Some peculiarities of turbulent mixing growth and perturbations at hydrodynamic instabilities

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The author presents a review of some experimental works devoted to the research of evolution of large-scale perturbations and turbulent mixing (TM) in liquid and gaseous media during the growth of hydrodynamic instabilities. In particular, it is shown that growth of perturbations and TM in gases is sensitive to the Mach number of shock wave; character of gas front penetration into liquid is not changed as the Reynolds number of flow increases from $5 \times 10^5$ to $10^7$; and change of the Atwood number sign from positive to negative causes stopping of gas front penetration into liquid, but mixing zone width is expanded under inertia.

1. Introduction

Hydrodynamic instabilities of Rayleigh–Taylor [1] and Richtmyer–Meshkov [2,3], as well as turbulent mixing (TM) owing to them, play an important role in many scientific areas: astrophysics, aero-hydrodynamics, gas dynamics, inertial thermonuclear fusion (ITF), etc. So, when compressing a thermonuclear target, growth of these instabilities causes violation of symmetry of the target shell convergence, mixing of its material and thermonuclear fuel. It strongly reduces the neutron release.

Investigations of growth of hydrodynamic instabilities and TM owing to them are both experimental and numerical. Model experiments allow investigation of physical regularities of these processes; calculations, which are tested in these experiments, allow taking account of their influence in real flows.
There are presently many experimental and numerical–theoretical works on this subject. But, unfortunately, there are still some unsolved problems in this field. In particular, unknown is influence of the Mach number of shock wave, the Reynolds number, substance compressibility, sign changing of the Atwood number, etc., on the mixing process.

The author of this work presents some results of experimental researches of these problems, which were obtained in RFNC-VNIIEF.

2. Influence of turbulent mixing on shock wave stability

When accelerating interface of gases by strong shock wave (SW; with Mach number \( M > 5 \)) or by a series of waves, which pass successively through gases, their compression can be several tens of times. It should cause approach of interface of gases to shock wavefront. The mixing zone, which takes place at the interface, can be ‘hampered’ by the shock wavefront. It can result in loss of its steadiness.

To investigate this situation, RFNC-VNIIEF arranged a laboratory technique, which allows performing researches in gases at SW Mach numbers of \( M \approx 10 \). Contrary to [4], it became possible to obtain these high Mach numbers in a shock tube with the use of detonation of gaseous explosive mixture (GEM) \((\text{C}_2\text{H}_2 + 2.5\text{O}_2)\) [5].

A scheme of the shock tube is presented in figure 1, variant Ia. The low-pressure chamber of the tube was filled with air; the measuring section was filled with gases with various compressibilities (\(\text{SF}_6\), \(\text{CO}_2\) or \(\text{He}\)) under atmospheric conditions.

Gases 1 and 2 were initially separated by polymer film with thickness of \( \approx 1 \mu\text{m} \). This film is decayed for several microseconds in experiments at \( M_1 > 5 \) (owing to high temperature and pressure). Density of its decay products is close to the density of compressed ‘heavy’ gas, and therefore they are not able to influence significantly the flow character.

The high-pressure chamber of the shock tube was filled with the mixture of acetylene and oxygen (GEM) under pressure \( P_0 \) (up to 9.5 atm). The flow was recorded by the schlieren method with use of a high-speed motion picture camera.

Figure 2 presents slit motion pictures of some experiments.
Figure 2. Slit motion pictures of experiments. SW, shock wave; C, curved front of SW; $X_1$, leading edge of TM zone; $X_2$, trailing edge of TM zone; IPI, initial position of interface; $A = (\rho_h - \rho_l)/(\rho_h + \rho_l)$, Atwood number; $\rho_h$, $\rho_l$, densities of ‘heavy’ and ‘light’ gases, respectively. (a) air–He, (b) air–CO$_2$, (c) air–SF$_6$, (d) air–SF$_6$, (e) air–SF$_6$ and (f) air–SF$_6$ (with thick separating film).

Based on them, it is possible to make the following conclusions.

— When SW falls down the interface at the same Mach numbers ($M_1$) in gas 2 (He, CO$_2$, SF$_6$), different Mach numbers ($M_2$) take place. In the experiments, which are presented in figure 2a–c, helium is $\approx$3 times compressed, CO$_2$ is $\approx$10 times and SF$_6$ is $\approx$17 times. As gas compressibility grows, the interface and, respectively, the leading edge of the mixing zone are approaching the shock wavefront.

