Mesoscopic systems: classical irreversibility and quantum coherence

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Mesoscopic physics is a sub-discipline of condensed-matter physics that focuses on the properties of solids in a size range intermediate between bulk matter and individual atoms. In particular, it is characteristic of a domain where a certain number of interacting objects can easily be tuned between classical and quantum regimes, thus enabling studies at the border of the two. In magnetism, such a tuning was first realized with large-spin magnetic molecules called single-molecule magnets (SMMs) with archetype Mn12-ac. In general, the mesoscopic scale can be relatively large (e.g. micrometre-sized superconducting circuits), but, in magnetism, it is much smaller and can reach the atomic scale with rare earth (RE) ions. In all cases, it is shown how quantum relaxation can drastically reduce classical irreversibility. Taking the example of mesoscopic spin systems, the origin of irreversibility is discussed on the basis of the Landau-Zener model. A classical counterpart of this model is described enabling, in particular, intuitive understanding of most aspects of quantum spin dynamics. The spin dynamics of mesoscopic spin systems (SMM or RE systems) becomes coherent if they are well isolated. The study of the damping of their Rabi oscillations gives access to most relevant decoherence mechanisms by different environmental baths, including the electromagnetic bath of microwave excitation. This type of decoherence, clearly seen with spin systems, is easily recovered in quantum simulations. It is also observed with other types of qubits such as a single spin in a quantum dot or a superconducting loop, despite the presence of other competitive decoherence mechanisms. As in the molecular magnet V15, the leading decoherence terms of superconducting qubits seem to be associated with a non-Markovian channel in which short-living entanglements with distributions of two-level systems (nuclear spins, impurity spins and/or charges) leading to 1/f noise induce $\tau_1$-like relaxation of $S_z$ with dissipation to the bath of two-level systems with which they interact most. Finally, let us mention that these experiments on quantum oscillations are, most of the time, performed in the classical regime of Rabi oscillations, suggesting that decoherence mechanisms might also be treated classically.

Keywords: mesoscopic systems; qubits: single-molecule magnets, superconducting, single electrons; quantum; classical; decoherence

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1. Introduction

The possibility to observe quantum phenomena at the macroscopic scale has been discussed from the earliest times of quantum mechanics (e.g. the Schrödinger’s cat paradox [1]). Experimental studies on ‘macroscopic quantum tunnelling’ started in the 1970s or 1980s—in particular, under the impulse of Leggett [2]. The first solid-state observation of tunnelling at the macroscopic scale was made on a micrometre-sized Josephson junction at IBM Yorktown Heights in 1981. It was shown that the ‘switching current’ of a junction, thermally activated at high temperature, becomes independent of temperature below a certain cross over temperature, in agreement with the expectations of quantum tunnelling where the barrier is crossed at constant energy. In magnetism, this subject started in the early 1970s and first consisted in observing whether the reversal of the magnetization of a macroscopic magnet, thermally activated at high temperature, could take place by quantum tunnelling at low temperature. After a relatively long ‘middle ground’ owing to the lack of systems with identical nanoparticles [3], the phenomenon of macroscopic quantum tunnelling in magnetism and its cross over towards the thermally activated regime at higher temperature was demonstrated with the nearly non-interacting \( S = 10 \) spins of the Mn\(_{12}\)-ac single-molecule magnet (SMM)\(^1\) [4,5] and later with the nearly non-interacting angular moments \( J \sim 10 \) of rare earth (RE) ions diluted in an insulator [6,7].

In this paper, we discuss the origins of hysteresis and of decoherence in the earlier mentioned superconducting and large-spin mesoscopic systems when they are in their ‘more classical’ or ‘more quantum’ regimes. Note that, when we say ‘classical’, we nevertheless assume spin quantization and a sufficiently large separation between the ground state and the first excited one. This is quite possible even with mesoscopic or macroscopic systems (large spins \( S \)) because this energy separation \( DS^2 - D(S - 1)^2 = 2DS \) is independent of the spin, the anisotropy energy per volume unit, which is the relevant quantity for a given material, being \( K = DS^2 \). Following this section, the paper is divided into a further nine sections: §2 deals with mesoscopic tunnelling of large spins; §3 explains the quantum reduction of classical hysteresis; §4 illustrates the classical Landau–Zener model; §5 narrates from relaxation to coherence; §6 describes Rabi oscillations and single-qubit decoherence in diluted RE ions; §7 gives information about Rabi oscillations and multiple qubit decoherence in an SMM; §8 outlines Rabi oscillations of single-electron spin in a quantum dot; §9 presents flux qubits and measurement of classical and quantum states; and §10 deals with Rabi oscillations and decoherence in superconducting qubits and §11 the conclusion.

\(^1\)This system, synthesized more than 20 years ago, was unknown to physicists until 1994. SMMs consist of relatively large arrays (single crystals) of identical magnetic molecules of about 1 nm carrying collective spins \( S \). Owing to the absence of intermolecular exchange pathways (the molecules are well isolated from each other), intermolecular interactions are extremely weak and result from long-range dipolar interactions only. They are equivalent to well-ordered lattices of nanomagnets.

\(^2\)This is very easy to see: \( K \) being given per unit volume (of the order of approximately 0.1–10 K, depending on the material), the anisotropy energy of a nanoparticle of volume \( V \) is \( E = KV \) proportional to \( KS \) and it must identify to the energy \( E = DS^2 \) obtained from the Hamiltonian. This shows that the energy separation between levels does not depend on the size of the spin but on the material.

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2. Mesoscopic tunnelling of large spins

We shall deal with two types of systems: the SMMs and the RE ions dispersed in a solid matrix. They are both with angular moments of the order of 10 corresponding to the mesoscopic scale in magnetism, although the former is a much more complex object.

The microscopic Hamiltonian of an SMM can be written as $H = \sum J_{ij} S_i S_j - \sum S_i H_i + H_a \ldots$. The first (second) term, where the summations are performed over intramolecular spins ($i, j = 1 \ldots N = \text{total number of the molecule spins}$), describes intramolecular (Zeeman) energy. The last term $H_a$ is the anisotropy Hamiltonian generally associated with single-ion anisotropy, the main effect of which is to favour either spin-up or spin-down states $\pm S_z$, i.e. to tend to suppress state mixing (terms with uniaxial symmetry $//Oz$) or, on the contrary, to favour in-plane spins, i.e. to tend to induce state mixing (transverse terms). The Hilbert space dimension $D_H = (2S + 1)^N$ (for a same magnetic species) is generally too large to enable exact diagonalization, unless approximations are made to reduce it. In the case of Mn$_{12}$-ac containing 4Mn$^{3+}$ and 8Mn$^{2+}$ spins, $D_H = (2 \times 3/2 + 1)^4(2 \times 2 + 1)^8 = 10^8$ can be lowered to $D_H = 10^4$ if spin pairs with the largest couplings $J_{\text{Max}}$ are ‘locked’, i.e. form a single spin equal to the algebraic sum of the two. Exact diagonalization can then be performed, giving energy levels at energy less than $J_{\text{Max}}$ only. Another approximation, more drastic but often sufficient, consists in assuming that all the molecular spins are locked ($J_{ij}$ being larger than all the other energies), forming the collective SMM spin $S = \sum S_i$. The first term in the Hamiltonian disappears, and the other terms become functions of $S$. In the case of Mn$_{12}$-ac with tetragonal symmetry, it can be written as

$$H = -DS^2_z - BS^4_z - C(S^4_+ + S^4_-) - g\mu_B S H.$$  \hspace{1cm} (2.1)

With $S = 10$, $D_H = 2S + 1 = 21$ is sufficiently small for easy diagonalization. This type of Hamiltonian is also valid for RE ions, the spin $S$ being replaced by the total angular momentum $J = L + S$ (although Stevens’s operators are here more adapted). $B \ll D$ gives two ladder energy levels of opposite spin orientations (doublets) separated by a parabolic energy barrier symmetrical in zero field, $E(\pm m) \approx -Dm^2 - Bm^4$ ($m = \langle S_z \rangle$). The well-known double-well energy barrier of classical physics can be obtained from $E(m)$ by plotting $E(\theta)$, where $\theta = \cos^{-1}(m/S)$ ($0 \leq \theta \leq \pi$) is the quantized tilt angle associated with semiclassical precessions of $(S, \langle S_z \rangle = m)$ with $m = \pm S, = \pm (S - 1), \ldots$ in each well (figure 1). This semiclassical description, valid when the spin orientations of each well do not ‘see each other’, becomes quantum when the levels are in coincidence (‘resonance’) with, at least, one non-zero transverse term ($C \neq 0$ and/or $H_{xy} \neq 0$); in this case, the different spin components ‘see each other’ and resonant tunnelling becomes possible. In the following section, we shall consider that the spins are in their classical regime (assuming semiclassical quantization; see footnote 2) when the Hamiltonian is with a barrier and without a transverse term, and in their quantum regime when the Hamiltonian is with at least one transverse term.

