The watt balance: determination of the Planck constant and redefinition of the kilogram

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Since 1889, the international prototype of the kilogram has served as the definition of the unit of mass in the International System of Units (SI). It is the last material artefact to define a base unit of the SI, and it influences several other base units. This situation is no longer acceptable in a time of ever-increasing measurement precision. It is therefore planned to redefine the unit of mass by fixing the numerical value of the Planck constant. At the same time three other base units, the ampere, the kelvin and the mole, will be redefined. As a first step, the kilogram redefinition requires a highly accurate determination of the Planck constant in the present SI system, with a relative uncertainty of the order of 1 part in 10^8. The most promising experiment for this purpose, and for the future realization of the kilogram, is the watt balance. It compares mechanical and electrical power and makes use of two macroscopic quantum effects, thus creating a relationship between a macroscopic mass and the Planck constant. In this paper, the operating principle of watt balance experiments is explained and the existing experiments are reviewed. An overview is given of all available experimental determinations of the Planck constant, and it is shown that further investigation is needed before the redefinition of the kilogram can take place. Independent of this requirement, a consensus has been reached on the form that future definitions of the SI base units will take.

Keywords: International System of Units; redefinition; international prototype; kilogram; watt balance; Planck constant

1. Introduction

The International System of Units (SI) is the most widely used system of units for measurements in commerce and science. The SI is based on seven base units (metre, kilogram, second, ampere, kelvin, mole and candela) from which other units are derived [1]. The SI was officially adopted by the General Conference on Weights and Measures (CGPM) in 1960, but has its origins in the Metre Convention of 1875.

The kilogram, the unit of mass, is the last base unit to be defined by a man-made object, the international prototype of the kilogram. This is a cylinder made of an alloy of 90 per cent platinum and 10 per cent iridium, cast in 1879 by

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Johnson Matthey, and kept at the International Bureau for Weights and Measures (BIPM) since then. It was adopted as the international prototype of the kilogram in 1889 during the first meeting of the General Conference and still serves today to define the kilogram.

Every measurement in the world expressed using the kilogram unit is ultimately traceable to the international prototype of the kilogram at the BIPM. Most Member States of the Metre Convention hold national prototypes which are compared from time to time against the working standards of the BIPM, which are traceable to the international prototype. Although this system has worked quite well until now, and ensures uniform mass measurements throughout the world, unit definitions that can be realized anywhere are preferable. The international prototype as a material object could also be damaged, with obvious negative consequences for mass metrology.

Three comparisons were carried out between the international prototype and the national prototypes, one in the 1880s, one in 1946 and one in 1989, and they show a trend towards larger mass values for most of the prototypes with respect to the international prototype, of approximately 50 µg over 100 years [2]. In relative terms, this corresponds to a change of 5 parts in 10^8 over 100 years. This observation might be interpreted as an indication that the international prototype loses mass. However, there is no clear explanation of the situation, because during the last century a more stable mass reference did not exist. It cannot be excluded that all prototypes show a common drift in addition to this relative drift, which cannot be detected by comparisons between the prototypes and which at present is completely unknown.

The definitions of several other base units depend on the kilogram; this is the case for the ampere, the mole and the candela. Typical measurement uncertainties in chemistry and photometry are such that a possible slight drift of the kilogram, and consequently of the mole and the candela, would go unnoticed. Practical electrical metrology has since 1990 been based on the use of the Josephson effect and the quantum Hall effect, together with conventional values of the Josephson constant, K_J-90, and the von Klitzing constant, R_K-90 [3]. Conventional values have been chosen, because the reproducibility of both effects is better than the knowledge of the Josephson constant K_J and von Klitzing constant R_K in SI units. The use of conventional values allows us to benefit from the very high reproducibility of the Josephson effect (parts in 10^{10}) and the quantum Hall effect (parts in 10^9) but, strictly speaking, takes electrical metrology outside of the SI. Realizations of electrical units based directly on the SI definition of the ampere suffer from comparatively large uncertainties: the ampere can be realized with an ampere balance with an uncertainty of 4 parts in 10^6 [4], the volt balance allows realization of the volt to within 3 parts in 10^7 [5] and the calculable capacitor realizes the farad to within 2 parts in 10^8 [6].