— In experiments with SF$_6$ at $M_2 \geq 8.4$, no optical clearance is observed between SW and the leading edge of TM ($X_1$); the mixing front is ‘hampered’ by SW (figure 2d,e). Here, SF$_6$ is more than 18 times compressed.

In zone C in experiments with SF$_6$ (figure 2d,e), local curving of the SW front is observed. Numerical calculations [6] of the experiments with SF$_6$ revealed that pressure pulsations in the SW front correspond to pressure pulsations at the leading edge of the mixing zone at high $M_2$. Pressure pulsations take place in TM zone owing to the vortex character of mixing (pressure field is irregular in vortices). This irregularity of the pressure field causes distortion of the SW front.

— As the Mach number $M_2$ grows, the optically observed width of TM zone increases.
It should be noted that no SW distortion is observed when a thick separating film (50 µm) is placed at the initial position of the interface. The thick film is not destroyed in this case, and TM does not grow actually at the interface (figure 2f).

3. Influence of compressibility of ‘heavy’ gas on growth of two-dimensional local perturbations

So-called local perturbations (LPs; occupying a small area), which are joints of the thermonuclear target shells, scratches, perturbations owing to different dynamics of laser pulses, etc. can also take place at interfaces of substances. Growth of these perturbations can cause asymmetry of convergence of the target shells.

Figure 3 shows motion pictures of growth of two-dimensional LPs as a sharp bend (figure 1, variant Ib) of the interface (step) and as a triangular groove (figure 1, variant Ic) at the air–SF₆ interface at various Mach numbers of the SW (i.e. at various compressions of SF₆).

The motion pictures show that a vortex grows from the perturbation as a step up to the time of experiment completion at $M_2 \approx 1.6$. At $M_2 \approx 4.2$ and higher, the lifetime of the vortex becomes shorter; the vortex transforms to a jet at $M_2 \approx 9.5$ that can be caused by deceleration of the bottom
branch of the vortex by the SW, which is located close to the interface. Perturbation as a groove grows as two vortices at $M_2 \approx 1.7$. Vortices are partially diffused by intensively growing TM zone when $M_2$ increases up to $\approx 4.2$, and an explicit growth of this perturbation is not observed at $M_2 \approx 9$, only the TM zone intensively grows. Growth of this perturbation is also decelerated by a neighbouring SW.

These experiments show that the character of perturbation growth is changed as compression of ‘heavy’ gas is increased. It should be taken into account in the numerical simulation of flows.

4. Influence of Mach number of shock wave on growth of mixing zone

It is known that turbulence is decaying with time passing at the Richtmyer–Meshkov instability. The mixing zone $L$ can, in particular, be described by the $k$-$\varepsilon$ model [7]. This model is the most successful and generally acknowledged, though it has disadvantages as well.

In [8], the following relations of time of transition from the stage of instability growth to TM $t_0$ and dependences of the mixing zone width $L$ on time were obtained for this model:

$$t_0 = \frac{Re_e \nu}{4(a_a A_+ U_k)^2}$$ (4.1)

and

$$L = \frac{4}{k} \left[ 1 + \frac{a_a |A_+| U_k^2}{5.5 p} (t - t_0) \right]^p.$$ (4.2)

Here, $k = 2\pi/\lambda$ denotes wavenumber; $U_I$, interface velocity; $\lambda$, wavelength; $A_+$, $a_a = a_0(1 - U/D)$ denote Atwood number and amplitude of perturbations after interaction of SW and the interface; $D$, velocity of shock wave falling down the interface; $\nu$, average kinematic viscosity coefficient of gases; and $p$, power exponent of turbulence attenuation.

In the k-$\varepsilon$ model, calculated value of the power exponent is $p \approx 0.3$ [7,8]. In experiments [9], which imitate compression of targets of ITF, the value $p = 0.5 \pm 0.1$ was obtained at the SW velocity $D \approx 40 \text{ mm } \mu\text{s}^{-1}$. The reasons for $p$ difference in [7–9] have no substantial explanation yet. The high value of $p$ in [9] can be associated with the high Mach number of the SW.

To reveal how the Mach number of the SW influences TM growth, a series of experiments were performed in the range of SW Mach numbers in ‘heavy’ gas from $\approx 2$ to $\approx 8$ [5]. Experiments were performed with the use of the shock tube presented in figure 1. TM growth was investigated at the following interfaces: air–SF$_6$ (Xe, CO$_2$), He–SF$_6$ and Ar–Xe. Figure 4a,b presents video frames of some experiments. Based on them, it is possible to see that the mixing zone width is increased as the Mach number of the SW grows.