In the classical regime, a spin $S$ can reverse by a succession of spin-phonon transitions going through the top of the barrier only, implying relatively large
temperatures. In the quantum regime, a spin $S$ can reverse at $0\text{K}$ by resonant tunnelling when the ground state $-|S\rangle$ is in coincidence with each one of the other states $|S-n\rangle$ (where $n \leq S$) under the effect of a field $H_z$ that shifts the two ‘ladders’ in opposite directions. When $E_S = E_{n-S}$, the successive coincidence fields are given by

$$g\mu_B\mu_0 H_{zn} = nD \left[ 1 + \left( \frac{B}{D} \right) \left( S^2 + (S-n)^2 \right) \right].$$

(2.2)
Tunnelling reversal occurs at \( n = 0 \) (zero field), \( n = 1 \) (first step), \( n = 2 \) (second step),... leading to a staircase hysteresis loop [5] (figure 1). As \( B \ll D \), the steps are almost equally separated and equation (2.2) becomes \( g_\mu_B H_m \approx nD \). Even weak dipolar interactions play an important effect here in broadening both the steps (inhomogeneously) and the tunnel windows (homogeneously), which makes the observation of tunnelling much easier [8,9]. If the field is fixed near a step (\( H_m \approx nD / g_\mu_B \), with \( n = 0, 1, \ldots \)) the magnetization decreases (time dependent) and quantum relaxation tends to diminish hysteresis. Between steps, the two ladders are shifted and tunnelling is impossible; the magnetization remains constant (in field and time) leading to hysteretic plateaus [4,5]. Such a staircase hysteresis loop (which figures out semiclassical quantization of the magnetization at the macroscopic scale) is completely different from a classical hysteresis loop characterized by a single step at the anisotropy field \( H_A = 2DS / g_\mu_B \) (approx. 10T in Mn\(_{12}\)-ac). Quantum relaxation simply short-cuts classical hysteresis at resonant fields \( H_m \), and may completely destroy it if transverse terms in the Hamiltonian are large enough.

These experiments strongly suggest that hysteresis is classical and that quantum dynamics simply ‘tries’ to reduce it through tunnelling events, but, in general, hysteresis persists in the quantum regime. This example shows that classical mechanics, including irreversibility, is a part of quantum physics, the contrary being not true. We shall try to develop these points in §3.

### 3. Quantum reduction of classical hysteresis

For the sake of simplicity, we shall start our discussion with the simplest Hamiltonian \( H = -DS^2_z - g_\mu_B (S_z H_z + S_x H_x) \), where the anisotropy barrier \( DS^2_z \) makes the regime more classical and the transverse field \( H_z \) (inducing tunnelling) more quantum. As in §2, we consider a large spin \( S \), at 0K, without any explicit reference to the spin-bath, which simply renormalizes the tunnel splitting through a multiplicative constant (revealing the effects of the environment) [8]. Let us now introduce another type of gap renormalization, very relevant in our context, as it depends exponentially on the spin.

The probability of the \( |S\rangle \rightarrow |n-S\rangle \) transition requires \( (2S-n) \) successive applications of the operator \( S_- \), each of them with probability \( p \sim (H_{xy}/D)^2 \). As the transition \( |S\rangle \rightarrow |n-S\rangle \) is associated with \( 2S-n \) such transitions, it will occur with probability \( p = (H_{xy}/D)^{2(2S-n)} \) because the population of the state \( \pm m \) before application of \( S_- \) is that of the state \( \pm (m-1) \) after application of the same operator. It is for this reason that, in the presence of a barrier \( DS^2 \), the tunnel splitting can be written as \( \Delta_{S,n-S} \sim DS^2 \) (\( H_z/D \))\(^{2S-n} \) [9,10]. Because the energy separation between spin states \( E(m = S) - E(m = S - 1) \approx 2DS \) is large and independent of the spin \( (K = DS; \text{ see footnote 2}) \), the ground states \( \pm S \) are sufficiently isolated from the other states to consider, at low temperature, a simple two-level Hamiltonian with effective spin 1/2 and \( \Delta_{S,S} = \Delta \sim DS(H_z/DS)^{2S} \). The corresponding renormalized Hamiltonian is (in reduced units)

\[
H = -H_zS_z - \Delta S_x.
\]
The Landau–Zener transition when $H_z = ct$ varies from $-\infty$ to $+\infty$ (say $|H_z| \gg \Delta$) [11,12] gives, after diagonalization of the time-dependent Schrödinger equation, the tunnelling probability

$$P_{LZ} = 1 - \exp \left[ -\frac{\pi (\Delta/\hbar)^2}{\gamma c} \right],$$

(3.2)

where $\gamma = g_z \mu_B/\hbar$ and $\hbar$ is the Planck constant. $P_{LZ}$ is the probability for spin reversal (adiabatic), whereas $1 - P_{LZ}$ is the probability for which the spin has not the time to reverse (non-adiabatic). The latter process, where the spin remains in its initial state, contributes to hysteresis, whereas the adiabatic process, with tunnelling spin reversal, contributes to suppress hysteresis (figure 2). If $S$ increases (decreases), $\Delta$ decreases (increases), and hysteresis increases (decreases). In the ‘pure quantum regime’ ($\Delta \to \infty$), the spin remains in the ground state and, even if $H_z(t)$ oscillates many times between $-\infty$ to $+\infty$, the spin adiabatically reverses back and forth without hysteresis, which is consistent with reversible quantum mechanics. In the ‘pure classical regime’ ($\Delta = 0$), the spin remains in its initial state and does not reverse, giving maximum (= classical) hysteresis. Examples of irreversible to reversible transition when an increasing transverse field is applied were given in ensembles of nearly non-interacting Mn$_{12}$-ac [13,14] and Ho (holmium RE ion; figure 3) [6,7,15]. In such a transition, the magnetization curve $M(H)/M_s$ switches from a broad staircase loop to the reversible magnetization curve $M(H)/M_s = dE/dH = H/(H^2 + \Delta^2)^{1/2}$, where $E(H)$ is the energy separation of the spin-up and spin-down levels of the Landau–Zener model. Clearly, these measurements on large-spin ensembles are self-averaged and therefore reproducible (they obey expression equation (3.2)). Single-shot experiments, sometimes performed on single-electron spins or on superconducting qubits, cannot be reproducible, except for those two purely classical or quantum limits.

We shall now discuss in more detail the nature of the hysteresis observed, in these large-spin systems (Mn$_{12}$-ac [4,5,14] and Ho based [6,7,15]), at temperatures low enough for the ground states $m = \pm S$ only to be concerned. A simple way to evaluate hysteresis consists in taking the area of the hysteresis loop $E_s = \int M_z \, dH_z$ where $H_z$ is between $\pm \infty$. This is the magnetic energy stored by the spin system, as in permanent magnets which are so useful in industry. As already mentioned, these systems are ensembles of two non-interacting levels with effective spins $1/2$ and renormalized transverse field $H_z/DS \to (H_z/DS)^{2S}$. Note that, as we consider the limit of low temperatures, the ground state $m = \pm S$ is represented by an effective $g$-factor proportional to $S$ (in the renormalization, it intervenes as a multiplicative factor only). In the classical description, the total magnetization fully reverses at the anisotropy field $H_A$, giving for the stored energy $E_{sc} = 4M_sH_A$, where $M_s = g\mu_BS = g_z\mu_B/2$ is the remanent magnetization along the $z$-axis. In the quantum description, the total magnetization can change at $H_z = 0$ (Landau–Zener transition in zero field) and $H_z = H_A$ giving $E_{sq} = 4pM_sH_A$, where $p = \exp(-\Delta^2/c)$ with $\Delta = DS^2(H_z/DS)^{2S}$. The relative decrease of the stored energy $(E_{sc} - E_{sq})/E_{sc} = 1 - p$ is directly related to the Landau–Zener probability. As expected, $\Delta \to 0$ does not modify classical hysteresis, whereas $\Delta \to \infty$ completely suppresses irreversibility, showing that Landau–Zener tunnelling reduces classical hysteresis through the quantum adiabatic process and
Figure 2. The two-level Landau–Zener model for a spin $S$ with projections $+m$ or $-m$. The zero-field splitting $\Delta$ is associated with the mixing of the spin-up and spin-down states whereas the mixing vanishes at fields $H_z \gg \Delta$, leading to a spin up or down. The arrows indicate the evolution of the state (bottom left) prepared with a spin down, at large negative fields. $P$ is the probability for adiabatic spin reversal ($\S 4$). (Online version in colour.)

Figure 3. Staircase hysteresis loop of Ho ions (RE with total angular momentum $J = 8$) diluted in a single crystal of LiYF$_4$ (where it partially substitutes Y ions) measured at 30 mK for different transverse fields $[6,7,15]$. The steps, corresponding to tunnelling as in figure 1, increase with the transverse field leading to hysteresis reduction. Hysteresis is already strongly reduced at the highest transverse field (190 mT) and disappears above a few hundred mT (not shown). (Online version in colour.)
preserves hysteresis through the classical non-adiabatic process. Time-dependent irreversibility and hysteresis loops $M(H)$ are determined through the probability average as for any measurable quantum quantity.

4. Classical Landau–Zener model and tunnelling

We now show that the solution equation (3.2) of the Landau–Zener model for a spin $1/2$ Hamiltonian (3.1) can also be obtained classically. For that, we shall recall why the quantum and classical dynamics of a spin $1/2$ are the same:

— In the quantum case, the time-dependent Schrödinger equation

$$i\hbar \frac{d|\Psi(t)\rangle}{dt} = H|\Psi(t)\rangle,$$

applied to Hamiltonian (3.1), leads to the differential equation

$$\frac{d^2b}{dt^2} = icb - (I^2 + (ct)^2)b,$$

where $b$ is the amplitude of the spin-up state (see [16]). As is well known, the initial condition with all spins down when $t \to -\infty$ leads to the solution $|b(t \to \infty)|^2 = 1 - \exp[-\pi(\Delta/h)^2/\gamma c] = P_{LZ}$.