The main shortcomings of the present SI system are the use of an artefact to define the unit of mass and the fact that the practical realization of electrical units is not based directly on the SI definition of the ampere but on conventional values for the Josephson and the quantum Hall effects. These are the main drivers for the planned redefinition of the kilogram and the ampere, which will remedy both problems. It is expected that the unit of thermodynamic temperature, the kelvin, and the unit of the amount of substance, the mole, will be redefined at the same time [7].
2. Definition of the kilogram based on a fundamental constant

Fundamental constants are, to the best of our present knowledge, constant in time and space, and, therefore, well-suited as a basis for a system of measurement units. The definition of the second has, since 1967, been based on the frequency of a transition of the caesium atom, and the metre has been defined, since 1983, by a fixed numerical value of the speed of light in a vacuum [1].

Since the international prototype of the kilogram might drift by about 50 µg per century, a definition based on a fundamental constant should allow realization of the kilogram with a relative uncertainty of the order of 1 part in $10^8$ or to within 10 µg. Such a new definition will present an advantage for periods of more than about 20 years.

The definition of the kilogram can be based on several different fundamental constants. In each case, an experiment is needed to establish a relationship between a macroscopic mass and the relevant constant. From the point of view of the existing experimental techniques, the most interesting are the Planck constant $h$, the Avogadro constant $N_A$ or an atomic mass $m_x$. The last two can be easily seen as equivalent since they are linked by the relationship

$$m_x N_A = A_r(x) M_u$$

with $A_r(x)$ being the relative atomic mass (with a relative uncertainty typically below 1 part in $10^9$) and $M_u = 1$ g mol$^{-1}$ being the molar mass constant (with no uncertainty).

A direct link between a macroscopic mass and the Planck constant can be established with a watt balance, as will be shown below, and between a macroscopic mass and the Avogadro constant by counting atoms in a nearly perfect silicon sphere [8]. The Planck constant and the Avogadro constant are linked to each other through the definition of the Rydberg constant by the following equation:

$$N_A h = \frac{A_r(e) c \alpha^2}{2 R_\infty} M_u.$$  

Since the relative uncertainty of the relative atomic mass of the electron $A_r(e)$ is $4.2 \times 10^{-10}$, that of the Rydberg constant $R_\infty$ is $6.6 \times 10^{-12}$, that of the fine structure constant $\alpha$ is $6.8 \times 10^{-10}$ [9] and the speed of light $c$ and the molar mass constant $M_u$ have no uncertainty, the Avogadro constant can be deduced from the Planck constant, and, vice versa, with a negligible additional uncertainty which is about twice the relative uncertainty of the fine structure constant, that is, $1.4 \times 10^{-9}$. Therefore, the choice of the constant which will serve as the basis for the new definition of the mass unit has no practical implications on the choice of the experimental method used for the realization of the kilogram. Independently of the choice of $h$ or $N_A$, the realization could be carried out either with a watt balance or by counting atoms in a silicon sphere.

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Considering only mass metrology, it appears to be most appropriate to base the new definition of the kilogram on a fixed numerical value of an atomic mass, because it is a quantity of the same type as the kilogram unit. One kilogram would then just be a specified large number of atoms of a certain type. However, fixing the numerical value of the Planck constant presents great advantages for electrical metrology. When $h$ is used to define the kilogram and the elementary charge $e$ is used to define the ampere, both the Josephson constant $K_J = 2e/h$ and the von Klitzing constant $R_K = h/e^2$ will become exactly known. Therefore, the need for the conventional constants $K_{J-90}$ and $R_{K-90}$, as discussed above, will cease and the Josephson and quantum Hall effects will become direct realizations of the SI.

The new definition of the kilogram could thus have the form ‘The kilogram, kg, is the unit of mass; its magnitude is set by fixing the numerical value of the Planck constant to be equal to exactly $6.62606X \times 10^{-34}$ when it is expressed in the unit $s^{-1} m^2 kg$, which is equal to $J s$’. $X$ stands for digits to be added to the numerical value at the time when the definition will be adopted. This definition is equivalent to the exact relation $h = 6.62606X \times 10^{-34} s^{-1} m^2 kg$. The value or size of the Planck constant is decided by nature, its numerical value is set by this definition and the second and the metre are defined separately. Therefore, the effect of this equation is to define the unit kilogram.

An important aspect for the redefinition of any unit is its continuity. The newly defined unit shall be of the same size as the previous unit so that the results of past measurements need not be changed. The unavoidable discontinuity shall be smaller than or at least comparable to the uncertainty, with which the unit can be realized. In the case of the redefinition of the kilogram, this requires a measurement of the Planck constant in the present SI system to determine the numerical value to be used. Another important aspect is that the future definition can be practically realized with a sufficiently small uncertainty. Experts in mass metrology who meet in the Consultative Committee for Mass and Related Quantities (CCM) estimate that the uncertainty for the realization of the kilogram should not be larger than 2 parts in $10^8$, mainly for requirements in legal metrology.