Figure 4c presents dependences of the mixing zone width on time, which were obtained in calculation by formula (4.2) and by experiment.

When calculating $L(t)$ by (4.2), the Reynolds number was $Re = 10^4$–$10^6$ and the kinematic viscosity coefficient was $\nu \approx (0.1 \times 10^{-5}) \text{ m}^2 \text{ s}^{-1}$.

The experimental results show that the power exponent $p$ grows from $\approx 0.3$ to $\approx 0.6$ in the $k$-$\varepsilon$ model as $M_2$ grows from $\approx 2$ to $\approx 8$. The $p$ growth can be explained by the fact that the extent of turbulence attenuation is less at high $M$ than that at low $M$.

To obtain the $p(M)$ dependence, researches of TM growth are required in a wider range of the Mach number of SW.

5. Turbulent mixing growth at gas–liquid interface

It is known that at the Rayleigh–Taylor instability, the depth of ‘heavy’ substance penetration into ‘light’ substance $h_{hl}$, ‘light’ substance penetration into ‘heavy’ substance $h_{lh}$ and the total width of
the mixing zone $H$ can be described by the following relations at the stage of TM in the self-similar regime:

$$h_{lh} = \alpha_1 Ag t^2; \quad h_{lh} = \alpha_2 Ag t^2; \quad H = \alpha Ag t^2,$$

(5.1)

where $A$, the Atwood number; $g$, acceleration; $t$, time; $\alpha_i$, constant, which characterizes rate of $h_i$ or $H$ growth. $\alpha_i$ is different in different sources.

So, there is $\alpha_2 \approx 0.07$ in the earlier experimental researches [10], there is $\alpha_2 \approx 0.05$ in the calculations [11]. In later calculations [12], $\alpha_2 \approx 0.04$ was obtained for ideal liquids, $\alpha_2 \approx 0.04$ was obtained in the experiments with intersoluble liquids [13], and $\alpha_2 \approx 0.03$ was obtained in the experiments at the gas–water interface at $g \approx 10^5 g_0$, gas temperature of $\approx 2000^\circ C$ and pressure of $\approx 400$ atm (i.e. at the overcritical state of the liquid surface layer) [14]. Reasons for the $\alpha_2$ scatter are not clear presently. It was supposed that $\alpha_2$ was reduced as the flow Reynolds number grew.

To investigate this problem, there is a large-scale facility, KU-210, which provides a flow with $Re \approx 10^7$ (figure 5).

In this facility, the liquid layer with weight of $\approx 3$ kg was accelerated by compressed gas. The acceleration value reached $g \approx 10^5 g_0$ (where $g_0 = 9.8 \text{ m s}^{-2}$), the layer displacement was 350 mm and the mixing zone width $H$ was 200 mm.

In the motion picture (figure 6a), the classic character of growth of the liquid–gas mixing zone is observed in these experiments, namely, gas penetrates into liquid in the form of bubbles increasing with time, and liquid penetrates into gas in the form of jets. Some bubbles ‘grow’ up to

![Figure 4](http://rsta.royalsocietypublishing.org/) Results of experiments with air–Xe interface (time is counted after SW arrival to IPI). SW, shock wave; TMZ, turbulent mixing zone; IPI, initial location of interface; $p$, power exponent by the $k$–$\varepsilon$ model; $\Rightarrow$, experiment. ($a,b$) Motion pictures and ($c$) $L(t)$ diagrams. (Online version in colour.)
Figure 5. Accelerating channel KU-210. 1, driver (steel); 2, manometer; 3, liquid; 4, sealing ring (rubber); 5, stud; 6, substrate (foam plastic); 7, measuring section (Plexiglas); 8, membrane (lavsan); 9, flange; 10, support; 11, electric contacts for triggering the recording equipment; 12, cotton wool; 13, concrete. (a) Facility scheme (sizes in millimetres); (b) photo of facility. (Online version in colour.)

30 mm in diameter at the end of recording. Also small bubbles occur in the TM zone. Secondary smaller bubbles are observed on large bubbles (figure 6c).

The dependences \( h_{lh}(2S) \) and \( Re(2S) \) are presented in figure 7. Slope ratio of the dependences \( h_{lh}(2S) \) to the abscissa axis \( \alpha_2 = \Delta h_{lh} / \Delta 2S \) characterizes dimensionless velocity (rate) of \( h_{lh} \) growth.