Now, it is also well known that (4.1) is equivalent to

$$i\hbar \frac{dS(t)}{dt} = [S(t), H(t)],$$

where $S(t)$ is the time-dependent vector-spin operator. This equation gives

$$\begin{align*}
\frac{dS_x(t)}{dt} &= H_z S_y, \\
\frac{dS_y(t)}{dt} &= -H_z S_x + \Delta S_z \\
\frac{dS_z(t)}{dt} &= -\Delta S_y,
\end{align*}$$

(4.4)

and

$$\frac{d\langle S_x(t)\rangle}{dt} = H_z \langle S_y \rangle,$$

$$\frac{d\langle S_y(t)\rangle}{dt} = -H_z \langle S_x \rangle + \Delta \langle S_z \rangle,$$

$$\frac{d\langle S_z(t)\rangle}{dt} = -\Delta \langle S_y \rangle,$$

(4.5)
This system of equations can in principle be solved, but, as equations (4.1) and (4.3) are equivalent, their solutions must be the same, i.e. the expectation value solutions of (4.5) are given by
\[ \langle S_z(\infty) \rangle = 1 - 2P_{\text{LZ}} = 2 \exp[-\pi(\Delta/\hbar)^2/\gamma c] - 1 \] and
\[ \langle S_y(\infty) \rangle = \langle S_y(\infty) \rangle = 0. \] Such Bloch equations can be used to blend the non-adiabatic dynamics of spins with the presence of a non-equilibrium phonon bath, by using an effective Hamiltonian and a relaxation term given by a phonon-bottleneck mechanism \[17\]. Note that eliminating the transverse spin components in equation (4.5) gives
\[ \frac{d^2}{dt^2} \langle S_z(t) \rangle = C(t) \int \langle S_z \rangle dt - (\Delta^2 + c^2 t^2) \langle S_z \rangle, \] the solution of which must also be \[\langle S_z(\infty) \rangle = 1 - 2P_{\text{LZ}}\] when time goes from \(-\infty\) to \(+\infty\).

Let us now take the classical torque equations
\[ \frac{dS_z(t)}{dt} = H_z S_y, \]
\[ \frac{dS_y(t)}{dt} = -H_z S_z + \Delta S_z \]
and
\[ \frac{dS_z(t)}{dt} = -\Delta S_y. \]

These equations are nothing else but the well-known Bloch equations \[18\] used everywhere in the physics of resonance phenomena, but with \(\tau_1 = \tau_2 = \infty\), i.e. without dephasing and dissipation, which are here treated separately.

Equations (4.5) and (4.6) are identical, which simply means that the classical spin components coincide with their quantum expectation values. The consequence is that the quantum equations (4.1), (4.4) and (4.5) and the classical one (4.6) must have the same solution, giving for the time-dependent components of the classical spin \(S(t)\), \(S_z(\infty) = S(1 - 2P_{\text{LZ}}) = S(2 \exp[-\pi(\Delta/\hbar)^2/\gamma c] - 1)\). In the limit \(t \to \infty\), classical spin precessions must average and \(S_z(\infty) = S_y(\infty) = 0\). This is the solution of the ‘classical Landau–Zener model’. As we did with equations (4.5), we can eliminate transverse spin components in equations (4.6), and this obviously gives the same expression \[\frac{d^2}{dt^2} \langle S_z(t) \rangle = C(t) \int \langle S_z \rangle dt - (\Delta^2 + c^2 t^2) \langle S_z \rangle, \] both expressions are very similar to (4.2), but with the following differences: (i) the variables \(\langle S_z(t) \rangle\) and \(S_z(t)\) of (4.5) and (4.6) are expectation values (or probabilities) while in (4.2) the variable \(b\) is a wave function amplitude and (ii) equations (4.5) and (4.6) contain an explicit integral \((c^2 t \int S_z dt + c^2 t \int \langle S_z \rangle dt,\) respectively) on a time-dependent probability over the single classical trajectory, whereas equation (4.2) contains the imaginary amplitude term \(icb\), instead. The difference (ii) is just a consequence of the difference (i), and solutions of quantum and classical differential equations must be the same.

Let us now assume that we perform an experiment to determine \(\Delta\), which can be the tunnel splitting of an SMM or of an RE ion. For that purpose, we sweep the field \(H_z\) from \(-\infty\) to \(+\infty\) at the rate \(c\), starting with a spin-down state. If a single experiment is performed on a single spin (single-shot experiment), the spin will collapse on one of the two states and the experiment will obviously not...
be informative unless $\Delta = 0$ or $\infty$ (i.e. when the probability of obtaining one or the other state is equal to 1). In order to have access to the probability $P_{\text{LZ}}$, one must either perform a single experiment on a large ensemble of spins (e.g. a single crystal, §2) or repeat the experiment many times on the same spin (e.g. a single spin in a dot or a single superconducting qubit, §§8–10).

In the case of a spin ensemble, the measurement of $S_z(\infty) = \sum S_{iz}(\infty)$ can equally be interpreted in the quantum or classical models, both giving $S_z(\infty) = S(1 - 2P_{\text{LZ}})$. In the case of a single spin, the result will be the same but, here, the classical spin $S_z(\infty)$ has no real existence, as it is built up experiment after experiment by summation of the results (one cannot measure it in one step using a magnetometer). Those may be the reasons why one may consider that the classical interpretation deals with large ensembles only, whereas the quantum interpretation can deal with both large and small ensembles. Nevertheless in the present context of an ergotic ensemble of spins, the global spin $S_z(\infty)$ can easily be imagined even if it is built up in event-by-event measurements. Note that this system is ergodic (in the sense that ensemble and time averages are the same), although single-spin events can be non-adiabatic and lead to hysteretic magnetization loops $M_z(H_z)$, when $H_z \gg \Delta$.

Let us now give a classical interpretation of the Landau–Zener model. According to the Bloch torque equations (4.6), the time evolution of the classical spin components $S_x$, $S_y$ and $S_z$ results from the superposition of two classical precessions. The first one, about $H_z = H_z u_z$, is dominant when $|H_z| \gg \Delta$, and the second one, about $H_x = \Delta u_x$, is dominant when $H_x \ll \Delta$, and in particular when $H_z \sim 0$ ($u_x$ and $u_z$ are unit vectors). The precession about the $z$-axis (with the constant of motion $+S_z$ or $-S_z$ implying no $S_z$ reversal) leads to a non-adiabatic irreversible motion (figure 4), whereas the precession about the $x$-axis (with the constant of motion $S_x$ implying periodic oscillations of $S_z$ and $S_y$) leads to an adiabatic reversible motion of $S_z$ in which it is periodically reversed (figure 4). In the range of interest, where $|H_z(t)| \sim \Delta$, the $H_z$ and $H_x$ precessions will compete within the time scale $\tau_c \sim \Delta/c$. If $\tau_c$ is much smaller than the $H_x$ precession period $\tau_x \sim 1/\Delta$, i.e. if $\Delta^2/c \ll 1$, the spin will not have the time to reverse in the $yOz$ plane perpendicular to $H_x$ and will keep with its initial direction with spin-down state (non-adiabatic motion, favouring hysteresis; figure 5). More precisely, when $H_z \sim H_x$ during the fast reversal of $H_z$, the spin starts to precess about $H_x$ but the corresponding rotation $\Delta s$ is very small because $H_z$ reaches very rapidly its up orientation with an amplitude $\gg H_z$ (figure 5). This $\Delta s$ induces a small decrease of $|S_z|$ only (proportional to $\Delta^2/c$) while $H_z$ changes its sign (non-adiabatic motion). In the opposite limit $\Delta^2/c \gg 1$, $H_z(t)$ will be slow compared with the $H_x$ precession period $\tau_c$ and the spin components of the $yOz$ plane will average so that $|S_z|$, constant of motion of $H_z$, at $t = -\infty$, will become the constant of motion of $H_z(t) + H_x$, at each instant during the slow rotation of this field (figure 6). It will follow this field and reverse with it adiabatically (reversible motion, ‘quantum mechanical’ reversibility; figure 6). This is a classical version of the adiabatic theorem [19]. Note that the combined precessions about the two perpendicular axes can be considered as Rabi oscillations with frequency $\Omega_R = \Delta$ (in reduced units) in the time interval $\delta t = 2\pi/\Omega_R c = 2\pi \hbar/c \Delta$ (where an instantaneous rotating frame can be defined). Outside this interval, the oscillations are modified because $H_z$ cannot be considered as a constant anymore.

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Figure 4. Schematic of the two basic precessions of a spin submitted to a time-dependent field rotating in a plane (here the plane is $xOz$), as in the Landau–Zener model. When $H_z \gg H_x$, the precession is about $Oz$, and the spin projection $S_z$ is a constant of motion; when $H_z \ll H_x$, the precession is about $Ox$, and the spin projection $S_x$ is a constant of motion. In the first case, the spin components $S_x$ and $S_y$ rotate at the frequency $\gamma H_z$ and, in the second case, the spin components $S_z$ and $S_y$ rotate at the frequency $\gamma H_x$ ($\gamma$ is the gyromagnetic factor). In the presence of an anisotropy barrier $-D S^2_z$ favouring the $+S_z$ or $-S_z$ components, the constant of motion is $+S_z$ or $-S_z$ depending on what the lowest level is, if they are not in coincidence (non-zero $H_z$). If they are in coincidence, both precessions are possible and this enables the precession about $H_x$, in which case $S_z$ can switch back and forth between $+S_z$ and $-S_z$ (classical interpretation of tunnelling).

