends on radial modes, while the ISW measurements depend on uncorrelated angular modes [20]. There is additional complementarity in which the power of the ISW effect lies on very large scales $k \ll 0.01 \, h \, \text{Mpc}^{-1}$, while the RSD signal lies on smaller scales.

6. Wide-angle redshift-space distortion effects

When a galaxy pair has a separation that is comparable with their distances, the plane-parallel approximation fails and equation (1.1) breaks down. Because of isotropy about the observer, the RSD signal for any galaxy pair depends on three variables, which can be taken to be the separation $r$, the angle that a galaxy pair forms with respect to the observer $\alpha$, and the cosine that the galaxy separation vector makes with the angle bisector (which we still denote as $\mu$ following plane-parallel theory). The wide-angle equivalent of equation (1.1) has been calculated [21–25], and has been shown not to deviate significantly from its ‘plane-parallel’ counterpart if the opening angle is less than $10^\circ$.

There are a number of effects that fall under the ‘wide-angle’ umbrella.

— Geometrical effects arise because of the triangle formed by the galaxies and the observer means that the standard RSD motions are no longer parallel.
Mode confusion arises owing to the coherent deviation between true and observed galaxy densities: for non-plane parallel RSD, coherent distortions in the radial density lead to appreciable effects when Fourier modes no longer evolve independently.

Sample effects occur when we consider the pair distribution covered by an actual galaxy survey. This introduces a weighting function in $\mu$ that needs to be applied before the observed clustering can be modelled—the standard assumption is that pairs are isotropically distributed, giving $\mu$ a uniform distribution $0 < \mu < 1$, will not hold in practice.

These three effects have been recently demonstrated using numerical simulations [26]. The effect of the sample distribution in $\mu$ is the most significant of these when applied to data [27].

7. Towards a robust analysis

We have seen in the preceding sections how there are a number of complications that apply to the basic model for RSD given by equation (1.1). The relative importance of these effects was recently analysed by Samushia et al. [27], who also considered the effects of nonlinear clustering in the matter density field, and how well we can model the expected radial galaxy density. Clearly, this is a scale-dependent issue, but for scales greater than $30h^{-1}$ Mpc, where the RSD signal is strongest, the nonlinear clustering effect is the strongest systematic, particularly around the baryon acoustic oscillation (BAO) scale. In the order of decreasing importance comes the survey geometry component of wide-angle effects, the FOG and the geometrical wide-angle effects. The radial galaxy density model is important, but its full impact was not assessed in this work. The Alcock–Paczyński effect is unimportant when considering only lambda cold dark matter ($\Lambda$CDM) background models, and including Wilkinson microwave anisotropy probe (WMAP) priors, but can be more important depending on the cosmological model to be fitted to the data.

As future surveys give increasingly accurate cosmological measurements, we need to be increasingly concerned that our treatment of the likelihood calculation itself is robust. Measurements of the correlations function become non-Gaussian as the expected number of pairs decreases, potentially leading to biases and incorrectly derived errors. However, the multi-pole moments of the correlation function are significantly better behaved [27], and provide more robust constraints. Equation (1.1) shows that the first three even multi-pole moments of the power spectrum contain all of the information in the linear power spectrum, and the same holds for the correlation function [3]. There is therefore a strong argument for also considering these multi-pole moments when including the perturbations to equation (1.1) suggested above. Note that there is evidence that all three moments should be included in any analysis [28].

Applying this analysis to measure $f(z)s_8(z)$ from the Sloan Digital Sky Survey (SDSS-II) luminous red galaxy (LRG) data, gives $f(z = 0.25)s_8(z = 0.25) = 0.3930 \pm 0.0457$ and $f(z = 0.37)s_8(z = 0.37) = 0.4328 \pm 0.0370$ when no prior is imposed on the growth rate, and the background geometry is assumed to follow a $\Lambda$CDM model with the WMAP + SN1a priors [27]. The standard
WMAP-constrained ΛCDM model with GR predicts $f(z = 0.25)\sigma_8(z = 0.25) = 0.4260 \pm 0.0141$ and $f(z = 0.37)\sigma_8(z = 0.37) = 0.4367 \pm 0.0136$, which is fully consistent with these measurements.

8. Predictions for future surveys

There are a number of ongoing galaxy surveys that aim to make interesting cosmological measurements from RSD, including the following.

(a) WiggleZ

WiggleZ [29] will use the AAΩ spectrograph on the 3.9 m Anglo-Australian telescope to measure redshifts for 240 000 emission-line galaxies over the redshift range $0.1 < z < 1.0$, with a median at $z = 0.6$. The sky area is approximately 1000 deg$^2$, and the target density is 350 deg$^{-2}$. Although the aim of the survey is to measure the scale of BAOs imprinted on the spatial distribution of these galaxies it has already made interesting RSD measurements [30]. The WiggleZ team has recently published RSD constraints from their survey [31].

(b) Baryon Oscillation Spectroscopic Survey

The Baryon Oscillation Spectroscopic Survey (BOSS) [32] is part of the SDSS-III project [33], and will observe 1 500 000 massive galaxies over 10 000 deg$^2$. Most of the galaxies observed are LRGs, but the high redshift colour cuts used were designed to apply a constant stellar-mass threshold; so there is a fraction of massive blue galaxies present. The target selection algorithm has been chosen to give an approximately constant number density for $0.05 < z < 0.6$. The project is 1 year into its 5 year timescale.