The target uncertainty for the determination of the Planck constant is therefore 2 parts in $10^8$. As already stated above, the Planck constant can be directly determined with a watt balance or indirectly, via the definition of the Rydberg constant, by counting atoms in a silicon sphere. The watt balance approach presents an advantage in that it can be carried out by a single laboratory. This allows for the development of several watt balances and the possibility of their comparison. The Avogadro approach requires a large international collaboration and this is unlikely to be repeated [8].

### 3. The principle of the watt balance

A watt balance establishes a relationship between a macroscopic mass $m$ and the Planck constant $h$. Whereas $m$ is the mass of a macroscopic object, the Planck constant is the fundamental constant of quantum physics, which describes the behaviour of the microscopic world. The watt balance therefore needs to establish
a link between the very different domains of the macroscopic and the microscopic worlds. This link is provided by two macroscopic electrical quantum effects: the Josephson effect and the quantum Hall effect.

The Josephson effect was predicted in 1962 by Brian Josephson [10] and first observed in 1963 by Sidney Shapiro [11]. The inverse AC Josephson effect is observed by irradiating a junction between two superconductors which are separated by a thin insulating barrier, a Josephson junction, with microwave radiation of frequency \( f \). The current of Cooper pairs tunnelling through the junction synchronizes with the applied frequency \( f \) and its harmonics. The voltage \( U_J \) over the junction is then quantized at the values

\[
U_J = n f \frac{h}{2e} = n f \frac{K_J}{e},
\]

where \( n \) is an integer quantum number, \( e \) is the elementary charge and \( K_J \) is the Josephson constant. The exact value of \( n \) and the specific voltage level are selected by choosing the value of a DC current through the junction.

The quantum Hall effect was observed experimentally in 1980 by von Klitzing et al. [12]. The quantum Hall effect is observed on samples which contain two-dimensional electron gases, as GaAs heterostructures, silicon metal–oxide–semiconductor field-effect transistors (Si-MOSFETs) and graphene. When such structures are subjected to very low temperatures and strong magnetic fields, the electronic states are grouped into separated Landau levels. The structure then exhibits a quantized Hall resistance

\[
R_H = \frac{1}{i} \frac{h}{e^2} = \frac{R_K}{i},
\]

where \( i \) is an integer quantum number and \( R_K \) is the von Klitzing constant.

The common property of both effects is that they establish a relationship between a macroscopic measurand, a voltage and a resistance, and fundamental constants, the elementary charge and the Planck constant. Both effects are nowadays widely used as standards for resistance and voltage metrology [13,14].

The watt balance experiment takes advantage of this property to relate an electrical power to the Planck constant. An electrical power \( P_{el} \) takes the form

\[
P_{el} = U_1 I = \frac{U_1 U_2}{R},
\]

if the current \( I \) is measured as the voltage drop \( U_2 \) over a resistance \( R \). The value of the resistance can be determined with respect to the quantized Hall resistance and the voltages can be measured with respect to a Josephson voltage standard. Therefore, the electrical power can be expressed as \( P_{el} = C_{el} f_1 f_2 h \), where \( C_{el} \) is the electrical calibration constant and \( f_i \) are the microwave frequencies of the two Josephson voltage measurements. The electrical power is then linked to the Planck constant.

Mechanical and electrical powers are of the same kind, they have the same unit, they can be compared with each other and they can be transformed into each other. The equations for mechanical power take different forms dependent on the physical phenomenon being described, but always depend on a mass \( m \) and other...
quantities, such as velocities and accelerations. One special form is \( P_m = mgv \), which describes the motion of a mass \( m \) with the velocity \( v \) against the direction of gravitational acceleration \( g \).

A watt balance compares an electrical power in the form derived above with a mechanical power, which leads to

\[
mgv = UI = C_{\text{el}} f_1 f_2 h. \tag{3.4}
\]

In principle, every experiment which converts electrical power into mechanical power could establish a link between a mass and the Planck constant, for example an electric motor lifting a mass. However, every direct energy conversion suffers from energy losses, which would need to be quantified at the level of several parts in \( 10^9 \), which is very demanding. Experiments of this type, in the form of magnetic levitation of a superconducting body, have been carried out but were abandoned for this reason [15]. Direct energy conversion should therefore be avoided.