We now check that such a classical interpretation based on figures 4–6 allows one to understand simple spin $1/2$ quantum simulations of the Landau–Zener model at different sweeping field rates (without barrier, $\Delta \equiv H_z$). The longitudinal and transverse components $S_z(t)$ and $S_x(t)$, as well as $S_y(t)$ obtained after averaging, are given in figures 7 and 8. At negative time where $|H_z| \gg H_x$, $S_z$ does not show significant oscillations, contrary to $S_x$ and $S_y$. This is normal as $S_z$ is the constant of motion of the $H_z$ precession. At positive time, $S_z$ shows important oscillations about a mean value that increases when $c$ decreases ($\Delta^2/c$ increases). These oscillations are a modulation of the nearly constant $S_z$ by much stronger $S_y$ (and $S_x$) oscillations owing to the precessions about $S_x$ (and $S_z$). These figures also show that, at smallest $c$, the adiabatic motions of $S_z$ and $S_x$ do not show significant oscillations even at positive times, which is a consequence of the fast spin components average in the $yOz$ plane.

We now shift to the case of a large spin $S$ with a barrier $DS^2_z$. The earlier mentioned discussion remains valid in this case, as the large spin $S$ can be represented by an effective spin $1/2$ in the renormalized transverse field $\Delta \sim DS(H_z/DS)^{2S}$ ($\S3$). Assuming this renormalization, we try a classical interpretation of the phenomenon of resonant tunnelling of an ensemble of spins. Classically, this effect is simply associated with the precession of spins about $H_x = \Delta \mathbf{u}_x$, where $S_z$ oscillates between $\pm 1/2$, as shown earlier in the absence of a
Figure 5. Classical interpretation of the non-adiabatic motion of the Landau–Zener model when the field $H_z$ switches from a large negative to a large positive value. At large negative $H_z$, the spin rotates about this field ((a), bold orbit), but when $H_z$ starts to change rapidly from $<0$ to $>0$ there is a short period of time ($\tau_c = \Delta/c$) during which $|H_z| \sim 0 < \Delta$, i.e. during which the spin rotates about $H_x$. This spin motion $\Delta s$ proportional to $\Delta \theta \sim \gamma \Delta \tau_c \sim \gamma \Delta^2/c$ implies a change of $S_z$ by the same order (small arrow $\Delta s$ along the transverse rotation about $H_x$). At large $c$, this change is small and takes place, while $H_z$ reaches large positive values for which the constant of motion is now approximately $S_z - \gamma \Delta^2/c$, where $\gamma \Delta^2/c \ll 1$ ((b), bold orbit) (classical interpretation of non-adiabatic Landau–Zener transition).

Figure 6. Classical interpretation of the adiabatic Landau–Zener transition when the field $H_z$ sweeps slowly from a large negative to a large positive value. At any value of $H_z$ (large negative, nearly zero or large positive), the spin rotates at time $t$ about the local field $H_z(t) + H_x$ very rapidly, so that the constant of motion is $S_x + S_z(t)$ at each instant $t$. In this case of low $c$, the precession period is such as $\tau_{yz} = 1/\gamma H_z \ll \tau_c$, giving, for $|H_z| \sim \Delta$, $\gamma \Delta^2/c \gg 1$.
barrier ($H_x = H_x u_x$; figure 4). In the presence of a barrier, the picture is the same: the periodic back and forth reversal, from one well to the other, of the classical variable $S_z$ results from the $H_x = \Delta u_x$ precession; the tunnelling rate is just the inverse precession period $1/\tau_x = \Delta/\pi$ (figure 4, in which $H_x = H_x u_x$ is replaced by $H_x = \Delta u_x$). In the presence of a sweeping $H_z$, the two classical precessions compete and if, say, $\tau_c \sim \tau_x$ (or $\Delta^2/c \sim 1$) the number of back and forth tunnelling events (oscillations) becomes smaller because the sign of $H_z$ changes in a time of the order of the $H_x$ half precession period $\tau_x/2$. Hysteresis becomes apparent when the sign of $H_z$ changes more rapidly ($\tau_c < \tau_x$ or $\Delta^2/c < 1$). When the precession about $H_x$ dominates ($\tau_c \gg \tau_x$ or $\Delta^2/c \gg 1$), the number of oscillations is large and this is because many $H_x$ half precessions can take place while $H_z$ changes sign. The main differences between these classical and quantum descriptions of tunnelling

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are the following: (i) the variables are not the same (a classical probability in the former case and a quantum amplitude in the latter one) and (ii) the former is periodic and the latter stochastic, which is quite normal as the probabilistic nature of the former results from the stochastic character of the latter; note that decoherence is not yet introduced in this model.

This classical interpretation of quantum tunnelling should allow one to interpret the non-stochastic consequences of quantum tunnelling. As an example, the fast oscillations of period \( \tau_x \ll \tau_c \) can be considered as a classical representation of quantum fluctuations. They act similarly to quantum fluctuations, as shown in, for example, the quantum annealing process where the width of the hysteresis loop is reduced by ‘quantum fluctuations’ (here the \( H_x \) spin precession) if the sweeping field is slow enough; full reversibility is reached when the latter dominates over the \( H_z \) precession. In a second example, if we imagine that \( S \) increases, then \( \tau_x \) decreases exponentially and vanishes with \( \Delta \) so that the system becomes classical (with unique constant of precession motion \( S_z \)).

Such a derivation of the quantum/classical transition, obtained from a classical model, may seem surprising. In fact, we must admit that, if this description is fully classical, its starting point, the renormalization of the transverse field accounting for the presence of a barrier \( H_t \rightarrow DS(H_x/DS)^2 \), cannot be derived classically. If we start from the classical equations of motion, this can be done only after integration of the imaginary-time action along all trajectories [21]; in that case, the renormalization appears with the WKB (Wentzel–Kramers–Brillouin) tunnelling rate obtained, proportional to \( \exp[2S\ln(H_x/DS)] \).

5. From relaxation to coherence

In the above section dealing with large spins, transition probabilities associated with tunnelling spin reversal were small, leading to quantum relaxation. One of the remarkable consequences of the spin-bath theory [22] was to predict that quantum relaxation is proportional to \( (\Gamma t)^{1/2} \) and not to \( \exp(-\Gamma t) \), as is usual in the absence of distribution. After quantum relaxation was studied in detail [9], the natural question was to know whether quantum dynamics of large spins could be coherent enough to show quantum oscillations. In this section, we describe rapidly how this was made possible with SMMs [23,24] and RE ions [25,26] and then we describe Rabi oscillation measurements and discuss decoherence mechanisms.

The most obvious reason for decoherence being long-range dipolar interactions, we first separated the magnetic species by diluting the RE ions in a non-magnetic matrix (Er in substitution of Ca in a CaWO \(_4\) single crystal [25,26]) or by the introduction of a surfactant with SMMs (\( V_{15} \) separated by the cationic surfactant dimethyldioctadecylammonium bromide (DODA) [23,24]). Er:CaWO \(_4\) has in-plane magnetic anisotropy and \( V_{15}:\text{DODA} \) is almost isotropic. We did not take the Ho and Mn\(_{12}\)-ac systems of previous sections because of their large barriers, which prevent spin-mixing and lead to low tunnel-transition probabilities, unless a large transverse field is applied to strongly enhance the tunnelling gap. Observing coherent states in such systems would require high transverse (intra-well transitions) or longitudinal (single-well transitions) magnetic fields (more than a few tesla), not available in our set-up. However, such experiments were
Rabi oscillations were measured in Er:CaWO$_4$ and $V_{15}$ by applying a long microwave pulse at 9.7 MHz with a transverse AC field of amplitude $H_1 \sim 0.1$ mT (along $Ox$ or $Oy$) in the presence of a static magnetic field $H_0$ reaching a few hundred mT along $Oz$. The mean spin components ($\langle S_z(t) \rangle$ or $\langle S_x(t) \rangle$ or $\langle S_y(t) \rangle$) of the ensemble were measured using spin–echo sequences (application of two consecutive $\pi/2$, $\pi$ pulses) at the end of a long microwave pulse of duration $t$. Further averaging was performed by repeating this measurement a few thousand times.

6. Rabi oscillations and single-qubit decoherence in rare earth ions

Rabi oscillations, measured on a single crystal of Er:CaWO$_4$ with approximately $10^{-5}$ Er/Ca substitutions, first show that the oscillations of the Er isotopes with or without nuclear spins ($I = 0$ and $I = 7/2$) are nearly the same, demonstrating that Er nuclear spins do not give decoherence, which is quite normal because RE hyperfine interactions are large enough so that nuclear and electronic angular moments are locked. In particular, the tunnelling effects (or classical precessions) of previous sections are associated with the total angular momentum $I + J$, which is constant. The oscillations observed are fitted to the expression $\langle M_z(t) \rangle = \langle M_z(t = 0) \rangle \exp(-t/\tau_R) \sin \Omega_R t$, where $\Omega_R$ is the Rabi frequency and $\tau_R$ is the Rabi damping time, which we showed to be in general significantly smaller than the spin–spin coherence time $\tau_2$ (which is also measured by spin–echo measurements, but without applying the first long microwave pulse, etc.; we shall see that this pulse, necessary to induce Rabi oscillations, can be an important source of decoherence). Extending these measurements to other microwave powers (i.e. different AC field amplitudes $H_1$) shows that $\tau_R$ decreases when $H_1$ increases and the number of oscillations $N(c)$, for a given Er concentration $c$, remains nearly unchanged, whereas, in the absence of damping by microwaves, $N$ can be increased at will with $H_1$. This decrease in $\tau_R$ is always limited by the spin–spin coherence time $\tau_2$ and can be represented by [25]: $1/\tau_R(c) \sim \Omega_R/N(c) + 1/\tau_2(c)$ when $\tau_R(c) \ll \tau_2$ and by a plateau in which $\tau_R(c) \rightarrow \tau_2(c)$ when $\Omega_R$ (or $B_1$) $\rightarrow 0$. The coherence time $\tau_2$, proportional to $1/c$, is limited by relatively well-understood spin-diffusion mechanisms because of long-range dipolar interactions between RE moments, but the fast decrease of $1/\tau_R(c)$ with $\Omega_R$ results from an inhomogeneous nutation frequency (distributions of $\Omega_R$, different from one spin to another) leading to destructive superposition when averaging a large number measurements or when measuring an ensemble of qubits (destructive interferences).