(c) VIMOS Public Extragalactic Redshift Survey

The VIMOS Public Extragalactic Redshift Survey (VIPERS) is an ongoing ESO large programme using VIMOS at the very large telescope to measure 100 000 redshifts for galaxies with red magnitude $I(AB)$ brighter than 22.5 over an area of 24 deg$^2$. Colour–colour pre-selection means that the survey focuses on $0.5 < z < 1.2$. VIPERS currently has approximately 30 000 redshifts, and is benefiting from a recent upgrade to the VIMOS instrument.

(d) Dark Energy Survey

The DES is building a 570 megapixel digital camera, DECam, to be mounted on the Blanco 4 m telescope at Cerro Tololo Inter-American Observatory. Starting in late 2011 and continuing for 5 years, DES will survey 5000 deg$^2$, reaching approximately 24th magnitude in SDSS $g$, $r$, $i$ and $z$ filters. This will be supplemented by $J$, $H$ and $K$ images from the ESO VISTA Hemisphere Survey. DES will rely on inferring galaxy distances from photometric redshifts, leading to large radial errors in galaxy positions. The RSD signal can still be measured, and can provide competitive $z \sim 1$ measurements of the growth rate [16].

1Higher than the 240 deg$^{-2}$ galaxies expected because of the redshift completeness.
(e) Proposed ground-based surveys

There is a clear ‘gap-in-the-market’ for high multiplex next-generation multi-object spectrographs (MOS) on 4 or 8 m telescopes, capable of taking thousands of spectra simultaneously of a field-of-view of many square-degrees. Such an instrument could undertake a survey of approximately $10^7$ $0.5 < z < 1.4$ galaxies over $5000–15\,000\,\text{deg}^2$. One of the most advanced proposals for such an instrument and survey is the BigBOSS concept [34], which recently submitted a proposal to the NOAO to build such an instrument for the 4 m Mayall telescope at KPNO. Similar instruments have been proposed for the William Herschel (WEAVE), VISTA (VXMS) and Blanco telescopes (DESpec). There have also been projects to build a wide-field MOS for 8 m class telescopes including HETDEX and the SuMIRE project for Subaru. In addition, Subaru already has a 400 fibre infrared spectrograph FMOS, which would be capable of undertaking a high redshift RSD-based survey.

(f) Proposed space-based surveys

As galaxy surveys increase in size, the cosmological measurements will become increasingly dependent on understanding observational systematics such as the seeing, sky brightness, extinction and scatter in target selection algorithms. Additionally, ground-based surveys are limited to approximately $15\,000\,\text{deg}^2$ unless they can either build two instruments, or move one instrument to telescopes in different hemispheres, and are limited to $z \lesssim 1.5$. Consequently, there is a strong case for a space-based satellite mission to robustly measure redshifts over approximately $15\,000\,\text{deg}^2$ out to $z \simeq 2$. The latest in a long line of space-based satellite mission concepts designed to measure dark energy is Euclid [35], an ESA Cosmic Visions satellite proposal, which includes a slit-less spectrograph capable of measuring spectra for more than $50\,000\,000$ galaxies over $15\,000\,\text{deg}^2$.

(g) Comparison of redshift-space distortion capabilities

In order to compare the RSD measurements that we can expect from the surveys described above, we use the Fisher matrix formalism of White et al. [36]. This determines the cosmic variance and shot noise errors expected for the anisotropic power spectrum given the volume and number density of galaxies in the plane-parallel approximation (equation (1.1)), and translates these errors through to the RSD signal. We consider BigBOSS as the standard for surveys that would be possible using a next-generation MOS instrument on a 4 m class telescope, and HETDEX for 8 m class projects. We compare against predictions for the 2dFGRS, SDSS-II main galaxy and LRG samples: in order to make the comparison robust, we apply the Fisher matrix forecasting technique even for these completed surveys where measurements have already been made. The relative size of errors expected for a flat concordance $\Lambda$CDM model is given in figure 1, which shows that there is a clear hierarchy of surveys, starting with current measurements and ending with the measurements possible using a space-based survey.
Figure 1. Fisher matrix predictions for $f \sigma_8$ measurement from RSD in various past, current and proposed surveys. Points that are assumed to be independent correspond to redshift bins. See text for details. (Online version in colour.)

9. Conclusions

RSDs have to be considered in any analysis that uses galaxy redshifts to make cosmological measurements. This is true for the ISW effect and angular clustering as well as more standard three-dimensional clustering measurements. As well as being a ‘contaminant’ for other observations, RSD offers tremendous potential for understanding cosmological structure growth. For models with anisotropic stress, RSD measurements, which only depend on time-like fluctuations, are complementary to measurements made through the ISW or weak lensing effects, as these depend on both time-like and space-like metric fluctuations. Assuming that GR holds on all scales, RSD makes competitive measurements of structure growth, which depends on cosmological expansion. Figure 1 shows that the potential offered by RSD will be realized by future surveys.

As with any observational technique there are issues that could turn into systematic errors if they are not corrected, and we have tried to review some of the major ones in this article. Clearly, in addition to the effort being put into future surveys, we need further theoretical support to ensure that the systematic effects are understood at a level where they do not impact on the measurements being made. Given recent progress, we should be optimistic that this will be achieved in time for the next generation of surveys.

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