The three necessities for the watt balance are therefore (i) the use of the Josephson and quantum Hall effects, (ii) the equivalence of electrical and mechanical power, and (iii) a clever measurement scheme to avoid direct energy conversion.

The watt balance experiment realizes these three principles. To avoid direct energy conversion, the experiment is carried out in two separate phases, the static phase and the dynamic phase. This approach was proposed in 1976 by Bryan Kibble from the National Physical Laboratory (NPL) [16].

In the static phase, the weight of a mass \( m \) subjected to gravitational acceleration \( g \) is balanced by the Lorentz force on a coil with current \( I \), hanging in a magnetic field such that the flux \( \Phi \) passes through it,

\[
mg = -I \frac{\partial \Phi}{\partial z}. \tag{3.5}
\]

The direction of the gravitational acceleration defines the vertical direction \( z \). Since a balance is only sensitive to vertical forces, even if small horizontal components of the Lorentz force exist, only the vertical flux gradient contributes to the force balance. In the case of a circular coil with wire length \( L \) placed in a horizontal, purely radial magnetic field \( B_r \) (figure 1), this equation becomes

\[
mg = I B_r L. \tag{3.6}
\]

In reality, the situation is more complex, because the mass \( m \) of the test mass needs to be isolated from the mass of the suspension, which typically is several times larger than \( m \). Therefore, a substitution principle is used. First a measurement is made with the test mass on the weighing platform and a current \( I \) is passed through the coil. The test mass is then removed and the direction of the current reversed. A counterweight is used such that the balance is in equilibrium in both situations. Therefore, the change of the mechanical force \( mg \) corresponds to a change in the Lorentz force of \( 2ILB_r \). The use of two currents of the same value but in opposite directions is advantageous, because the Joule heating is the same in both cases.

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Figure 1. Static phase of the watt balance measurement. The weight of the test mass $m$ is balanced against the Lorentz force on a coil with wire length $L$ and current $I$ hanging in a magnetic field with a radial flux density $B$.

$$F_{\text{el}} = ILB$$

$$F_{\text{in}} = mg$$

Figure 2. Dynamic phase of the watt balance experiment. The coil is moved vertically with velocity $v$ through a magnetic field with the flux density $B$, which leads to an induced voltage $U$.

$$U = BLv$$

In the dynamic phase, the coil is moved vertically at the velocity $v_z$ through the same magnetic field as in the static phase. A voltage is induced which is given by

$$U = -\frac{\partial \Phi}{\partial t} = -v_z \frac{\partial \Phi}{\partial z}. \quad (3.7)$$

In the case that the velocity is not purely vertical, analogous contributions to the induced voltage from the horizontal movement exist. In the case of a horizontal coil in a radial magnetic field (figure 2), this simplifies to

$$U = v_z B r L, \quad (3.8)$$

where $L$ is again the wire length on the coil.

If the velocity at the position where the static measurement is made is purely vertical, and if the magnetic field and the alignment of the coil with respect to the magnet do not change between the static and the dynamic measurement, equations (3.5) and (3.7) or equations (3.6) and (3.8) can be combined by eliminating the terms $\partial \Phi / \partial z$ or $B r L$. The resulting equation

$$UI = mgv \quad (3.9)$$

is in the form of equation (3.4). The left-hand side of this equation is an electrical power, the right-hand side a mechanical power, which explains the name of the watt balance experiment. As both phases are carried out separately, the current

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I and the voltage $U$ are not present at the same time, and the power in this sense is a virtual power; the same holds true for the mechanical power. There is no direct energy conversion, which is of importance as discussed above.

As described above, the electrical quantities are measured using the Josephson effect and the quantum Hall effect, which lead to the measurement equation

$$mgv = UI = C_{el} f_1 f_2 h,$$

identical to equation (3.4).

Watt balance experiments can be interpreted in another way, by making use of the definition of the Rydberg constant $R_\infty$

$$R_\infty = \frac{\alpha^2 m_e c}{2h},$$

where $\alpha$ is the fine structure constant, known with a relative uncertainty of 6.8 parts in $10^{10}$ [9], and $c$ is the speed of light, with no uncertainty because its numerical value is fixed in the definition of the metre [1]. The Rydberg constant is known with a relative uncertainty of 6.6 parts in $10^{12}$ [9]. Since the uncertainties of $\alpha$ and of $R_\infty$ are much smaller than that of $h$, the mass of the electron $m_e$ can be obtained from this equation with the same relative uncertainty as that of $h$.