Such single-qubit decoherence is easily reproduced by quantum simulations and analytically in the simplest cases in which only one mechanism is taken into account. Distributions of $\Omega_R$ come from distributions of the transverse Landé factor $g_{xy}$ (often called the $g$-factor) and/or of the microwave field amplitude $H_1$ (in the Hamiltonian they always appear with their product $g_{xy}H_1$). These distributions give an increase in the Rabi damping rate $1/\tau_R$ proportional to $\sigma \Omega_R$ (figure 9), where $\sigma$ is the distribution width of $g_{xy}$ (or $H_1$) [20,24–26]. Distributions of the longitudinal terms $g_z$ and/or $H_0$ give a similar but different
Figure 9. Damping rate $c_R = 1/\tau_R$ of Rabi oscillations obtained from quantum simulations [20] in the presence of weak disorder of the $g$-factors, inducing Lorentzian distributions of $g_x$, $g_y$ and $g_z$. The bullets, squares and triangles are for the distribution widths $\gamma_x = \gamma_y = 10^{-3}$ and $\gamma_z = 10^{-3}$, $2 \times 10^{-3}$ and $3 \times 10^{-3}$. The solid line is a fit of experimental data (not shown, see [25]). (Online version in colour.)

effect: as it affects $\tau_R$ in the limit $\Omega_R \rightarrow 0$ only, it contributes to the measured value of the coherence time $\tau_2$ (the measurement of which is not preceded by a long microwave pulse) despite the fact that such single-qubit decoherence $\tau_{R0}$ has nothing to do with the $\tau_2$. Note that this $\tau_{R0}$ can be smaller (even much smaller) or larger than $\tau_2$, suggesting that single-qubit decoherence must be eliminated before trying to interpret spin–spin decoherence times, or more exotic decoherence effects. In our system Er:CaWO$_4$, the distributions of $g_x$ and $g_y$ are due to very small unavoidable random crystal-field components, single crystals even of excellent quality, being deformed at relative scales less than $10^{-3}$, which is small but sufficient to give sizable decoherence. However, one cannot exclude other sources of disorder such as an extremely weak gradient or slow time fluctuations of $H_0$ or $H_1$. We also showed that single-qubit decoherence depends on the size of the measured crystal (at the sub-millimetre scale), and this is because of non-homogeneous $H_1$ in the cavity [20]. A non-perfect cavity may also induce fluctuations of $H_1$ with a mean standard deviation $\sigma \sim 1/\omega$, giving a small single-qubit decoherence $1/\tau_R \sim \sigma \Omega_R \sim \Omega_R/\omega \ll 1$ except for very large $H_1$.

This study suggests that single-qubit decoherence associated with quenched transverse or longitudinal disorder is general. In particular, it should not necessarily vanish with the number of qubits and could persist with single-qubit measurements if $H_0$ and $H_1$ are not stable in time, as measurements are repeated and averaged. This is, for example, the case with single-electron spins in a GaAs quantum dot where the Ga nuclear spin configurations evolving slowly in time are different at each measurement, contributing differently to $H_0$ (see §8). Note that, in this case, even if decoherence comes from the average of the single-qubit states measured, it originates from multiple pairwise dipolar couplings of the single qubit with nuclear spins, and, in that sense, it is not really single-qubit decoherence.
7. Rabi oscillations and multiple qubit decoherence in a single molecular magnet

Here, the Rabi oscillations were not measured in a single crystal as in the previous case, but on a frozen liquid with well-defined $V_{15}$ molecules (small molecule distortions) separated by the surfactant DODA. Being in an amorphous medium, the molecules, with axial symmetry, are with random relative orientations. Nevertheless, with $g$-factor distribution being smaller than $\Delta g = 0.02$, their isotropic character is not crippling (but still gives some single-qubit decoherence; see later text). Our main purpose in this section is to describe the existence of a new, microwave-field-dependent decoherence mechanism demonstrated by Shim et al. [24]. This is a multi-qubit decoherence mechanism induced by proton nuclear spins, and may be similar to decoherence by spin impurities in superconducting qubits (§10). Rabi oscillations of $V_{15}$–DODA were measured, between the $S = 3/2$ states of the first excited $V_{15}$ multiplet, at different concentrations and microwave powers [23,24]. The curves obtained show a fast decrease at short times owing to the dephasing of spin-packets with different Larmor frequencies and are followed by damped Rabi oscillations [23,24]. The fast initial decreasing is associated with $g_z$-factor distribution and slow-varying modifications of Zeeman splitting by nuclear spins (we shall see a similar effect in the decoherence of a single-electron spin in a dot; §8). The Rabi curves are fitted to the expression $j_0(Q_{R}t)e^{-t/\tau_{R}}$. The first term $j_0(z) = \int_{-\infty}^{\infty} j_0(z) dz$ comes from the distribution of Larmor frequencies within the electron paramagnetic resonance line and the second one $e^{-t/\tau_{R}}$ comes from the damping of oscillations related to decoherence. As shown in Bertaina et al. [23] and Shim et al. [24], oscillations almost disappear near
\( \Omega_R/2\pi = 10 \text{ MHz} \), proving the existence of a strong decoherence mechanism. In fact, the full evolution of the damping rate \( \tau_R^{-1} \) versus \( \Omega_R/2\pi \) in the range 20–60 MHz shows (figure 10) a broad decoherence peak in the range \( 8 \text{ MHz} < \Omega_R/2\pi < 15 \text{ MHz} \). Superimposed on this decoherence window, a much smaller contribution to \( \tau_R^{-1} \sim 0.02\Omega_R \) comes from the random distribution of the Landé factor as with the Er system (single-qubit decoherence; §6).

The interpretation of this decoherence window starts with the observation that it falls in the vicinity of the Larmor frequency of protons \( \omega_N \) in the field \( H_0 \). A model was constructed [24] showing that \( \tau_R \) is directly associated with electronic/nuclear spin cross-relaxation in the rotating reference frame (in which the main Larmor frequency \( \omega = \gamma H_0 \) does not play a role). This model leads to a Hamiltonian in the rotating frame with the form \( H = \Omega S_x + \omega_x(t)S_x + \omega_z(t)S_z \), where the spin operators are defined along appropriate quantization axes [24],

\[
\Omega = \sqrt{\varepsilon^2 + \Omega_R^2} \quad \text{is the nutation frequency of the central spin and} \quad \varepsilon = \omega_e - \omega \quad \text{is the shift of precession frequency} \quad \omega_e \quad \text{of the central spin from} \quad \omega.
\]

The term \( \Omega S_x \) represents the interaction of the central spin with the microwave field in the rotating reference frame. The time-dependent perturbation terms involve local random fields induced by the couplings between nuclei and central spins (\( I_j \) and \( S \), respectively; an integration over central spins with different \( \varepsilon \) is performed afterwards). These random fields //\( O_x \) (transverse fluctuations) and //\( O_z \) (longitudinal fluctuations) are coupled to the \( S_x \) and \( S_z \) operators:

- The first one \( \omega_x(t) \), which is aligned in the transverse direction of the central spin quantization axis, consists of \( S_x I^I_{+(-)} \) and \( S_x I^I_{+(-)} \) terms and results mainly in spin-dephasing because longitudinal or transverse nuclear-spin flips induce modifications of the main Rabi term \( \Omega S_x \). This mechanism of decoherence without dissipation is relevant for spins far from resonance only (with \( \varepsilon \sim \Omega_R \); [24]).

- The second one \( \omega_z(t) \) is longitudinal and consists of \( S_z I^I_{+(-)} \) and \( S_z I^I_{+(-)} \) terms. The first ones are not so important but the second ones give cross-relaxations in which transverse nuclear-spin flips induce longitudinal flips of central spins. This decoherence mechanism generates a polarization transfer and, in the presence of the field \( H_0//O_z \), a back and forth energy transfer between the electronic and the nuclear sub-systems. However, such a back-and-forth energy transfer between large ensembles of spins cannot be fully reversible because its period is much larger than the damping time \( \tau_R \). As a consequence, it is associated with energy dissipation. These resonant processes occur only when Larmor nuclear spins and Rabi frequencies coincide (\( \omega_j \sim \Omega \)).

These two types of decoherence mechanisms may also be described classically, on the basis of the physical picture involving two orthogonal precessions (§§3 and 4). Here, these precessions are with frequencies \( \omega_x(t) \) and \( \omega_z(t) \) randomly distributed in space and in time, the spatial distribution of super-hyperfine interactions also evolving in time. The classical picture shows an ensemble of central spins (SMMs), each one in precession about its local magnetic field (//\( O_x \) and \( O_z \)) fluctuating in space and time. The former \( \omega_x(t) \) modifies the

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Rabi precession frequency $\Omega$ and leads to simple ‘wave decoherence’ when the different precessions add (ensemble measurements). The latter $\omega_z(t)$ gives residual slow precessions in the rotating frame ($\omega_j - \omega$). As these distributions overlap, they feed each other (classical resonances with beating) and as the beating periods are, at least for a fraction of spins, much larger than $\tau_R$ the energy flow to the nuclear spin-bath cannot fully flow back to the electronic spin-bath.