Two parts are observed for the \( h_{lh}(2S) \) dependences. The rate of mixing zone growth is high in the first part (0 mm < 2S ≤ 100 mm). It is lower in the second part (2S > 100 mm). In the first part, according to numerical analysis [15], turbulence is not in the self-similar regime. Therefore, we will not consider this part. The second part is longer; it can be considered as self-similar. In this part, the average value of \( \alpha_2 \) is 0.075 ± 0.005 (\( \alpha_1 \approx 0.26, \alpha \approx 0.33 \)). The asymmetry ratio of the zone is \( k = \alpha_1 / \alpha_2 \approx 3 \).

In the experiments, the Reynolds number was determined as \( Re = H \cdot \sqrt{A \cdot g \cdot H / v} \) (where \( A \approx 1 \) is the Atwood number, \( v \) is the kinematic viscosity coefficient of water). Based on them, it is possible to see that, if \( Re \leq 10^5 \) and \( \alpha_2 \approx 0.11 \) (in the first part), the value of \( \alpha_2 \) is decreased to 0.075 (in the second part) as \( Re \) grows up to \( \approx 5 \times 10^5 \). If \( Re \) grows further up to \( \approx 10^7 \), then the average value of \( \alpha_2 \) is not changed.
Figure 6. Video frames of experiments. TMZ, zone of turbulent mixing; S, layer displacement; R, reference mark; J, joints of measuring sections; X₁, substrate bottom; X₂, front of gas penetration into liquid; X₃, front of liquid penetration into gas. (a) Motion picture of experiment and (b) increased image of frames.

Figure 7. Results of experiment processing. (Online version in colour.)
Therefore, $\alpha_2 \approx 0.07$ at the gas–liquid interface (in the conditions of heterogeneous mixing) at $g \approx 10^3 g_0$ and the Reynolds number $5 \times 10^5 < Re \leq 10^7$; i.e. it coincides with results of the earlier experiments. Thus, the reduction of $\alpha_2$ to $(0.03–0.04)$ cannot be explained by the increase of only the Reynolds number. The value $\alpha_2 \approx 0.03$ can be a particular case of particular experiments and calculations. There is need for researches with search for conditions when $\alpha_2$ is decreased.

6. Growth of hemispherical local perturbation at gas–liquid interface

According to [16], hemispherical LP turns to the self-similar regime of growth with time at unstable interface. It is shown in [17,18] that, as the initial radius of this perturbation is growing (from $R \approx 0.5 \text{ mm}$ to $R \approx 3 \text{ mm}$) with presence of TM zone at the gas–liquid interface, rate of its penetration into liquid grows approximately 2 times, i.e. self-similarity is absent. Experiments [17]
and calculations [18] were performed at relatively low displacements of IF ($S \approx 40$ mm). In what way will this perturbation grow at large displacements of the interface? For clearing up this question, additional researches were performed at the large-scale facility KU-210 (figure 5).

Low-strength jelly of gelatine water solution was used as the liquid, the same as in [17]. Gelatin concentration in the solution was $\approx 2.2\%$. Strength of this jelly was less than $0.05$ kg cm$^{-2}$, and the coefficient of dynamic viscosity was less than $200$ cP. It behaves as a liquid under a pressure higher than $3$ atm.

At deliberately unstable (upper) surface of the jelly, LP was specified as a hemispherical groove with radius of 2, 3 or 6 mm. The jelly layer was accelerated by compressed air. The layer acceleration was $(6-9) \times 10^2 g_0$, the Reynolds number reached $4 \times 10^4$ and the layer displacement was $300$ mm. The flow recording was performed by high-speed video camera in transmitted light.

Figure 8a presents frames of the motion picture for LP growth with $R \approx 6$ mm; figure 8b presents the dependences $h_{lp}(2S)$ and $h_{lh}(2S)$. (Here, $h_{lh}$ is the penetration of the leading edge of the TM zone into the jelly; $\beta_i = \Delta h_i/\Delta S$ is the coefficient, which characterizes growth rate of $h_{lp}$ or $h_{lh}$.)

It is possible to see from the motion picture and the graphs that

Figure 9. Scheme of experimental device (sizes in millimetres).
Figure 10. (a–e) Results of experiments with SF$_6$–water interface. St, substrate; P, piston; TMZ, turbulent mixing zone; $S$, distance travelled by layer; LP, random local perturbation.

— LP, similar to that in [17], initially grows as a quasi-circular bubble, which takes the lead over the TM front. As time passes, the bubble mouth is partially closed, and a shaped jet is formed, which impacts against the bubble pole. The bubble becomes a mushroom shape (figure 8a).