A watt balance can therefore be seen as an experiment which determines the mass of the electron by comparing it with a macroscopic test mass. In fact, this is the most accurate method available to determine the electron mass, which is therefore known with the same relative uncertainty as the Planck constant, that is, 5 parts in $10^8$ [9].

The measurement equation (3.10) shows which quantities need to be measured in a watt balance experiment. Detailed descriptions can be found in the review articles [17,18]. The electrical calibration constant $C_{el}$ includes the determination of the induced voltage and the current-related voltage drop using a Josephson voltage standard, and the calibration of the resistor needed for the current measurement against the quantized Hall resistance. The resistance calibration needs to be carried out only occasionally because high-quality resistors are very stable in time. The Josephson voltage standard forms an integral part of the experiment because both the induced voltage in the dynamic phase and the current-related voltage drop in the static phase are time dependent and need to be measured in real time. A quantized and accurately known Josephson voltage is opposed to the voltage to be measured, and the small difference is determined with a voltmeter. The frequencies $f_1$ and $f_2$ are those of the microwaves that irradiate the Josephson junctions. The coil velocity $v$ is obtained by interferometry. It is important that the measurements of velocity and induced voltage are well synchronized, because this leads to a high rejection of vibration-induced noise in both signals. The value of the gravitational acceleration $g$ needs to be known at the centre of mass of the test mass, which is inaccessible once the experiment is set up. One technique to achieve this is to establish a map of the variation of $g$ with a relative gravimeter in the laboratory before the watt balance is installed. In addition, the absolute value needs to be known at least at one point. The absolute value of the gravitational acceleration at the centre of mass of the test mass can then be obtained by interpolation. A correction needs to be applied for the gravitational effect of the watt balance itself. To determine
Figure 3. The NPL mark II watt balance (courtesy of NPL). Top, the balance beam; left, the suspension with the coil hanging in the air gap of the magnet (bottom); right, an auxiliary magnet and a coil, used to tilt the balance beam.

a value for the Planck constant, the mass needs to be calibrated with respect to the present SI. Later, after the redefinition, the fixed numerical value of the Planck constant will become the basis for the determination of the mass.

4. The existing watt balance experiments

In this section, the main characteristics and distinctive features of the existing watt balance experiments are described. More details can be found in the review articles [17,18]. The published results of the experiments are reviewed and compared in the following section.

The development of a watt balance at the NPL in the UK started soon after the proposal of the two-phase operation, described above, by Kibble in 1976 [16]. This experiment used a very heavy permanent magnet (6 tonnes) to produce a uniform magnetic field, into which an 8-shaped moving coil was placed. A large balance beam resting on a knife edge was used for the force measurement. The final result of this experiment, with a relative uncertainty of 2 parts in 10^7, was published in 1990 [19]. At the same time, plans for an improved apparatus, the NPL mark II watt balance, were presented (figure 3). This apparatus uses the same balance beam, but with a new magnet, which produces at its top and bottom two radial magnetic fields of 0.42 T, of opposite sign, inside circular gaps. The coil consists of two separate coils, which are placed in both gaps and which are connected in opposition to eliminate external electromagnetic perturbations. A Michelson interferometer is used for the measurement of the velocity of about
1.3 mm/s$^{-1}$. The movement of the coil is generated by tilting the balance beam. The test masses are made of gold-plated copper (1 and 0.5 kg) and of silicon (0.5 kg). A result was published in 2007 [20] with a relative uncertainty of 6.6 parts in $10^8$. The NPL then decided to stop this project and the experiment was transferred to the National Research Council (NRC) in Canada in mid-2009 [21]. A possible systematic error was discovered before the experiment was shutdown at the NPL, but the limited time available did not allow an in-depth analysis. As a consequence, the uncertainty was increased to 2 parts in $10^7$ (I. Robinson 2010, personal communication). It is believed that, after analysis and elimination of this effect, it should be possible to approach a relative uncertainty of a few parts in $10^8$ with this apparatus. Reassembly at the NRC started at the beginning of 2010 and measurements have been made at the level of several parts per million. To improve accuracy, parts of the balance suspension are being rebuilt and the knife edges replaced. The team plans to make sub-$10^{-7}$ measurements in mid-2011.