This decoherence mechanism by nuclear spins in the presence of microwaves has two main channels: (i) a Markovian channel where electro-nuclear flips (or transverse $\omega_x(t)$ precession) involve $S_x$ only, i.e. induce simple decoherence, and (ii) a non-Markovian channel where decoherence, associated with electro-nuclear cross-relaxations (or longitudinal $\omega_z(t)$ precessions), involves $S_z$ relaxation with dissipation ($\tau_1$-like process). Simple local field estimations suggest that such cross-relaxation in a resonant microwave field observed in the SMM $V_{15}$ should take place in other proton-abundant SMMs such as Fe$_8$ and Mn$_{12}$ within a more or less broad frequency window [24]. Note that, in our $V_{15}$ system, most protons come from the surfactant DODA.

Regarding Fe$_8$, recent experiments showed that Rabi oscillations can be seen without dilution if the applied field is large enough [27]. In this case, decoherence is dominated by SMM magnetic dipole–dipole interactions (the effects of which decrease with the temperature/high-field ratio), whereas, in $V_{15}$, this type of decoherence was suppressed by dilution. The interesting point is that such decoherence is collective and brings in collective short-living spin-wave excitations in the paramagnetic state [27,28]. Recently, it was shown that Fe$_8$ orders magnetically below 0.6 K through long-range dipole–dipole interactions [29] implying spin-waves with much longer life-times and probable suppression of Rabi oscillations below this temperature. The microwave-field-dependent decoherence by protons in $V_{15}$ cannot be tested in these Fe$_8$ experiments because they were done outside the decoherence window, given in Shim et al. [24]. In the next section we give an example of decoherence of a single-electron spin in a quantum dot. Roughly speaking we shift from measurements with ensemble averages to measurements with time averages.

8. Rabi oscillations of single-electron spin in a quantum dot

These systems have been intensively studied during past years. The experiments are difficult and require stable longstanding groups, with continuous flows of researchers to learn how to make reproducible experiments based on highly specialized technology, and how to transmit the knowledge, as in any self-organized system. This is also the case with superconducting qubits.

The experiment described here (figure 11) consists in two independent but close GaAs quantum dots, each one associated with a (more or less charged) gate enabling them to shift their electronic states independently [30,31]. Owing to Coulomb interactions between electron charges, these states are well separated and an electron can tunnel from one dot (say the left one) to an empty state of the other dot (the right one) only if the two states are in coincidence; otherwise, nothing happens (Coulomb blockade). Electron spins also can prevent tunnelling (spin blockade) and this occurs when the tunnelling spin is parallel to the spin already present in the dot so that the Pauli principle is not satisfied. Both...
blockade effects are used to isolate an electron spin in a dot (say a spin-up in the left dot), then to manipulate it by applying microwaves, and finally to allow it to tunnel in the right dot, at a given time $t$. The tunnelling current will be finite only when the spin, in the course of its nutation, is down, allowing one to determine, after repeating the experiment (several thousand times), the probability associated with each electron-spin state at time $t$ in the left dot.

Coherence times were obtained by performing Ramsey measurements (application of two-phase coherent $\pi/2$ pulses of frequency $\omega + \Delta \omega$, where $\omega$ is the Larmor frequency, and separated by a delay $\Delta t$ during which spin precession about the $Oz$-axis is free). The switching probability displays decaying oscillations of frequency $\Delta \omega$, which corresponds to the ‘beating’ of the spin precession with the external microwave field. The envelope of the oscillations yields the coherence time $\tau_{\text{Ram}} \sim 30$ ns. The main source of decoherence comes from the numerous
nuclear spins present in natural Ga (dipolar interactions between spins in the two dots are negligible) as singlet and triplet states entangle with nuclear spin states, the separation $J_\text{st}$ of the former being of the order of the distribution width of hyperfine energy $\sigma$ of the latter. During the free spin motion in the left dot, nuclear spin configurations evolve in time owing to short-living electro-nuclear entanglements of the spin-bath [8] with, however, a slow evolution of the local field they create on the electron spin, between two different measurements ($\tau_\text{NS} \sim \text{ms}$ to $\text{s}$). With the read-out time $\tau_\text{RO} \sim 2\mu\text{s}$, the local field appears quasi-static within each measurement, but different from one measurement to the next one, leading to a distribution of Larmor frequencies. The integration time $\tau_\text{INT} \sim 2\text{s}$ being much larger than $\tau_\text{NS}$, the integrated signal drops rapidly, giving a coherence time of $\tau_\text{Ram} \sim 30\text{ns}$. Rabi oscillation measurements, using the spin–echo technique, show several oscillations with $\tau_\text{R} \sim 1\mu\text{s}$ actually much larger than the $30\text{ ns}$ of Ramsey interferences. This suppression of decoherence and the fact that the number of oscillations increases with the microwave field, instead of remaining constant as is the case with spin ensembles (single-qubit decoherence by microwaves; §§6 and 7), are at first sight surprising because the decoherence of single-electron spins, resulting from the addition of out-of-phase single-spin oscillations, should be of a single-qubit type. The explanation is very simple: with spin ensembles, we had two types of decoherence, a longitudinal one associated with $g_x$ and $H_1$ factors or $H_0$ distributions. Rabi damping comes from the latter (giving $1/\tau_\text{R} \sim \Omega_\text{R}/N$ proportional to $\Omega_\text{R}\sigma$ [20]), while here, with the quantum dot, we have only $H_0$ distributions (or possibly $g_z$-factor distributions if successive electrons occupy different defect sites) and such a longitudinal disorder affects the coherence times in the limit $\Omega_\text{R} \to 0$ only, giving $\tau_\text{R0} = h/\sigma$ (approx. $30\text{ ns}$) independent of $\Omega_\text{R}\sigma$ [20,25]. Such a longitudinal decoherence should be, at least, partially suppressed by spin–echo measurements where spins rotate clockwise and then anticlockwise in the $xOy$ plane under the effect of a $\pi/2$, $\pi$ pulse in the presence of (distributed) $H_0/Oz$ explaining the larger value of $\tau_\text{R} \sim 1\mu\text{s}$. Furthermore, in spin ensembles, disorder affects both $g_{x,y}$ and $g_z$ factors or $H_1//Ox$, whereas in single-electron dots, $g$-factors and the microwave field amplitude $H_1$ are in principle well defined, explaining why, in this case, the number of oscillations remains constant (does not decrease when $H_1$ increases); it is the static field $H_0//Oz$ only which is modified by quasi-static nuclear spins. Finally, let us note that quantum simulations with only $g_zH_0$ distribution [20] always show that, in the limit of infinite time, the Rabi oscillations vanish at a positive non-zero $\langle S_z(t \to \infty) \rangle$ value given by the width $\sigma$ of the distribution. This is also easy to understand; the differences between the microwave frequency and the local field act, in the local frame, as a static field distribution $//Oz$, and, even after Rabi oscillations (precession about $H_1$) are damped, the spins remain in precession only about the $z$-axis (these calculations are without the explicit dissipation term).

Before ending this part, we should mention that the transverse field components of nuclear spins resulting from short-living electro-nuclear entanglements also have an effect in the spin–echo measurements, leading to measurement errors. The small dynamical component of this field modifies the amplitude of the $ac$-field $H_1$, so that after a $\pi/2$ (or $\pi$) microwave pulse, the length of which is calibrated to the value of $H_1$ (it is proportional to $1/H_1$), the rotations will be different from $\pi/2$ (or $\pi$). In the earlier mentioned example, it was
shown that the spin rotates by approximately 130° instead of 180°, leading to a shortening of $\tau_R$. The best way to increase coherence in such single-electron dots is to ‘disentangle’ them from nuclear spins, e.g. by applying a large field. One might also decrease $\sigma$ or increase $\Delta$ (by at least one order of magnitude). This would require more refined nanostructure or replacement of Ga by another element. It is clear that it would be even better if this element was nuclear spin-free. Several candidates, on the basis of new forms of carbon or silicon, are considered.

In the next section, we describe quantum measurements and coherent oscillations in superconducting qubits (micrometre scale). Here the mesoscopic scale is much larger than with spin, superconductivity being a quantum phenomenon at the macroscopic scale.

9. Flux qubits, measurement of classical and quantum states

The first experimental coherent manipulation of superconducting qubits follows the line opened in Tsukuba, Japan [32], Delft, The Netherlands [33,34] and Saclay, France [35,36]. In Chiorescu et al. [33,34], taken as an example here, the qubit is formed of a small superconducting loop with three Josephson junctions (figure 12), fed by a current $I_b$ that modifies slightly the phase across them $(\gamma_1 + \gamma - \gamma_2 = 2\pi\phi/\Phi_0)$ and therefore the flux, given essentially by the applied longitudinal magnetic field $H_0$. The loop and the SQUID are connected and entangled, during the measurement only. Here, the Josephson energy $E_J = \Phi_0 I_c/2$ (energy barrier of the junction) is much larger than the Coulomb energy ($E_J \gg E_c$) and we have a phase qubit sensitive to phase fluctuations and much less to charge fluctuations. At the anti-crossing point (also called the ‘magic point’), we have the symmetric and antisymmetric quantum superposition of two phases with a time-dependent wave function $|\psi\rangle = a(t)|\uparrow\rangle + b(t)|\downarrow\rangle$, classically equivalent to two supercurrents rotating clockwise and anticlockwise. This scheme is identical to that described in §§3 and 4, where the dynamics of a large spin with a barrier is interpreted with the quantum or classical Landau–Zener model. In this section, we shall use the same approach.