— If the layer is displaced for $2S < 100$ mm, as the initial radius of LP grows, the velocity of its growth ($\beta_{LP}$), the same as in [17,18], increases (figure 8b).

— In the layer displacement range $100$ mm < $2S < 600$ mm, values of $\beta_{LP}$ become close for the investigated values of $R$, i.e. the perturbation growth process gets the self-similar regime. Time of perturbation getting the self-similar regime increases, as its initial size grows. Some differences of $\beta_{LP}$ at $100$ mm < $2S < 600$ mm can be caused by different overlap of LP channel by a neighbouring zone of mixing. The relation of the average values is
\[ n = \beta_{lp}/\beta_{lh} \approx 1.4, \text{ i.e. LP always takes the lead over the leading edge of the mixing zone.} \]

Growths of these perturbations in shells of ITF targets can cause them to break.

7. Influence of Atwood number sign changing on mixing growth

In the case of the Rayleigh–Taylor instability, the situation is possible when a ‘light’ substance is compressed up to the extent that its density becomes higher than the density of a ‘heavy’ substance, that is, the Atwood number changes its sign. What would be the behaviour of the TM zone in this case? This situation was simulated in [19]. It is presented in this section.

Scheme of the experimental device is presented in figure 9. The device is a gas gun. Water was poured into the substrate. Volume between the piston and the water layer was filled with SF₆ under pressure of \( \approx 9 \) atm, and the GEM chamber was filled with the mixture \( \text{C}_2\text{H}_2 + 2.5\text{O}_2 \) under pressure of \( \approx 4 \) atm. After detonation of GEM, the membrane was destroyed, the piston was accelerated and it compressed the gas below it. When the gas pressure reached a pressure higher than the critical pressure \( (P \approx (30–100) \text{ atm}) \), the substrate flange was cut, and the water layer together with the substrate were brought vertically down. The Rayleigh–Taylor instability took place at the interface of compressed gas and water. Pressure in SF₆ reached 500 atm and temperature reached 100°C. The Atwood number was determined in the experiments with the condition of adiabaticity of the gas compression process.

It is possible to see from the motion picture of the experiment (figure 10a) that fine fractions prevail in the TM zone structure. There are no evident jets and bubbles observed; a random LP does not grow at \( 1.7 \text{ mm} < S < 17 \text{ mm} \).

SF₆ penetration into water \( h_{lh} \) stops after some time. Then it becomes smaller slightly and it starts growing again (figure 10d,e). This behaviour of \( h_{lh} \) is caused by the fact that the Atwood number is reduced from \( \approx 1 \) to \( \approx -0.2 \) in the experiments, then it grows up to \( \approx 0.8 \) (figure 10c). SF₆ is compressed to the density that exceeds the water density. The situation occurs when acceleration is directed from a heavy substance to a light substance, where the interface of the substances should be stable. But the mixing zone \( H \), which was formed at \( A > 0 \), continues expanding slowly in SF₆ under its own inertia. Therefore, change of the Atwood number sign does not cause a stop to total mixing zone growth but just reduction of the liquid density in it.

8. Conclusion

— When the interface is accelerated by SW, high compression of ‘heavy’ gas causes:

(i) approach of the leading edge of the TM zone to the SW and interaction between vortices of the zone and the wavefront that results in its distortion;

(ii) change of character of LP growth.

— Attenuation of turbulence is approximately 2 times reduced as the Mach number of the SW grows from \( \approx 2 \) to \( \approx 8 \).

— In the case of the Rayleigh–Taylor instability growth at the gas–liquid interface:

(i) the self-similar constant, which characterizes the rate of penetration of the gas front into the liquid, is not changed and it is \( \alpha_2 = 0.075 \pm 0.005 \) as the Reynolds number grows from \( 5 \times 10^5 \) to \( 10^7 \);

(ii) a LP, which was specified at the unstable surface of the liquid layer as a hemispherical groove, grows in the shape of quasi-circular bubble, which takes the lead over the front of the TM zone. Time of when this perturbation gets the self-similar regime of growth is increased as its initial size grows; and

(iii) when the Atwood number sign changes from positive to negative, penetration of gas front into liquid stops, but the mixing zone, which was formed at the positive Atwood number, continues expanding into gas under its own inertia.
The work results enrich knowledge in the field of hydrodynamic instabilities and they can be used for testing numerical techniques.

References