Briefly after the proposal of the watt balance concept, the National Institute of Standards and Technology (NIST) in the USA began to construct a watt balance. This apparatus used an electromagnet to generate the magnetic flux. A result was published in 1989 with a relative uncertainty of 1.3 parts in $10^6$ [22]. This was followed by the development of a second apparatus which uses two large superconducting solenoids wired in opposition to create a radial magnetic field of 0.1 T (figure 4). As a consequence of the size of the solenoids, the whole experiment is about 6 m high. Instead of a balance beam, a large balance wheel of 0.61 m diameter was chosen. Rotation of the wheel leads to vertical movement of the coil. The test masses of 1 kg are made of Au and Pt–Ir. The first result from this apparatus was published in 1998 with a relative uncertainty of 8.7 parts in $10^8$ [23,24]. The experiment was then largely rebuilt to eliminate many of the previous error sources, but the base concept stayed the same. Further results were
published in 2005 [25] and in 2007 [26] with relative uncertainties of 5.2 parts in $10^8$ and 3.6 parts in $10^8$, respectively, all results being consistent. Since then, many tests for possible systematic errors have been made, to increase confidence in the correct operation of the instrument. Recently, a project to build a new watt balance for future mass dissemination was started at NIST.

The Federal Office of Metrology (METAS) in Switzerland started the development of a watt balance in 1997. This experiment is characterized by two original ideas [18]. Instead of a 1kg test mass, a 100g mass is used, which reduces the forces by a factor of 10 and leads to a significant size reduction of the apparatus, in particular of the magnet. The METAS experiment is at present the only one to use a uniform magnetic field between two flat pole pieces, all others use radial fields. The second distinctive feature is that, in the dynamic phase, the coil is disconnected from the balance and moved by a separate mechanical system. This allows the balance to be always in an equilibrium position and avoids problems of hysteresis, but this requires a coil transfer between the balance suspension, for the static measurement, and the mechanical translation ‘seesaw’ system, for the dynamic phase. The work on the METAS watt balance has now led to a published result [27] with a relative uncertainty of 2.9 parts in $10^7$. This uncertainty is dominated by alignment issues and the present apparatus has reached its limits. A new project, based on the experience of the present experiment, has been started with the objective to reach a relative uncertainty close to 1 part in $10^8$.

The Laboratoire National de Métrologie et d’Essais in France commenced a watt balance project in the year 2000 and its development started in 2002 [18,28]. Its distinctive feature is that the force comparator is moved together with the coil by a guiding stage. The guiding stage is mechanically very rigid, which ensures a close to vertical movement of the coil. A motorized translation stage moves the guiding stage at a velocity of 2 mm s$^{-1}$. A two-stage velocity control system is used to control the velocity of the coil very precisely. The permanent magnet produces a radial field of about 0.9 T in the centre of the air gap. The pole faces have been machined at micrometre accuracy to ensure that the variation of the magnetic field in the vertical direction is within 1 part in $10^4$. Most components of the instrument now exist and the watt balance is presently being assembled. The first measurements from this watt balance are expected at the end of 2011. The objective is to have a result in 2014, which can be taken into account in the fixing of the numerical value of the Planck constant.

At the BIPM, development of a watt balance began in 2003. The BIPM approach is to carry out the static and dynamic phases simultaneously [29,30], but to avoid direct energy conversion, the importance of which is discussed above. The derivation of the watt balance measurement equation (3.9) depends on the term $\frac{B_r L}{L}$ being constant between both phases; otherwise, corrections are necessary. To achieve this requires a constant magnetic field and the same alignment and position of the coil in both phases. When both phases are carried out at the same time, these requirements are relaxed. Separation of the induced voltage from the resistive voltage drop, which results from the current flow, is required. One way to achieve this is to use a superconducting coil, in which the resistive voltage drop does not exist. A feasibility study for a future cryogenic experiment has therefore been started at the BIPM. The present work is focused on the development of a room temperature experiment. In this experiment, movement of the coil is
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Driven by an electrostatic motor which is part of the coil suspension. Parasitic power losses, for example owing to friction in the suspension or to magnetization changes of the magnet yoke, are compensated by the motor. A closed magnetic circuit is being developed which will screen the coil in its position in the air gap from external electromagnetic perturbations. A mechanical system for automatic correction of deviations of the coil trajectory from purely vertical has already been developed. Since early 2010, measurements of the Planck constant have been carried out, at present with an uncertainty of about 5 parts in $10^5$. In 2011, the experiment will be moved to a new laboratory, where it will be placed in a vacuum system installed on a concrete block to reduce vibration. It is expected that the uncertainty will then be reduced to about 5 parts in $10^6$. The long-term goal is to reach a level of several parts in $10^8$ around 2015.