The quantum state of a phase qubit is generally measured by the application of a sweeping magnetic flux starting at time $t$, from the anti-crossing point. The principle of the measurement is the following: after initialization, a microwave pulse of frequency $\hbar \omega = \Delta$ is applied (note that this $\Delta$ is different from the one of previous sections), inducing Rabi oscillations of the qubit phase. Their measurement, at time $t$, requires the determination of the probability associated with say $b(t)$. This is done in three stages, as follows. (i) The mixed state existing at the time $t$ is transported (with weak modifications) from the magic (avoided level crossing) point to the SQUID read-out point by application, at time $t$, of a longitudinal flux shift ($\gg \Delta$) created by a fast current $I_b$ ramp (figure 12). As in the Landau–Zener model, $|\psi\rangle(t) = a(t)|\uparrow\rangle + b(t)|\downarrow\rangle$ is projected on one state or the other with a probability depending upon the sweeping-rate/anti-crossing–splitting ratio (adiabatic or non adiabatic). (ii) The current associated with this state is maintained constant ($I_b$ plateau) for the time required for the electronic response. The $I_b(t)$ pulse height and length are set in order to optimize the distinction of the switching probability between the two states. The time $\delta t$ taken by the $I_b$ ramp to project $|\psi\rangle = a(t)|\uparrow\rangle + b(t)|\downarrow\rangle$ into $|\downarrow\rangle$ should be small.
Figure 12. (a) Scanning electron micrograph of a flux qubit (small loop with three Josephson junctions) and the attached SQUID (large loop with two big Josephson junctions of critical current) [33]. Evaporating Al from two different angles with an oxidation process between them gives the small overlapping regions (the Josephson junctions). Arrows indicate the two directions of the persistent current in the qubit. (b) Schematic of the on-chip circuit [33] where crosses represent the Josephson junctions. The SQUID is shunted by two capacitors to reduce the SQUID plasma frequency and is biased through a resistor to avoid parasitic resonances in the leads. Symmetry of the circuit is introduced to suppress excitation of the SQUID from the qubit-control pulses. The MW line provides microwave current bursts inducing oscillating magnetic fields in the qubit loop. The current line provides the measuring pulse $I_b$ and the voltage line allows the read-out of the switching pulse $V_{out}$. The $V_{out}$ signal is amplified, and a threshold discriminator (dashed line) detects the switching event at room temperature. (c) Landau–Zener representation showing the supercurrent rotation states (or equivalent spin states) which superpose at $I_b = 0$ (avoided the crossing point). While the qubit oscillates at the Rabi frequency (under microwaves offrequency $D/h$), the qubit state at time $t$ is measured through the application of a fast current ramp $I_b$ inducing a non-adiabatic transition preserving the current (spin) state at time $t$. This example of measurement based on the (Landau–Zener) wave function collapse is interpreted classically in figure 13. (Online version in colour.)

enough, and on the basis of the earlier mentioned classical argument ($\S$4) such as $\delta t \ll 2\pi/\Omega R c =$ time interval during which an instantaneous rotating frame can be defined (non-adiabatic motion). (iii) The last stage consists in the read-out that monitors whether the SQUID—the phase of which is now entangled with the qubit supercurrent phase—has switched or not to its normal state. More precisely, during the $I_b(t)$ plateau, the SQUID fed by a current $I_{sq}$ that is smaller than but almost equal to its critical current $I_c$ will or will not switch to its normal state, depending on the value of the current circulating in the loop at instant $t$ ($I(t)$ proportional to $|b(t)|^2$), and adding to $I_{sq}$. The SQUID switches only if $I(t) + I_{sq} \geq I_c$. After the measurement is repeated, the proportion of such events will give the instantaneous macroscopic current (or phase), proportional to $|b(t)|^2$. Note that this method, based on the quantum approach, implies a large number of measurements at each moment $t$, but in principle this is not necessary in the classical picture, as long as $I(t)$ could be determined with enough accuracy (not obvious). In this case, $I_{sq}$ should not reach a plateau, but should increase continuously until the SQUID switches to its normal state. Therefore, the value of $I_{sq}$ gives information about $I(t)$ proportional to $|b(t)|^2$. 

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In this experiment, the SQUID is a macroscopic object that can play the role of the Schrödinger cat: alive, if the SQUID is in its superconducting state; dead, if it is in its resistive state. The qubit (in a superposition state when \( I_b = 0 \)) and SQUID phases are entangled, the former playing the role of the ‘killing particle’ (emitted by the radioactive element). At time \( t \), such a killing particle will be more efficient if the supercurrent \( I(t) \) associated with its quantum state is larger, and will lead to the death of the cat if \( I(t) > I_c - I_{sq} \) (the death probability increases with the ‘bad health’ of the cat when \( I_c - I_{sq} \) is small, and with the ‘vigour of the killing particle’ when \( I(t) \) is large). In the opposite case, where \( I(t) < I_c - I_{sq} \), the SQUID (cat) remains superconducting (alive) after the measurement (\( I_b \) pulse and ramp). Even if the cat is entangled with its poison, from the moment a measurement is performed, it either remains alive—or it dies—depending on whether its health is good enough—or not—to withstand a more or less vigourous killing particle. The entanglement only imposes that the result of the experiment is not predictable, as is usual with a Landau–Zener experiment (here the probability \( P(t) \) is periodic owing to the presence of driving microwaves).

One can say that the macroscopic spin \( S_z(t) \), proportional to the instantaneous qubit current \( I(t) \) (through the qubit area), is obtained by the measurement of the flux going through the SQUID at time \( t \) after initialization. In this classical approach, the Rabi oscillations of the phase qubit can be represented with the two spin precessions in §4 and figure 4 as shown in figure 13. The first precession about the \( z \)-axis induces the rotation of the spin vector \( S_{xz} \) at the frequency \( \omega = \Delta \) in the \( xOy \) plane. The second one, about the transverse microwave field \( H_1//Ox \), induces the rotation of the spin vector \( S_{yz} \) (projection of the total spin in the \( yOz \) plan) at frequency \( H_1 \) proportional to \( \Omega_R \) (Rabi precession). This vector represents the quantum state at time \( t \) and its longitudinal component \( S_z(t) = S_{z0} \sin \Omega_R t \) is equal, within a constant, to the classical spin-down probability \( |b(t)|^2 \). This Rabi precession at \( H_z = 0 \) (magic point) is measured by the application, at time \( t \), of a fast sweeping field \( H_z(t) \) inducing a precession about the \( z \)-axis (\( I_b \) pulse). The interesting point is that this precession starts with the instantaneous \( S_z(t) \) resulting from the \( H_x \) precession (i.e. the mixed state at this moment \( t \)), and it retains it since \( S_z(t) \) becomes instantaneously the constant of motion of the precession about \( H_z \gg \Omega_R \) (condition \( \delta t \ll 2\pi/\Omega_R c \) given above). This is another example (figure 5 and §5I and 5) of projection of a mixed state to the quantization axis, seen from the classical side. As \( S_z(t) \) is a probability representing the qubit current \( I(t) \), it also represents the vigour of the killing particle, which oscillates between 0 and its maximum value, at the period \( \Omega_R \). The SQUID, entangled with the qubit, also oscillates at its own frequency, and the beating oscillations of the two (SQUID–cat and qubit–poison) may be considered as coupled oscillations of the cat’s health and the vigour of the poison (classical version of entanglement). The cat remains alive during these oscillations, unless the latter prevails over the former, meaning the death of the cat (\( S_z(t) \) is proportional to \( I(t) > I_c - I_{sq} \)). The irreversible transition of the SQUID–cat from its oscillating–living state to its resistive–dead state suppresses all SQUID–cat oscillations destroying the beating (entanglement) and the Rabi precession (superposition). Entangled and superposed states being dynamical—they are suppressed by an irreversible transition to a static state.

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Figure 13. Classical interpretation of the wave function collapse of a phase superconducting qubit (Figure 12). At $I_b = 0$, the qubit is within the avoided level crossing and oscillates at the Rabi frequency. In this two-precession model, this oscillation corresponds to the rotation of the spin vector $S_{xy}(t)$ (bold) about $H_x$ (vertical circle); the components $S_z$ and $S_y$ oscillate, showing in particular tunneling oscillations of $S_z$ when $I_b = 0$. If we want to measure the spin state $S_{xy}(t)$ at time $t$, we apply the current ramp $I_b$, which has to be fast enough so that modifications of the spin state during the measurement are negligible. In that case, the spin rotation $\Delta \theta \sim \gamma \Delta \tau_c \sim \gamma \Delta^2/c$ must be very small (as in Figure 5), implying that the change of $S_z$ remains negligible ($\Delta^2/c \ll 1$) while the constant of motion changes from $Ox$ (when $H_x \gg H_z$ proportional to $I_b \sim 0$) to $Oz$ (when $H_x \ll H_z$ proportional to large $I_b$). Before the application of $I_b$, $S_z$ is given by the vertical vector within the vertical circle, and after the application of $I_b$, it is given by the bold vertical vector along the $Oz$ axis; they differ by approximately $\gamma \Delta^2/c = 1$. This fast change of the constant of motion from $S_z$ to $S_z$ transmutes the instantaneous spin component $S_z(t)$ (resulting from the rotation of $S_{xy}(t)$ and characterizing the quantum mixed state) into the continuous constant of motion $S_z$ about $H_z$.