The experiment conducted by the National Institute of Metrology, China, follows a different approach which does not require a dynamic measurement phase [31,32]. The electromagnetic force on the coil is created by the magnetic field of a second coil, aligned parallel to the coil which is suspended from the balance. The force equation, equivalent to equations (3.5) and (3.6), is then given by

$$mg = \frac{\partial M}{\partial z} I_1 I_2,$$

where $M$ is the mutual inductance between both coils, $\partial M/\partial z$ is the variation of the mutual inductance with the coil separation $z$ and $I_1$ and $I_2$ are the currents in both coils. Using a special multi-coil system, it is possible that over a certain range of coil separations the force, or $\partial M/\partial z$, is nearly constant. Equation (4.1) can then be integrated between two coil separations $z_1$ and $z_2$ which lie within the range of nearly constant force,

$$[M(z_1) - M(z_2)] I_1 I_2 + mg(z_2 - z_1) = \int_{1}^{2} \Delta f_z(z) \, dz. \quad (4.2)$$

The first term on the left-hand side of equation (4.2) is the change of magnetic energy between both positions, the second is the change of potential energy of the test mass. The term on the right-hand side of equation (4.2) corresponds to the small change of force with coil separation. Because this experiment compares energies instead of power, as is the case for the other watt balances, this experiment is called a joule balance.

Up to now the focus has been on the determination of the mutual inductance between both coils. A new method based on direct digital synthesis has been developed. The relative standard deviation of the mean of repeated measurements is of the order of 1 part in $10^7$. Since the measurements of mutual inductance are made at AC, the result needs to be extrapolated to DC. The uncertainty of the extrapolated value is estimated at less than 1 part in $10^6$. Other aspects of the experiment are also being developed; for example, the balance necessary for the force measurement and an optical system to determine the vertical position of the moving coil. It is planned to use a superconducting fixed coil to increase the magnetic flux density.

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5. Overview of the available results for the Planck constant

An overview of determinations of the Planck constant available in 2006 can be found in the publication on the Committee on Data for Science and Technology (CODATA) fundamental constants adjustment [9]. Figure 5 shows these results and the more recent ones in chronological order.

Values for $h$ can be obtained directly from watt balance experiments but also indirectly from measurements of various other constants, as discussed in Mohr et al. [9]: the gyromagnetic ratio of the proton (indicated in figure 5 as ‘gamma-p’), the Faraday constant, the Josephson constant (indicated as ‘volt’, because it is obtained with a voltage balance) and the Avogadro constant. In figure 5, all watt balance results are shown by filled diamonds, results obtained by other electrical measurements by open diamonds and results based on the silicon sphere technique by filled squares. The value for the Planck constant obtained in the last 2006 CODATA fundamental constants adjustment is indicated by the solid line. This value is largely dominated by the 2005 NIST watt balance result.

The most interesting feature of figure 5 is the discrepancy of 8 parts in $10^7$ between the Avogadro 2005 result, obtained with spheres made of silicon with natural isotopic composition, and the watt balance results. For several years, this has been considered to be the greatest obstacle in the redefinition of the kilogram. Recently, it has been found that there may be an error in the mean molar mass measurement of natural silicon, from which the spheres were made. This effect is now being investigated, but it is too early to tell how large this error is and if a correction can be applied retrospectively.
The two techniques which allow the Planck constant to be determined with the smallest uncertainty (several parts in $10^8$) are the watt balance and the silicon sphere technique, which has spheres made of isotopically enriched $^{28}$Si. Recent results obtained by these two techniques are shown in figure 6.

The latest 2010 Avogadro result is closer to the watt balance results than the Avogadro result of 2005 (figure 5), but its uncertainty is much smaller. This leads to a difference between the Avogadro 2010 result and the latest NIST watt balance result, which is statistically highly significant. The difference corresponds to nearly four times the combined standard deviation of both results.

The NPL result has not changed between 2007 and 2010, but its uncertainty has increased considerably because of a systematic effect found shortly before shipping the experiment to the NRC. After investigation of this effect at the NRC, it is expected that this instrument is again capable of achieving relative uncertainties of a few parts in $10^8$. The very recent result from METAS agrees with all other experiments, as a result of its relatively large uncertainty.

At the present time, the main problem in the redefinition of the kilogram is the significant discrepancy between the results of the silicon sphere approach and those obtained by the NIST watt balance.

6. Status of the redefinition of the kilogram

As already described above, it is planned to redefine four of the seven base units of the SI: the kilogram, the ampere, the kelvin and the mole [7]. The decision for this change will be made by the CGPM, which meets in Paris every 4 years; the next meeting is in October 2011. The resolutions of the CGPM are prepared by the International Committee for Weights and Measures (CIPM), which meets annually at the BIPM. The CIPM receives scientific advice from its
Consultative Committees, which comprise world experts in the different fields of metrology.