If, for example, the mixing was maximum with $S_z(t) = 0$, then the transmuted constant of motion would be $S_z = 0$ and the measuring SQUID would not be affected, remaining in its superconducting state (alive cat; §9). If, on the contrary, the state was not mixed at all with $S_z = 0$, the measuring SQUID would switch to its resistive state (dead cat; §9). In the measurement procedure, a threshold value $S_{zc}$ is defined so that the SQUID switches to its normal state only if $S_z > S_{zc}$, showing that the SQUID–cat will be alive only if the qubit spin state (poison strength) is larger than $S_{zc}$. Note that this method, based on the quantum approach, implies making a large number of measurements at each moment $t$, but this is not necessary in the classical picture.

10. Rabi oscillations and decoherence in superconducting qubits

Let us now describe some experiments, taking the example of Chiorescu et al. [33,34]. As the case may be, a spin–echo sequence (to eliminate decoherence by quenched disorder) is or is not used before probing the state. The Rabi oscillations measured without a spin–echo show a relatively large number of
oscillations $N$ and a Rabi time $\tau_R \sim 150\,\text{ns}$, limited by transverse fluctuations. This time is multiplied by a factor of 4 if the spin–echo technique is used, showing the presence of longitudinal fluctuations at scales smaller than 10 MHz (the measurement time scale). Such longitudinal fluctuations must be associated with the energy splitting (Zeeman like) resulting from fluctuations of the maximum qubit persistent current $I_p$ and/or from the phase shift across the junction $\gamma_q$ (in the spin case, equivalent to $g_z$, $H_0$). Contrary to what is seen with spins (§§6 and 7) the ratio $N/\Omega_R$ is constant, showing that transverse disorder, associated with slow fluctuations of the tunnel splitting $\Delta$, must be small (as with large spins with an effective $g_{xy}$ factor, a barrier is present). This flux qubit also gives a rather short coherence time and a Ramsey time $\tau_2 \sim 30\,\text{ns}$ and $\tau_{Rm} \approx 20\,\text{ns}$. The consequence of that, $\tau_2 < \tau_R < \tau_1$, was in fact also observed with other types of superconducting qubits [35,36]. Quantum simulations performed with $10^4$ non-interacting spins (then applicable to repeated measurements on a single qubit) [20] show the different effects of longitudinal and transverse fluctuations on the Rabi time. Above a threshold value of $h_p$ (or $\Omega_R$), $1/\tau_R$ increases linearly owing to a $g_{xy}$ disorder, whereas in the limit of $h_p = 0$ (no microwaves) $1/\tau_R(h_p = 0) = 2\pi \Gamma$ where $\Gamma$ is the width of the distribution function of $g_q$ (in units of $F_0$, the Larmor frequency [20]). In between the two, the value of $1/\tau_R$ results from the competition between both types of disorder and it can be much smaller than $1/\tau_R(h_p = 0)$ depending on the compared values of the $g_z$ and $g_{xy}$ distribution widths. The $1/\tau_R$ discontinuity appearing at $h_p = 0$ is related to the fact that in this limit the transverse disorder is no longer relevant [20]. This shows that the $\tau_R > \tau_2 = \tau_R(h_p = 0)$ observed may simply be explained in terms of slow single-qubit longitudinal fluctuations. We cannot exclude a pairwise origin for such fluctuations, as this is the case with the single-electron spin in a box, where decoherence is dominated by a quenched disorder of super-hyperfine interactions (§8). More generally, short qubit entanglements with distributions of spin or charge two-level systems, leading to a $1/f$ noise, may give single-qubit decoherence. Other fluctuations (such as the fluctuations in control parameters, bias currents and voltages, losses in the dissipative circuit element) may also play a role. However, a more probable source of decoherence may result from longitudinal flips of the $S_z$ spin component (associated with circulating supercurrents, in the classical picture) induced by cross-relaxations of the qubit with distributed two-level systems as with the single molecule $V_{15}$ (§7). In the present case, not only nuclear spins, but also impurity spin states localized at the superconductor–insulator interfaces or at the superconductor surface, or non-controlled charges are relevant. As with the spin $1/2$ $V_{15}$ SMM, the integration over a distribution of Larmor precessions should give access to a (non-exponential) time evolution of the qubit $\langle S_z(t) \rangle \sim \exp -P(\Omega t)$. In such a non-Markovian mechanism, $\langle S_z(t) \rangle$ damping is associated with a ‘one-way’ dissipation to the spin-bath. Because the distances between impurity spins (localized in normal regions, in the proximity of superconducting ones) and supercurrents, are widely distributed (above approximately 0.2 nm), their dipolar interactions could easily cover broad energy ranges up to the GHz. Furthermore, dipolar fields of approximately $10^{-3} - 10^{-4}$ T could induce enough Cooper pair breakings in superconducting Al to explain longitudinal decoherence in terms of supercurrent fluctuations, if the superconducting/normal interfaces approximately 1–2 nm, which is a good order of magnitude for oxide surface roughening. Lastly, Rabi
damping by microwaves can be induced through the entanglement of the qubit with collective Cooper pair excitations of the SQUID (plasmons) \[33,34\]. The energy spectrum of plasmons being similar to that of an harmonic oscillator, the number of entangled states is very large and strong decoherence occurs if their energy separation \(m\Delta + n\hbar\omega_p\) (\(m\) and \(n\) are integers) falls within the microwave frequency ranges \(\hbar(\omega \pm \delta\omega)\). Such decoherence resulting from the qubit/measuring tool coupling is accidental, but, if it occurs, it can be really drastic \[33,34\], as it leads to energy dissipation towards the plasmon bath.

Other types of superconducting qubits have been elaborated (e.g. the quantronium or the transmon). The former is a Cooper pair box qubit, where the single junction associated with the basic Cooper pair box is split into two nominally identical junctions in order to form a superconducting loop. The relative values of the Josephson energy \(E_J\) and the charge energy \(E_c\) can be varied through the charge number \(N_q\) to move between charge and flux qubits. In particular, at \(N_q = 1/2\), \(I_b = 0\) and \(\varphi = 0\), the ‘Zeeman energy’ of the order of \(E_J\), is stationary with respect to \(N_q\), \(I_b\) and \(\varphi\), making the system immune to first-order fluctuations of the control parameters (the second-order charge noise is dominant). Manipulation of the quantum state is thus performed at this optimal point. The latter is a flux qubit where the large ratio \(E_J \gg E_c\) results in an exponential suppression of the \(1/f\) charge noise. This has been observed experimentally and yields homogeneous broadening, negligible pure dephasing and long coherence times up to 3 \(\mu\)s, but never exceeding a few microseconds.

After important improvements, superconducting qubits still suffer from their macroscopic character implying the existence of several non-controlled decoherence mechanisms. The latter can be divided into two main categories: those associated with spins and those associated with charges, both intervening with more or less intensity depending on the ratio \(E_J/E_c\). Coherence times of superconducting qubits are always with \(\tau_R \gg \tau_2\). Very low \(\tau_2\) are actually due to the longitudinal fluctuations that are not suppressed by the spin–echo measurement method, above 10 MHz. The relatively large \(\tau_R \sim 150\) ns probably comes from the absence of transverse fluctuations generally responsible for Rabi damping. It is reasonable to admit that these fluctuations are nothing else but a longitudinal \(1/f\) noise resulting from broad distribution of simple two-level systems (spins or charges) in dipolar interactions with the qubit. The associated relaxation time \(\tau_1\), upper bound for the Rabi decay time, should not be associated with spin–phonon transitions as usual, but with dissipative phase or charge cross-relaxations implying \(S_z\) (depending on the ratio \(E_J/E_c\)). Finally, the fact that ‘the Rabi time \(\tau_R \sim 150\) ns is multiplied by a factor of 4 if the spin–echo technique is used’ shows the simultaneous presence of slow longitudinal fluctuations (scales smaller than 10 MHz). All that strongly suggests that spin and/or charge cross-relaxations induce a broad longitudinal \(1/f\) noise spectrum (well below and well above 10 MHz) with much weaker transverse noise.

11. Conclusion

In the first step, we recalled recent experiments on the dynamics of relatively large spin \((S \sim 10)\) with vanishingly small tunnelling gap \(\Delta\). Although they are often called ‘classical’, these spins show quantum tunnelling with quantum relaxation
and quantum coherence. The relaxation times, proportional to $\Delta^2$, are at our time scale and so are easy to measure (minutes, hours, days). Coherence times can reach the microsecond scale if the spins are sufficiently distant from each other. After having indicated why the classical and quantum dynamics of an effective spin $1/2$ are the same, we showed how the Landau–Zener model can be treated classically. This provides us with the possibility to (i) explain in a very simple way the probabilistic origin of hysteresis in these quantum systems and (ii) give a classical interpretation of different phenomena associated with ensembles of spins, including quantum tunnelling, relaxation and coherence.

In the second step, we analysed the main decoherence mechanisms of large-spin systems and extended the analysis to other types of qubits (single spin in a quantum dot and superconducting qubit). The decoherence of Rabi oscillations, by the microwaves which create them, is always observed with ensembles of spins and comes from quenched transverse disorder. The effect of quenched longitudinal disorder is different: it does not damp the Rabi oscillations but contributes to $\tau_{20}$, the coherence measured in the absence of microwaves and Rabi oscillations. Multi-qubit decoherence is observed in the molecule spin system $V_{15}$ and apparently in superconducting qubits also. In both cases, different types of short entanglements [8] between the qubit and environmental two-level systems are present, some with decoherence only and some with decoherence + dissipation (more relevant). In short, irreversibility can always be present in these quantum systems, either with long time scales (relaxation with hysteresis) or with much shorter time scales (coherence with dissipation).

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