The CCM has discussed the redefinition of the kilogram and recommends that the following conditions are met before the kilogram is redefined:

— At least three independent experiments, including work both from watt balance and from International Avogadro Coordination projects, yield values of the relevant constants with relative standard uncertainties not larger than 5 parts in $10^8$.
— At least one of these results should have a relative standard uncertainty not larger than 2 parts in $10^8$.
— For each of the relevant constants, values provided by the different experiments should be consistent at the 95% level of confidence.

The first condition is at present fulfilled by the NIST 2007 watt balance result [26] and the Avogadro 2010 [8] result. An interesting question in this respect is in how far the NIST 1998 result [23] can be seen as independent from the later results, because the apparatus had been considerably modified. However, its uncertainty of 8.7 parts in $10^8$ is too large to fulfill the first condition. The target uncertainty of the second condition has, so far, not been reached by any experiment. The International Avogadro Coordination Group plans to reach 2 parts in $10^8$ by the end of 2011 or early 2012. The third condition is violated by the significant discrepancy between the NIST watt balance and the Avogadro result. From the point of view of mass metrology, it is therefore too early to make any decision on the redefinition of the kilogram at the meeting of the CGPM in 2011.

The CCM has also pointed out that it is necessary to have a sufficient number of facilities that realize the new kilogram definition with a relative standard deviation of not larger than 2 parts in $10^8$. The CCM will develop a *mise en pratique* which will specify how the new definition can be realized in practice. One of the difficulties which is still to be addressed is how to ensure uniformity of mass calibrations in the future. Even if several watt balances with relative uncertainties as small as 2 parts in $10^8$ did exist, it would be necessary to organize, at least during an initial period, periodic comparisons between them.

The Consultative Committee for Units is responsible for the development of the SI. During recent years, it has discussed the planned redefinitions of four base units and has been in contact with the relevant Consultative Committees. Although it is too early to proceed with the redefinition in 2011, a consensus has now been reached on the form of the new SI. This will be presented to the CGPM in 2011. This proposal expects to redefine

— the kilogram by fixing the numerical value of the Planck constant $h$;
— the ampere by fixing the numerical value of the elementary charge $e$;
— the kelvin by fixing the numerical value of the Boltzmann constant $k_B$; and
— the mole by fixing the numerical value of the Avogadro constant $N_A$.

All definitions will be of a form in which the unit is defined indirectly by specifying explicitly an exact value for the related fundamental constant. An example for the kilogram is provided above.
7. Conclusions

The international prototype of the kilogram has fulfilled its role very well since it was sanctioned by the first CGPM in 1889. There are, however, indications that its ‘absolute’ mass has not been perfectly stable, although it is by definition always 1 kg. Because measurements are becoming more and more precise this instability will be a problem in the future. Therefore, preparations are being made to redefine the kilogram with respect to a fixed numerical value of a fundamental constant. It is also the intention at the same time to redefine the ampere, the kelvin and the mole. Several constants could be chosen for the kilogram redefinition but, owing to the advantages it presents to electrical metrology, the Planck constant was selected. The first step towards a redefinition is the determination of the numerical value of the Planck constant in the present SI, with an uncertainty of the order of 1 part in $10^8$.

Watt balances establish an experimental link between the Planck constant and a macroscopic mass. Watt balance experiments rely on the equivalence of mechanical and electrical energy and on the use of two macroscopic quantum effects: the Josephson effect and the quantum Hall effect. Watt balance experiments at different stages of development exist in several National Metrology Institutes. Up to now only two experiments have achieved measurement uncertainties below 1 part in $10^7$, one at the NIST and one at the NPL, but these results are not in agreement. Very recently another technique used to determine the Planck constant, via the Avogadro constant, by ‘counting’ the number of silicon atoms in a nearly perfect silicon sphere, has led to another discrepant value.

Experts in mass metrology have recommended that several conditions must be met before the kilogram can be redefined, and these are at present not yet fulfilled. However, the form of the new SI has already been defined, so that in the future all seven SI base units will be based on fixed numerical values of constants. The exact time when the new definitions are officially adopted by the CGPM will depend on the progress of future work to determine the Planck constant and on its evaluation in the relevant expert committees. The CGPM is scheduled to meet in 2011 but owing to the quadrennial cycle of these meetings the next occasion to decide on the redefinitions will be in 2015.

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